Elektrodynamiikan sovellutuksia
Applications of electrodynamics

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Lectures: Monday 10-12 (D106), Friday 14-16 (D105); Jan 17 - Mar 4.

Exercises: Thursday 8-10 (D114); Jan 27 - Mar 3 (6 times).

Home exam: problems will be (tentatively) delivered in the last lecture. Dead-line for returning: to be agreed.

Approximate grade limits: see the study guide (theoretical physics). The weight of weekly exercises is 1/3 and of the home exam 2/3. Bonus: presentation of a solution in exercises on blackboard is worth an extra point (at most one per time). Getting 5/6 of the maximum points of weekly exercises yields the weight 1/3. Exceeding the 5/6 threshold gives extra points.

Assumed background knowledge: electrodynamics and related mathematical tools (mathematical methods of physics I-II = FYMM I-II or corresponding courses strongly recommended). The level of this course is suitable for students close to finishing M.Sc. degree and for Ph.D. students. Exercises comprise an important part of the course and require quite a lot of work.

This is not a complete text book, but some topics or additional examples may be discussed in more detail during the lectures. These lecture notes surely contain several misprints. The reader should correct them. Please tell the lecturer about them too.

Home page: http://www.ava.fmi.fi/˜viljanen/eds2005/; paper copy available in the Kumpula library.

Chapter 1

Foreword

1.1 What is this course?

What belongs to applications of ”classical” electrodynamics? Examples of Finnish research topics are the following presentations given in the latest national seminar of electromagnetics (Sähkömagnetiikka 2004, some titles in Finnish):

- 1D spherical elementary current systems and their use for determining ionospheric currents from satellite measurements
- Sähkökoneen kenttäyhtälöiden kytkeminen piiriyhtälöihin ja lohkokaaviomalleihin
- Electromagnetic description of complex, very complex, and metamaterials
- Optimization of an electrical machine using distributed computation
- Electromagnetic modelling of complex composite structures with the surface integral equation method
- Field test of a transient EM-system on sea ice
- Sähköstatiikan kuvateorioita epäisotrooppisille aineille
- Fast integral equation methods in computational electromagnetics
- Electrostatics in coupled microfluidic simulations
- Sähkömagneettisten kenttien ja aaltojen laskemisesta tietokoneella - missä tiede ja käytäntö menevät

- Modeling of macroscopic electrical parameters of materials with disordered microstructure
- Fast multipole boundary element method for acoustic scattering problems
- Numerical model for assessment of exposure to pulsed magnetic field from a mobile phone
- Modeling of electrostatically actuated structures
- Electromagnetic deformation method

Problems encountered in electrodynamics can be divided into direct and inverse problems. We define the basic form of a "direct problem" as follows: sources of the electromagnetic field are known as well as electromagnetic parameters of materials present in the problem. The task is to determine the electric and magnetic fields as functions of time and space. The solution is in principle straightforward. A typical inversion problem deals with a situation where the electromagnetic field is known in some region. The task is to determine its sources and/or electromagnetic parameters of materials present in the problem. This is often very difficult and may not have a stable nor a unique solution.

This course concentrates on the direct problem, because it is necessary to manage before going to inversion tasks. We will mostly consider cases where the space is divided into uniform regions, of which one contains the primary sources of the field, or where the primary field is given without explicitly describing its sources. The basic procedure is to solve the Maxwell equations in each region and then to apply boundary conditions. If the primary source is explicitly given it is typically a current, since especially in technical application, an electric current is easier to control than a charge distribution.

It is worth listing the arguments for deriving closed form solutions of the Maxwell equations in the computer age[1]:
• Closed form solutions provide insight in electromagnetic field behaviour and immediately reveal the influence of parameters to the field.
• Closed form solutions are forever. Once a problem is solved, it is solved. This is contrary to a new numerical solution technique which is most often open for improvements. Once an improvement is found, the old technique looses its usefulness.
• Closed form solutions are good benchmark cases for numerical simulation techniques or for the calibration of measurements.

[1] Olyslager, F. and I. Lindell, the Radio Science Bulletin No. 305, June 2003. Available on WWW: http://www.ursi.org/RSBissues/RSBjune03.pdf

• Constructing closed form solutions gives a certain satisfaction, which is perhaps the most important motivation.

Anyhow, we will also need some numerical skills in this course.

1.2 Notations

Bold symbols denote vector quantities. We use SI units throughout the text.

$\mathbf{A}$ vector potential
$\mathbf{B}$ magnetic field (magnetic induction)
$\mathbf{D}$ electric displacement
$\mathbf{e}_i$ unit vector in the Cartesian coordinate system
$\mathbf{E}$ electric field
$\mathbf{H}$ strength of the magnetic field ("$\mathbf{H}$ field")
$\mathbf{J}$ current density [A/m^2]
$\mathbf{K}$ surface current density [A/m]
$\mathbf{\Pi}$ Hertz vector

b wave number in Fourier space
c vacuum light speed (*exactly* 299792458 m/s)
k wave number
$K = \sqrt{b^2 - k^2}$ generalised wave number
T period

δ skin depth; delta function
θ polar angle in spherical coordinates; Heaviside step function
ϵ permittivity
ϵ_0 vacuum permittivity (*exactly* $1/(\mu_0 c^2)$)
μ permeability
μ_0 vacuum permeability (*exactly* $4\pi \cdot 10^{-7}$ TmA^{-1})
ρ charge density
σ conductivity [Ω^{-1}m^{-1}]
ω angular frequency ($= 2\pi/T$)

Conventions used in Fourier transforms: if $f(\mathbf{r})$ is a function of space then its transform is

$$F(\mathbf{b}) = \frac{1}{(2\pi)^{3/2}} \int f(\mathbf{r}) e^{-i\mathbf{b}\cdot\mathbf{r}} d^3\mathbf{r} \tag{1.1}$$

where the integration is performed over the whole space. The inverse transform is

$$f(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \int F(\mathbf{b}) e^{+i\mathbf{b}\cdot\mathbf{r}} d^3\mathbf{r} \tag{1.2}$$

Similarly, if $f(t)$ is a function of time then its transform is

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{+i\omega t} dt \tag{1.3}$$

and inversely

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{-i\omega t} d\omega \tag{1.4}$$

Note that there are several other conventions in literature concerning the 2π factor and the sign in the exponential term. Especially, many references given in these lectures call $e^{+i\omega t}$ as the harmonic time dependence, whereas here we use $e^{-i\omega t}$, following the practise of the course of electrodynamics at the University of Helsinki.

1.3 Orders of magnitudes

Even a theoretical physicist should know some numeric values or orders of magnitude of different quantities. Because several examples during this course deal with the geomagnetic field and its variations, the following list deals with them.

Earth's conductivity. $10^{-5}...10\ \Omega^{-1}\mathrm{m}^{-1}$ in the depth of 0...500 km, generally increasing with depth.

Seawater conductivity. $0.5...7\ \Omega^{-1}\mathrm{m}^{-1}$.

Earth's permeability: in a large scale equal to the vacuum permeability.

Earth's permittivity: 5...10 times the vacuum permittivity (at low frequencies).

Conductivity of the air: $10^{-13}\ \Omega^{-1}\mathrm{m}^{-1}$ close to the surface, $10^{-10}\ \Omega^{-1}\mathrm{m}^{-1}$ at the height of 40...50 km, then rapidly increasing to $10^{-4}\ \Omega^{-1}\mathrm{m}^{-1}$ at the height of 90 km.

Permeability and permittivity of the air: practically equal to the vacuum values.

Magnitude of the geomagnetic field at the surface: 24000 nT (magnetic equator) to 66000 nT (magnetic poles). The (geographic) north component of the field in Helsinki is about 15000 nT, east component 1700 nT, vertical component 50000 nT, declination 6.5 degrees to the east, and inclination 73 degrees (in 2004). Declination is presently increasing with the rate of one degree per 10 years.

Amplitudes of rapid geomagnetic variations in time scales from seconds to hours: 0.1 nT (micropulsations) to 1000 nT (magnetic storms). The largest time derivatives are of the order of 10 nT/s when calculated from 1 s recordings.

Horizontal electric field at the earth's surface due to geomagnetic variations: 0...10 V/km.

Ionospheric currents. Horizontal currents flow at the altitude of 100...120 km. The total current may be some millions of amperes during magnetic storms.

1.4 Abbreviations

EIT = exact image theory
FMI = Finnish Meteorological Institute
GIC = geomagnetically induced current
HUT = Helsinki University of Technology

1.5 References

The basic electrodynamics assumed as preknowledge is covered by the lectures by Koskinen and Viljanen (www.ava.fmi.fi/˜viljanen/ed2004/; paper copy available in the Kumpula library). Other useful elementary books are, for example,

• van Bladel, J., Electromagnetic Fields, Hemisphere Publishing Corporation, 1985.

• Cronström, C. ja P. Lipas, Johdatus sähködynamiikkaan ja suhteellisuusteoriaan, Limes ry., 2000.

• Jackson, J. D., Classical electrodynamics, 3rd edition, John Wiley & Sons, 1998.

• Jones, D.S., The theory of electromagnetism, Pergamon Press, 1964.

• Panofsky, W. and M. Phillips, Classical Electricity and Magnetism, Addison-Wesley, 1964.

• Stratton, J.A., Electromagnetic Theory, Harper & Row, 1941.

Some books use cgs units, so be careful.

We will also deal with some examples presented in scientific journals. References are given in the corresponding sections in these lecture notes.

Quite a remarkable amount of mathematical skills is also required. The tools covered by "Fysiikan matemaattiset menetelmät I-II" are expected. Recommendable text books about physical calculus are

• Arfken, G.B. and H.J. Weber: Mathematical Methods for Physicists, 4th ed., Academic Press 1995.

• Morse, P.M. and H. Feshbach, Methods of Theoretical Physics, Part I and II. McGraw-Hill Book Company, 1953.

Useful table books are

• Abramowitz, M. and I. Stegun, Handbook of Mathematical Functions, Dover, 1972 (or a newer edition).

• Gradshteyn, I. and I. Ryzhik, Tables of Integrals, Series and Products, Academic Press, 1980 (or a newer edition).

The use of computers in home exercises is naturally allowed. Some mathematical manipulations are surely easier with programs able to symbolic computation. However, human intelligence is still necessary. Some exercises require application of simple numerical methods or graphical illustrations.