Applications of elektrodynamics, spring 2005

Exercise 1 (Thursday 27.1., return answers until 16:00 on Monday 24.1.)

1. Assume that the electromagnetic field depends spatially only on the x and z coordinates, and that there are no external sources in the uniform medium. Show that the field decomposes then into two uncoupled triplets of components. In one of them the magnetic field has only the y component, and in the other one the electric field has only the y component.

2. Derive the wave equations of the vector and scalar potentials in a uniform conducting medium. Show that

$$\nabla \cdot \mathbf{A} + \mu\sigma\varphi + \mu\epsilon\frac{\partial\varphi}{\partial t} = 0$$

is a possible gauge condition.

3. The phase of the wave number

$$k = \sqrt{\omega^2\mu\epsilon + i\omega\mu\sigma}$$

with the time-dependence $e^{-i\omega t}$ is defined so that

$$Im(k) > 0,\, \omega > 0\,;\, Im(k) < 0,\, \omega < 0$$

Consider the phase convention in the case of a harmonic time-dependence $e^{+i\omega t}$ and draw illustrative figures in both cases.

4. a) Express the electric and magnetic fields with the electric Hertz vector in a uniform conducting medium.
b) Express the fields with the magnetic Hertz vector.

5. Derive the Poynting theorem for time-harmonic fields. Interpret physically its real and imaginary parts.

6. a) Show that the magnetic field can be expressed in the form $\mathbf{B} = \nabla f \times \nabla g$.
b) Show that f and g are constants at the field lines.
c) Let $f(x, y, z) = -B_0 L \ln\cosh(z/L)$ and $g(x, y, z) = y$. Calculate the magnetic field and the current density, and illustrate them schematically.

Vastaukset saa kirjoittaa suomeksi.