Applications of electrodynamics, spring 2005
Home exam

Return answers until 14:15 on Friday, March 4 (beginning of the last lecture). Include your student number too.
The four (4) best solved problems are taken into account.
You should solve the problems yourself, but the use of literature (books, journals, WWW, etc.) is allowed. Please mention the possible references.
You may use computer for manipulation of formulas, numerical calculations and plots.

Vastaukset saa kirjoittaa suomeksi.

1. An electric dipole at the xy-plane with a constant dipole moment magnitude $\mathbf{p}$ rotates with a constant angular frequency $\omega\mathbf{e}_z$.
(a) Determine the spherical components of the electromagnetic field far away from the dipole.
(b) Determine the angular distribution of the average radiated power.
(c) Calculate the total radiated power.

2. A plane wave ($\mathbf{k} = k\mathbf{e}_x$) arrives at an infinitely long perfectly conducting circular cylinder (radius a). The electric field of the wave is parallel to the axis of the cylinder, and has an amplitude E_0. Calculate the total surface current on the cylinder. The cylindrical wave expansion for the incoming electric field is useful:

$$e^{ikx} = e^{ikr\cos\phi} = \sum_{n=-\infty}^{\infty} i^n J_n(kr) e^{in\phi}$$

3. A resonant cavity consists of an empty space between two perfectly conducting, concentric spherical shells. The smaller one has an outer radius a and the larger one an inner radius b. Assume that there is no ϕ-dependence. Then in the TM mode the non-vanishing field components are B_ϕ, E_r, E_θ.
a) Continuing from lectures, show that the azimuthal magnetic field is

$$B_\phi(r,\theta) = (C_1 j_l(kr) + C_2 n_l(kr)) P_l^1(\cos\theta)$$

where $k = \omega/c$.
b) Using the boundary condition of E_θ, derive the transcendental equation for the characteristic frequencies of the cavity for arbitrary l.
c) For $l = 1$ show that the characteristic frequencies are given by

$$\frac{\tan kh}{kh} = \frac{k^2 + 1/(ab)}{k^2 + ab(k^2 - 1/a^2)(k^2 - 1/b^2)}$$

where $h = b - a$.
d) Solve the equation graphically to find the lowest frequency for $l = 1$. Use $a = 6370$ km and $b = 6470$ km.

4. Magnetotelluric sounding is based on the measured transverse horizontal components of the electric and magnetic fields at the earth's surface. From the measurements it is possible to calculate the surface impedance $Z(\omega)$ as a function of the angular frequency ω. Assuming a layered structure of the earth and plane wave fields, it is meaningful to define the apparent resistivity as

$$\rho_{app}(\omega) = \frac{|Z(\omega)|^2}{\omega \mu_0}$$

a) Justify the formula of the apparent resistivity in a simple way.
b) What (qualitative) conclusions of the earth's conductivity can you draw from Fig. 1?

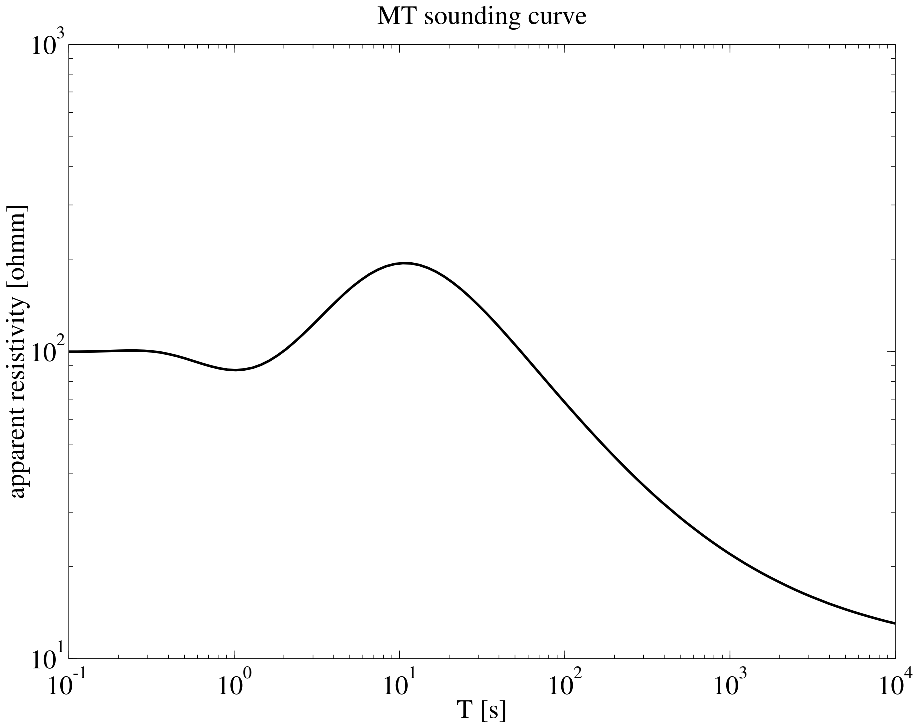

Figure 1: Apparent resistivity as a function of the period $T = 2\pi/\omega$.

5. Figure 2 shows the geomagnetically induced current along the Finnish natural gas pipeline at Mäntsälä, and the time derivative of the northward component of the magnetic field (dX/dt) at Nurmijärvi 30 km from Mäntsälä. From theoretical modelling we know that GIC at Mäntsälä is mostly produced by the eastward electric field. Assuming a plane wave magnetic field incident in a uniform earth, we can estimate GIC by

$$GIC(t) = -C \int_{-\infty}^{t} \frac{g(u)}{\sqrt{t-u}}\, du$$

where $C > 0$ is constant and $g = dX/dt$. (This formula also shows why it is appropriate to compare GIC with $-dX/dt$.)
a) There is quite a symmetric double peak in $-dX/dt$ around 05:56 (from about -4 nT/s to +5 nT/s). At the first glance, we might assume a similar behaviour for GIC with the maximum just after 05:56 being larger than the preceding minimum. However, this is clearly not the case. Explain this with the plane wave model by selecting a simple function to approximate dX/dt around 05:56.
b) After 05:57, the average GIC is clearly positive, although the average $-dX/dt$ is about zero. Can you explain this?

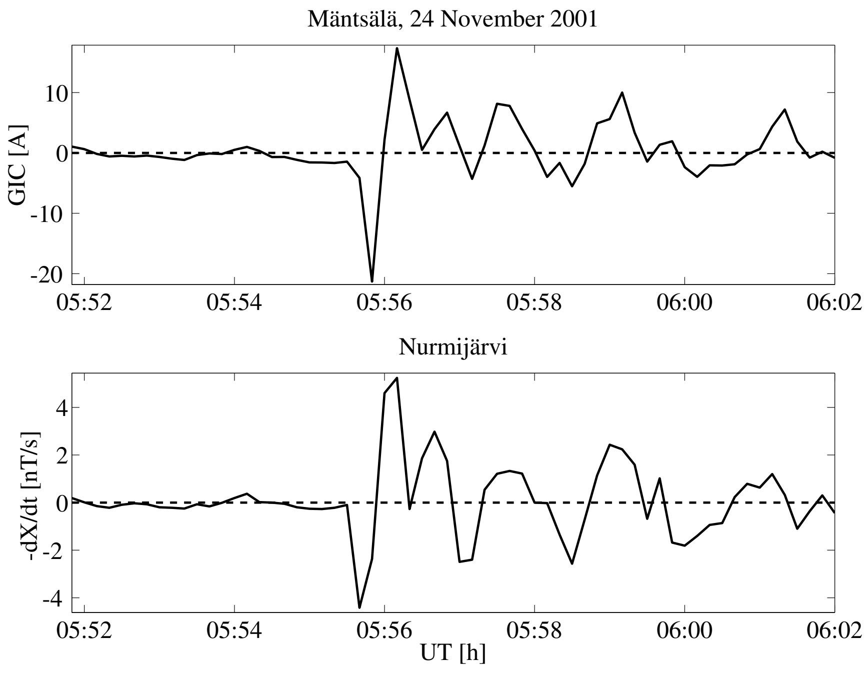

Figure 2: GIC at Mäntsälä and $-dX/dt$ at Nurmijärvi.