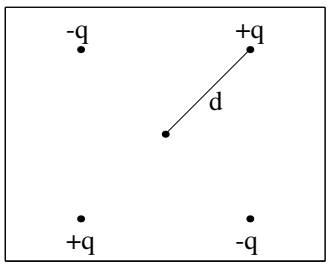
Applications of electrodynamics, spring 2005

Exercise 2 (Thursday 3.2., return answers until 16:00 on Monday 31.1.)

1. A radiating quadrupole is constructed by setting charges $\pm q$ at the corners of a square which circulates with the angular frequency ω around an axis crossing the centre of the square. Calculate the quadrupole moment, the angular distribution of the radiating power and the total power ($kr \gg 1$).



2. a) Show that the time-averaged radiation power per unit solid angle of a centre-fed linear antenna of length d is

$$\frac{dP}{d\Omega} = \frac{I^2}{8\pi^2\epsilon_0 c} \left| \frac{\cos(\frac{kd}{2}\cos\theta) - \cos(\frac{kd}{2})}{\sin\theta} \right|^2$$

   b) Show that at the long-wavelength limit ($kd \ll 1$) this reduces to the dipole result.
   c) Consider the special cases $kd = \pi$ and $kd = 2\pi$. Plot the angular distributions of the radiated power.

3. The angular distribution of the radiated power by a moving charged particle is

$$\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2\epsilon_0 c} \frac{\left|\mathbf{n} \times ((\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})\right|^2}{(1 - \mathbf{n}\cdot\boldsymbol{\beta})^5}$$

   Study a linear motion in which velocity and acceleration are parallel.
   a) Determine the direction of the maximum intensity.
   b) Sketch the radiation pattern for very slow and very high velocities.
   c) Calculate the total radiated power.

4. Study the same particle as in problem 3, but in the case of an instantaneous circular motion with velocity and acceleration perpendicular to each other.
   a) Express the intensity in the usual polar coordinates with velocity parallel to the z-axis and acceleration parallel to the x-axis.
   b) Calculate the total radiated power.