AGU JGR galley style, v3.0, 29 Oct 93

Perpendicular particle simulation of the E region Farley-Buneman instability

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Abstract. We study the stabilization and nonlinear quasi–steady state of the E region Farley–Buneman waves. We limit ourselves to the two–dimensional, fully perpendicular case. We use a two–dimensional explicit particle simulation code with neutral collisions. Since both perpendicular coordinates are included, we can study the dependence of temporal power spectra and backscattered power on flow angle. Unlike fluid simulation codes in general, the present method is not limited to small ambient electric field values. It is found that the phase velocity roughly obeys the linear theory: no saturation to the ion acoustic speed is seen. However, the spatial power spectrum is asymmetric as a function of the flow angle if the electric field is above the Farley–Buneman threshold. This nonzero flow angle propagation is the most important new result. The most intense waves propagate at the $\mathbf{k} \cdot \mathbf{E} < 0$ edge of the linearly unstable sector, with phase velocity close to the ion acoustic speed. No evidence of anomalous perpendicular wave heating is found.

Introduction

The simplest theoretical framework for the E region plasma irregularities in the equatorial and auroral electrojets is the two–dimensional linear two–fluid model [*Sudan et al.,* 1973; *Fejer et al.,* 1984]. The basic model can be extended by including three categories of effects, namely kinetic, three–dimensional, and nonlinear. If one includes them all, one has in principle constructed a fully realistic model. In this context we regard the nonlinear effects as being taken into account only if a numerical simulation technique is being used. In Table 1 we list some relevant works starting from first principles. In addition, there have been several attempts to understand the nonlinear aspects by theoretical means [*Sudan,* 1983a,b; *St-Maurice and Laher,* 1985; *St-Maurice,* 1987; *Robinson,* 1986; *Robinson and Honary,* 1990]. We have not included these theoretical papers in Table 1, but they can be compared with simulation results [*Keskinen et al.,* 1979; *McDonald et al.,* 1974; *Ferch and Sudan,* 1977; *Machida and Goertz,* 1988a; *Janhunen,* 1992]. Henceforth, we shall abbreviate *Machida and Goertz* [1988a] as MG.

Table 1

At present we are not able to properly study fully realistic models which would imply three–dimensional

Paper number 94JA00206.
0148-0227/94/94JA-00206$05.00

particle simulations. Therefore we have to limit ourselves to frameworks where one of the three aspects (kinetic, three–dimensional, nonlinear) is disregarded.

We use an orthogonal coordinate system with z parallel to the geomagnetic field, x parallel to the electric field, and y parallel to the $\mathbf{E}\times\mathbf{B}$ electron drift. A two–dimensional simulation has been carried out previously in the yz plane [MG; K. Schlegel and H. Thiemann, Particle–in–cell simulations of the modified two stream instability, accepted to *Annales Geophysicae,* 1994]. Please notice that our coordinate labeling differs from MG. The yz simulation plane contains aspect angle dependency but does not include flow angle effects. The present paper deals with xy simulation. That is, we are interested in flow angle effects and are ignoring parallel dynamics. Both types of two–dimensional simulation are subsets of an ideal three–dimensional model and are in a sense complementary to one another. The present paper tries to fill a gap in this sense. We also emphasize that the possibility to use xy simulation is rather unique to the E region; for example, in fully ionized magnetized plasma, such a geometry would not be nearly as feasible as yz simulation.

The organization of the paper is as follows. We first describe the simulation method, devoting a rather extensive discussion to neutral collisions (the appendix). After a brief review of linear theory we go on to analyze the numerical results and to discuss energetics, turbulence levels, phase velocities, and spatial power spectra. The discussion section contains an interpretation of the phase velocities and spatial power spectra in terms of linear theory. A later paper is devoted to a more thorough discussion of the geophysical implications of these results.

Model

Plasma

We employ a two–dimensional magnetized perpendicular electrostatic explicit particle code with bilinear weighting (cloud in cell (CIC)) in charge assignment and force calculation [*Birdsall and Langdon,* 1985]. The boundary conditions are purely periodic. The parameters are summarized in Table 2.

Table 2

We have N_e electrons and $N_i = N_e$ ions, with positions $\mathbf{x}_e$, $\mathbf{x}_i$ and velocities $\mathbf{v}_e$, $\mathbf{v}_i$. The particles obey the equations of motion

$$\frac{d\mathbf{x}_{\alpha p}}{dt} = \mathbf{v}_{\alpha p} \tag{1}$$

$$\frac{d\mathbf{v}_{\alpha p}}{dt} = \frac{q_\alpha}{m_\alpha}\left[\mathbf{E}\left(\mathbf{x}_{\alpha p}\right) + \mathbf{v}_{\alpha p}\times\mathbf{B}\right] \tag{2}$$

where $\alpha = e, i$ and $1 \leq p \leq N_e$. The equations are discretized in time by the second-order-accurate leapfrog method.

An externally applied electric field is used to drive the plasma unstable. In addition, a force term simulating the zeroth–order electron density gradient can be added; it is effectively like an electric field, but it is not the same for the ions and for the electrons. Ideally, the zeroth-order gradient should be simulated by arranging and maintaining a small gradient in the actual density of the particle clouds, but this would probably require a too large number of particles to be feasible.

Neutral Collisions

Neutral collisions couple to the plasma dynamics at E region altitudes. The most important processes are elastic ion–neutral and both elastic and inelastic electron–neutral collisions. The neutral background can be considered as a fixed thermal population for the timescales (fraction of a second) under study; a significant heating or acceleration of the neutral fluid takes at least tens of minutes. The computations are carried out in the rest frame of the neutrals. Discussion of various collision schemes and their implementation can be found in the appendix. For the ions the inelastic collisions need not be considered in a qualitative computation since the ions and neutrals have comparable masses.

However, inelastic electron–neutral collisions must be retained, for they provide the dominant electron cooling mechanism. There is some uncertainty and even debate on the real inelastic cooling rate [*D'Angelo and Merlino,* 1987]. For our simulations we rather arbitrarily choose $l_F^{\mathrm{inel}} = 7\ l_F$, where l_F is the elastic mean free path. Each inelastic collision is assumed to "reset" the electron so that the after–collision mean drift is zero and the temperature equals T_n. The resulting electron temperatures depend strongly on l_F^{inel}, but in this paper it suffices to use a value that produces roughly the observed electron heating [*Williams et al.,* 1992] as a function of the electric field. In reality, both l_F^{inel} and l_F depend strongly on altitude; we are not trying to model this quantitatively here. The choice of l_F^{inel} also depends on the mass ratio (*K. Schlegel,* private communication, 1993).

Dimensionless Variables

In electrostatic particle simulation the natural length and time units are the electron Debye length $\lambda_{De}^{(0)}$ and the inverse electron plasma frequency ω_{pe}^{-1}, respectively. This $\lambda_{De}^{(0)}$ is computed with the electron temperature T_e equal to the neutral temperature T_n, so after electron heating the actual λ_{De} will be somewhat larger. We use a spatial grid spacing equal to the electron Debye length $\lambda_{De}^{(0)}$ in both x and y, this is different from MG, who used a larger grid spacing especially in the parallel direction. Otherwise, our parameters are equal to those employed by MG. We summarize them in Table 2. The last two parameters, the neutral thermal velocity u_n and the Debye length $\lambda_{De}^{(0)}$, define the scaling from dimensionless code parameters to physical ones, and they can be changed at will without rerunning the simulations. All plots appearing in this paper use these particular scaling values. For example, all phase velocities scale linearly with u_n, the neutral thermal velocity.

Linear Theory

The fluid linear theory has been extensively reviewed elsewhere [*Farley,* 1985; *Fejer et al.,* 1984; *Sudan et al.,* 1973], and so we just quote some results we will be using later. One version of the dispersion relation reads, with no gradient drift term,

$$\omega - \mathbf{k} \cdot \mathbf{V}_d = \frac{\psi}{\nu_i} \left[\omega(i\omega - \nu_i) - ik^2 C_s^2 \right], \qquad (3)$$

where

$$\psi = \frac{\nu_e \nu_i}{\Omega_e \Omega_i} \left(1 + \frac{\Omega_e^2}{\nu_e^2} \sin^2 \alpha \right). \qquad (4)$$

Here θ is the flow angle, α is the aspect angle, and other symbols have their usual meaning. Assuming $|\mathrm{Im}\ \omega| \ll |\mathrm{Re}\ \omega|$, we readily obtain the solution

$$\mathrm{Re}\ \omega = \frac{kV_d \cos\theta}{1+\psi} \qquad (5)$$

$$\gamma = \mathrm{Im}\ \omega = \frac{\psi}{1+\psi} \frac{1}{\nu_i} \left[\left(\frac{kV_d \cos\theta}{1+\psi} \right)^2 - k^2 C_s^2 \right]. \qquad (6)$$

Kinetic theory [*Ossakow et al.,* 1975; *Schlegel,* 1983] should be used for small wavelengths. With the parameter values we are using in the simulations, however, the difference between fluid and kinetic phase velocities is smaller than the inaccuracies of our simulations.

The usual kinetic theory predicts nearly zero or even negative growth rates for the purely perpendicular waves, although waves with slightly nonzero aspect angle have $\gamma > 0$. Our simulations however produce growing waves even in the perpendicular plane. There are many possible reasons why the simulations are in this sense not in agreement with kinetic theory. The simulation has both elastic and inelastic collisions, whereas the theory only has BGK collision term. It may also be that even the thermal fluctuation level in the simulation is high enough to slightly break the linearity assumption.

Results

Runs

We made a series of six runs for different electric field values. The dimensionless electric field ranged from 3 down to 0.5 with 0.5 step. Unit field corresponds to approximately 20 mV/m in the auroral zone. The first run ($E = 3$) was initialized with a random state that was Maxwellian in velocity space and uniform plus a very small sinusoidal perturbation in position space. The following runs were initialized with the final state of the preceding run, for example, the $E = 2$ run was initialized with the state that the $E = 2.5$ run had reached. This set of basic runs contained no zeroth order electron density gradient. Each run took about 8 hours of Cray X–MP single–processor CPU time.

Our stepwise lowering of the electric field is not to model some physical process, but we use such a scheme simply because the final state reached by a run with nearest electric field is a priori the best approximation for the initial state. The initialization with a sinusoidal perturbation is not necessary, we could have used a completely random initial state. The initial wave has mode number $(k_x, k_y) = (0, 2)$, and it damps away during the first quarter of the run. It should be noted that the initial conditions have much less importance in the present simulation than in fully ionized plasmas, with which electrostatic particle simulations are most often used. This is because the neutral collisions efficiently randomize the system all the time.

The length of each run was 5000 ω_{pe}^{-1}, or 25 000 time steps. In order to make physical conclusions it is of course important to know which phenomena have their characteristic timescales shorter than the length of the run. In our case it turns out that the thermal relaxation times are well shorter than the run length. All thermal–related quantities, including neutral heating rates, can thus be measured in their saturated states, and well–defined values as a function of the electric field can be achieved. On the other hand, the fluctuations in the longest wavelength spatial Fourier mode amplitudes occur at timescales not much smaller than the length of the run. This makes it more difficult to estimate the turbulence level.

In Figure 1 we plot the plasma temperatures and drifts as a function of time for one particular electric field ($E = 2.5$). The ion temperatures and drifts are throughout the runs well defined, and their dependence on the electric field is only weak. The electron temperature drops from the value 3.7 (the $E = 3$ run final state) down to 2.9. In the average drifts the changes are smaller and the relaxation times are shorter. **Figure 1**

The ion temperature and ion drifts in Figure 1 show a small clusterlike fluctuation, which deserves a little explanation, although it is physically insignificant. It is a numerical effect related to the computation of a fresh set of Gaussian random numbers at every 2000th time step. Namely, we do not generate new normal deviates each time we need a random neutral velocity, but we use random indexing to a table of precomputed Gaussian deviates. The size of this table, 10 000, was apparently "small" enough to make the effective neutral temperature and drift a bit different from the continuum value. The extra fluctuation was no longer observable when the size of the table was increased to 100 000.

The plasma temperatures are shown as a function of the driving electric field in Figure 2. The data has been gathered by averaging from the latter half of each run. The electric field divided by the threshold (which is proportional to the ion acoustic velocity) is also shown, we see that the Farley–Buneman threshold is exceeded at about $E = 1.3$.

Figure 2

Energetics

During the run we kept track of the energy exchanged in all charged–neutral collisions by computing the difference between the scattered and incident kinetic energies. The most important energy exchange mechanisms are electron–neutral inelastic and ion–neutral elastic collisions. The plasma component gains energy from the external electric field E_0 and sooner or later passes it to the neutrals. The associated T_n increase can be neglected, as well as charged–charged collisions, since the neutral number density is much larger than the electron density.

The energy stored in the plasma (kinetic plus potential) is quite small in the sense that the stored energy could maintain the neutral heating only a few ion collision times, or about one ion Larmor period, if the external field was switched off. The plasma potential energy is even smaller: it could maintain the neutral heating for only a few electron cyclotron periods.

The energy equation is

$$< \mathbf{j} \cdot \mathbf{E} >= dW_k/dt, \qquad (7)$$

where W_k is the total (plasma plus neutral) kinetic energy and the angular brackets denote spatial averaging: $< \ldots >= (1/V) \int dV(\ldots)$. Here $\mathbf{j}$ and $\mathbf{E}$ are the total microscopic current density and electric field, respectively. Let us divide these vector fields to average and perturbation parts: $\mathbf{j} = \mathbf{j}_0 + \mathbf{j}_1$ with $\mathbf{j}_0 =< \mathbf{j} >$, $< \mathbf{j}_1 >= 0$, and similarly for $\mathbf{E}$. We obtain

$$< \mathbf{j} \cdot \mathbf{E} >= \mathbf{j}_0 \cdot \mathbf{E}_0 + < \mathbf{j}_1 \cdot \mathbf{E}_1 > . \qquad (8)$$

Should we call $< \mathbf{j} \cdot \mathbf{E} >$ or $\mathbf{j}_0 \cdot \mathbf{E}_0$ the energy input to the system? The answer depends on how the external field is maintained, but we choose $\mathbf{j}_0 \cdot \mathbf{E}_0$ because it is the work done by the external electric field only: $< \mathbf{j} \cdot \mathbf{E}_0 > = \mathbf{j}_0 \cdot \mathbf{E}_0$. The distinction between $< \mathbf{j} \cdot \mathbf{E} >$ and $\mathbf{j}_0 \cdot \mathbf{E}_0$ is important if we consider the Poynting theorem (in electrostatic approximation):

$$\frac{d}{dt}\frac{1}{2}\epsilon_0 < \mathbf{E}^2 > + \frac{1}{\mu_0} < \nabla \cdot (\mathbf{E} \times \mathbf{B}) > = - < \mathbf{j} \cdot \mathbf{E} > . \quad (9)$$

The Poynting flux term, which is essentially the energy input, cannot be computed unless the $\mathbf{E}_0$ maintenance mechanism is known in more detail. Here we *define* it to be equal to the work done by $\mathbf{E}_0$:

$$-\frac{1}{\mu_0} < \nabla \cdot (\mathbf{E} \times \mathbf{B}) > = \mathbf{j}_0 \cdot \mathbf{E}_0. \quad (10)$$

Then we may write the energy equation in the form

$$\mathbf{j}_0 \cdot \mathbf{E}_0 = \frac{d}{dt}\frac{1}{2}\epsilon_0 < \mathbf{E}_1^2 > + \frac{dW_k}{dt}, \quad (11)$$

which has a simple interpretation: the work done by the external field increases the mechanical energy of the neutral component and the plasma, while the irregularity electric field acts as a small energy accumulator or "buffer." Separating the kinetic energy as $W_k = W_{ke} + W_{ki} + W_{kn}$, we obtain for time averages

$$< \mathbf{j}_0 \cdot \mathbf{E}_0 >_t \approx < \frac{dW_{kn}}{dt} >_t \quad (12)$$

because the plasma terms have negligible time averages. Here $< \ldots >_t = (1/T) \int_0^T dt(\ldots)$.

The total neutral heating rate dW_{kn}/dt is shown in Figure 3. In Figure 4 we have computed the saturated values (taking the average of the heating from the latter half of each run) as a function of the electric field. We have six data points, so it makes sense to try different low–degree polynomial fits. It turns out that a parabolic fit is already fairly good, although there is a small E^3 tendency also present. This fit is displayed together with the simulation data in Figure 4. The E^3 part comes from the electron contribution; the extra power of E is probably because the electron temperature is itself roughly linearly proportional to E. The heating curve (Figure 4) depends on the amount and nature of inelastic collisions put into the model.

Figure 3

Figure 4

We made one run with the Poisson equation not solved, that is, the charged particle mutual interactions switched off. The result was the absence of waves, of course, but the electron and ion temperatures and neutral heating rates changed only slightly. Thus the electron heating is the classical frictional heating in our model. It has been proposed [*D'Angelo and Merlino,* 1987] that the observations may be explained by this heating alone. On the other hand, if we assume the existence of anomalous wave heating [*Schlegel and St-Maurice,* 1981; *St-Maurice et al.,* 1981], then the simulation provides indirect evidence that the process must involve the parallel wave component [*St-Maurice and Laher,* 1985], since $k_{\|}$ is not included in the simulation.

Turbulence Level

As pointed out earlier, the turbulence level is a very difficult quantity to measure from this kind of particle simulation. In any present–day particle code we have much more thermal noise than in the real situation. Another reason is the spatial smallness of the computational box, which prevents any mode coupling with long wavelength waves.

To estimate the thermal fluctuation level, let us assume that we have N_x grid points along one spatial direction, and N_C electrons per grid cell. Thus the total number of electrons is $N_e = N_C N_x^D$ where D is the spatial dimensionality ($D = 2$ in our case). Assume we are interested in thermal fluctuations in a coarse–grained subregions containing N_1^D grid cells. In one subregion, there are $n = N_C N_1^D$ electrons, and the statistical fluctuation level will be about $1/\sqrt{n}$. Then we choose N_1 in such a way that this "internal" fluctuation level is of the same order as the relative discretization error made when the total box is divided into N_x/N_1 subregions, along one spatial direction. The discretization error will be about N_1/N_x. Equating $1/\sqrt{n}$ with N_1/N_x yields

$$\frac{1}{\sqrt{n}} = N_e^{-\frac{1}{D+2}} \tag{13}$$

for the fluctuation level, which is independent on N_x.

In the present simulation we have $N_e = 380\ 000$, $N_D \approx 23$, $N_x = 128$, and eq. (13) yields about 4% level. Notice that this estimation was carried out ignoring the electrical coupling of the plasma, but this may not be too bad an assumption because of the thermalizing neutral collisions. Using higher–order particle clouds does not much reduce the above estimate since the averaging is already carried out over more ($N1 \approx 5$) cells than the extent of a feasible interpolation cloud.

In computing the turbulence level we should in principle integrate over all spatial Fourier modes. But as the smaller wavelengths are increasingly badly corrupted by thermal noise, this makes little sense. In Figure 5 we plot the turbulence level as integrated over 6 modes that satisfy the condition $|k| \leq 2$. For this choice, the thermal level appears to be 3–4 %, which is in a good agreement with the estimate (4%) obtained above.

Figure 5

The series of runs starts with $E = 3$ from a random, "almost empty" state. At $t = 2500$ the turbulence level rises above the thermal fluctuations, and by $t = 5000$

it reaches 9 % value. Although for $5000 \leq t < 10\ 000$ the electric field is lower, $E = 2.5$, the turbulence level keeps on rising, and at $t = 10\ 000$ it reaches its maximum of over 20%. The next run has $E = 2$, and this is sufficient to cut the growth of the turbulence and initiate a downward trend. At this point we have also followed the time development of another $E = 2$ branch which has the ambient density gradient scale length of $L = 50\ 000\ \lambda_{De}^{(0)}$ and it is seen that the turbulence level is higher than in the $L = \infty$ case. We will not however analyze further the effect of L in this paper. For the main branch the downward trend continues until the turbulence level drops below the thermal noise. This happens at about $t = 20\ 000$, when E is dropped form 1.5 to 1. At this point the theoretical Farley–Buneman threshold at $E = 1.3$ is also crossed, thus we have an agreement with simulation and theory, within simulation accuracy. For $t > 20\ 000$ we have again computed another branch with $E = 2$, $L = \infty$, and we see that $E = 2$ is sufficient to initiate a new upward trend. On the basis of Figure 5 we might predict that a run length of 50 000 or 100 000 time steps could be sufficient to observe the properly saturated turbulence level.

Phase Velocities

Much easier than to obtain the turbulence level is to compute the temporal power spectrum. Figure 6 displays one example: the spectrum of the ($k_x = 0, k_y = 1$) spatial Fourier mode (the longest wavelength mode in the $\mathbf{E}\times\mathbf{B}$ direction). A general feature of these spectra is that the peak Doppler shift is usually in good agreement with the fluid theory prediction (equation (3)). As already mentioned, they are also in agreement with kinetic theory [*Ossakow et al.,* 1975; *Schlegel,* 1983], since the kinetic theory phase velocity (not displayed) is only about 8% above the fluid result for these parameter values. These spectra are quite narrow (type 1). However, care must be taken when comparing the spectral widths with coherent radar results since a radar such as STARE (Scandinavian Twin Auroral Radar Experiment) measures an average over many heights, as well as a horizontal and temporal average. All this can only make the radar spectrum wider than the simulated one.

Figure 6

A summary of the simulated Doppler velocities as a function of the electric field is displayed in Figure 7. In Figure 7a we have plotted the heated ion acoustic velocity as a function of the electric field. Figures 7b–7d each show a comparison between the ion acoustic velocity (open circles), the fluid theory prediction (solid line), the approximated fluid theory (dashed line), and the simulated value (stars), for three different modes. By fluid theory we mean eq. (3), whereas the approximated fluid theory ("cosine law") refers to eq. (5). The

Figure 7

results are very similar for all modes having enough excitation to rise above the thermal level. The phase velocity is in a good agreement with the fluid theory, but the cosine law is not a bad approximation either. The accuracy of our numerical results does not, unfortunately, allow for a comparison between kinetic and fluid theories. Figure 8 has the same data as Figure 7, but the horizontal axis is now the electric field relative to the Farley-Buneman threshold.

Figure 8

To get a more global view of the phase velocities, Figure 9 displays the phase velocity in the $\mathbf{k}$ plane as a contour plot. This is compared with the dashed contours which display the corresponding result for the fluid cosine law. The same information is also in Figure 10, where we have plotted the phase velocity as a function of the flow angle θ for a few $|\mathbf{k}|$ values. The conclusion is that the simulated phase velocity map looks qualitatively similar to the cosine law and, most importantly, is nearly symmetric with respect to x axis (the ambient electric field direction). *Sudan* [1983a,b] proposed that the phase velocity should be equal to C_s or given by the cosine law, whichever is smaller. This C_s limited cosine law (equation 14 below) clearly would not be in agreement with the simulation.

Figure 9

Figure 10

The mode powers as a function of $\mathbf{k}$ are shown in Figure 11 for one electric field value ($E = 2$). The most important feature of this power map is its large asymmetry with respect to x. The most excited mode has about 45 deg. flow angle. The same data are also shown in Figure 12 with a different plotting method. This is, of course, very different from both fluid and kinetic linear theories, which are symmetric with respect to the flow angle. We return to this point later in the Discussion section, where we try to give a heuristic model for the nonzero flow angle propagation.

Figure 11

Figure 12

Since the radar will see the most intense mode most easily, it may make sense to ask what is its phase velocity. Which mode has most power depends on the electric field. Figure 13 displays this information, and it seems that interpreted this way, the phase velocity remains rather close to the ion acoustic velocity. This point will be central when we interpret the results in the Discussion section.

Figure 13

The flow angle asymmetry can also be visualized by computing the "center of mass" mode in the $\mathbf{k}$ plane as a function of the electric field (Figure 14). Notice in Figure 14 that the $E = 3$ point is a bit out of the main trend probably because the turbulence had not had enough time to develop (cf. Figure 5).

Figure 14

Consistency Checks

We have performed several kinds of consistency checks in order to assure that the conclusions made are not due to some numerical effect. We list some of the checks here. If we turn off the internal electric field and inelastic collisions, we recover the classical drifts and temperatures for both ions and electrons. We routinely check the Poisson solution by computing the residual, we also check that the charge assignment phase conserves the total charge.

In order to test the sensitivity of the results to the choice of mass ratio, one run was performed with mass ratio 400. The investigation confirmed that the conclusions presented in this paper are not affected by the mass ratio either.

In the present simulation, both ions and electrons are moved using the full equations of motion. We made a variant of the code in which the electrons are pushed in the guiding center approximation. The results did not change qualitatively, in particular the flow angle shift was again generated.

Latest Results: Three–Dimensional Modeling

Although it was mentioned in the introduction that a three–dimensional particle simulation is beyond our reach, we have nevertheless been able to develop a three–dimensional code which produces the flow angle shift in the perpendicular plane. We used a 32x32x64 grid and 600 000 electrons. The model was "32 times MG" in the sense that all characteristics were equivalent to MG, except the inclusion of the third dimension along the electric field. Also the collision algorithm was like MG rather than the constant mean free path algorithm used in the two–dimensional runs discussed in this paper. We started the run with completely random initial state with no sinusoidal perturbations added. The initial phase of the simulation was very similar to MG, the flow angle distribution being symmetric, but at later times a strong flow angle shift similar to Figure 11 developed. It dominated the aspect angle effects at the end of the run.

This three–dimensional study was carried out to confirm our two–dimensional findings, not to obtain new physical results. For routinely three–dimensional work our current computing capabilities are still too small, and many features can be more accurately studied using the two–dimensional code.

Discussion

We did not observe appreciable wave heating in our model. Instead, the electron temperature is just determined by a balance between frictional heating and inelastic (rotational) cooling. In reality, such a situation would lead to only very minor T_e enhancement, but we reduced the inelastic collision frequency so that our electron temperatures agree qualitatively with observations. The anomalous collision frequency is obtained as $\nu_e^* \approx 2\nu_e$ for $E \approx 2$. Since $T_e \approx 2$, $\nu_e^* \approx \sqrt{2}\nu_e$ is expected just because of more frequent collisions due to increased thermal speed. Our conclusion is that if the Farley–Buneman waves are responsible for the observed anomalous electron heating, then the heating must involve nonzero aspect angle effects (i.e., parallel wave component), maybe as proposed by *StMaurice and Laher* [1985] and *StMaurice et al.* [1981]. This is why we reduced the inelastic collision frequency: to keep the heated T_e even in the absence of parallel dynamics.

The phase velocity is not saturated to C_s but instead there is a flow angle asymmetry in the power spectrum, which causes the most intense mode having a phase velocity approximately equal to C_s (Fig. 13). The most intense mode satisfies $\mathbf{k} \cdot \mathbf{E} < 0$. This is what the simulation produces, let us now discuss it in terms of linear fluid theory. Recall that both fluid and kinetic linear theories are manifestly symmetric with respect to θ, so they cannot directly explain the flow angle asymmetry.

Equations (5) and (6) are valid for a small amplitude, large wavelength wave propagating in a quiescent plasma with homogeneous electron density. Whenever eq. (6) predicts that γ is positive for any $\mathbf{k}$, then the relation is, strictly speaking, not valid as the propagation environment can no longer remain homogeneous but becomes turbulent. However, even in this case the mode has some average phase velocity and we may turn things around and require that the average phase velocity is given by eq. (5). At the same time, if we assume that the state is quasi–steady, we must require $\gamma \leq 0$. This can be regarded as the "definition" of a quasi–steady state. After taking these generalizing steps, the quantities appearing on the right–hand sides of eq. (5) and eq. (6) must be re-interpreted, meaning that they must somehow contain the contribution of the turbulent environment in which our test wave is propagating.

So we must arrange $\gamma \leq 0$ in eq. (6). We list the four logical possibilities. Notice that with all these possibilities we have the result $|\mathrm{Re}\,\omega| \leq C_s$:

1. We may grow C_s, i.e. postulate that the plasma is heated so much that the drift is no longer "supersonic". However, in reality the C_s enhancement is not sufficient; this can be explicitly found from incoherent scatter measurements where the electric field (or V_d) and the electron temperature can be measured simultaneously.

2. ψ (equation 4) may be larger due to anomalous collisions. *Sudan* [1983a,b] assumed that ν_e effectively gets replaced by ν_e^*, which is just enough to make $\gamma = 0$ for the linearly most unstable ($\theta = \alpha = 0$ in the approximate fluid theory) mode. This prescription leads to the

C_s limited cosine law,

$$\mathrm{Re}\,\omega = \min\left(kC_s, \frac{kV_d\cos\theta}{1+\psi}\right). \tag{14}$$

3. ψ might also grow simply if nonlinear effects cause the largest amplitude waves propagate at a nonzero aspect angle α [*St-Maurice,* 1987]. This is conceptually different from *Sudan*'s [1983a,b] theory since it does not provide for a direct stabilization mechanism for all $\mathbf{k}$ vectors but instead invokes mode coupling to "repulse" the irregularities from the linearly most unstable $\mathbf{k}$ space regions. *Robinson* [1992] has discussed this and Sudan's theory in a unified formalism and concluded that cross–field anomalous diffusion is impossible to distinguish from the consequences of nonzero $k_{\|}$ propagation unless new and sophisticated experiments are employed.

The "wavetower tilting" mechanism [*Janhunen,* 1992], which also has nonzero aspect angle, is a still different possibility since it uses a definite "$\mathbf{k}$ repulsion" mechanism, namely the altitude dependence of the collision frequencies and thus the phase velocity in the auroral zone. It is likely, however, that the tilting mechanism is relevant for longer wavelengths (more than one meter, say) than we are studying in this paper.

4. Similarly to nonzero aspect angle propagation, it is as logical to propose nonzero flow angle propagation to achieve $\gamma \leq 0$. In equation form this "flow angle stabilization" then requires that the most important mode has flow angle defined by

$$\cos\theta = \frac{C_s}{V_d}(1+\psi) \tag{15}$$

for $V_d \geq C_s(1+\psi)$. In other words, the most intense waves should propagate at the edge of the linearly unstable sector, or cone.

It is now clear that in the present simulation the flow angle stabilization (item 4) is the relevant mechanism, with the addendum that the sign of θ appears to be fixed by $\mathbf{k}\cdot\mathbf{E} < 0$. On the average, eq. (15) gives slightly larger θ than the simulation, but the result is nevertheless well within the "error bars" of the simulation. In order to obtain the flow angle shift accurately from the simulation, the run should be very long, perhaps containing 100 000 time steps. Notice that eq. (15) is an accurate solution of eq. (3), since at the edge of the unstable sector we by definition have $0 \approx |\gamma| \ll |\mathrm{Re}\,\omega|$.

The use of fluid theory expressions in explaining our simulation results can be criticized. Our simulations are taking place in a short–wavelength regime, where the fluid theory growth rate estimate is no longer in agreement with kinetic theory [*Ossakow et al.,* 1975; *Schlegel,* 1983], although the phase velocity still is. Therefore computing the unstable sector from fluid theory would seem to be not correct. A more complete discussion of this question is outside the scope of this paper. The question is not at all trivial; for instance, it is not clear whether the "standard" kinetic theory [*Ossakow et al.,* 1975; *Schlegel,* 1983] is sufficient, or should we really use more realistic collision terms [*Stubbe,* 1989]. Stubbe's theory, however, leads to extremely complicated expressions, which have thus far not been evaluated numerically to our knowledge. However, for whatever reason, the fluid theory seems to be in a reasonable agreement with the simulations, and therefore it it logical to try to understand the nonlinear aspects of the simulations in terms of the fluid theory at first.

If flow angle stabilization is a real effect, it could perhaps be observed with a bistatic coherent radar system. We would probably need independent measurements of the electric field at least one point, however, and we should be able to remove aspect angle effects, since the radars usually have different height sensitivity profiles. Recently, some of the ROSE campaign (Rocket and Scatter Experiments) rocket flights showed very asymmetric angular power distributions [*Schlegel,* 1992], but it is yet unclear whether these are related to the flow angle stabilization or not.

Summary

We have studied the nonlinear Farley–Buneman waves in the plane perpendicular to the magnetic field using particle simulation techniques. Preliminary three–dimensional simulations have also been performed, and they show good agreement with the perpendicular two–dimensional runs.

The phase velocity is in a good agreement with both fluid and kinetic theory, the predictions of these theories are close together with the parameter values studied. The phase velocity did not saturate to the ion acoustic velocity, but was approximately proportional to the electric field.

For electric fields clearly above the threshold the most intense waves propagated at nonzero flow angles. The shift in the wave vector of the most intense mode was antiparallel to the electric field. The magnitude of the flow angle shift could be approximately reproduced by requiring the fluid theory growth rate to vanish and solving for the flow angle. The most intense mode had a phase velocity close to C_s.

We saw practically no perpendicular wave heating. Thus it is likely that if we need the plasma waves to explain electron heating in the E region, the heating must be related to the parallel wave component.

Appendix: Neutral Collisions

Elastic collisions between charged particles and neutrals are relatively easy to implement. We use a hard-ball model for all particles. We discuss two collision schemes, the first one (algorithm A) keeps the collision frequency constant, whereas the second (algorithm B) has constant mean free path.

Algorithm A (constant collision frequency method) works as follows. At each time step, collide each particle with a neutral with a small, fixed probability. The following is a definition of algorithm A, using a C–like language. Function Random(a, b) is assumed to return a uniform random float between a and b, and GaussianRandom() generates a normally distributed random float with unit standard deviation and zero mean. Other symbols are meant to be self–explanatory.

```
/* Constant coll. frequency algorithm (A) */
InelasticProbability= 1−exp [−(Δt/l_F^inel)u_thermal];
ElasticProbability= 1 − exp [−(Δt/l_F)u_thermal];
for (p = 1; p < N_e; p++) {
   v_n =GaussianRandom();
   if (Random(0,1) <InelasticProbability)
      v_p = v_n;
   else if (Random(0,1) <ElasticProbability) {
      b =Random(−1,1);
      b =sign(b)√|b|;
      u_⊥ = 2b√(1 − b²)/(1 + m/m_n);
      u_∥ = (m/m_n + 2b² − 1)/(1 + m/m_n);
      w = v_p − v_n;
      v_p = v_n + u_∥ w + u_⊥ ẑ×w;
   }
}
```

As written, algorithm A would be slow since all particles have to be processed every time step. In practice, we achieve the same result by scanning through a randomly chosen fraction of particles and colliding them with neutrals. The fraction of particles collided is equal to $\nu\Delta t$, where Δt is the time step and ν is the collision frequency. This variant of algorithm A is fast because the fraction is small, but fast particles will have larger mean free paths than slow ones.

In algorithm B (constant mean free path) we process all particles and collide each with a neutral with a probability that depends on its instantaneous velocity. In this way, the mean free path is the same for fast and slow particles. The definition is as follows:

```
/* Constant mean free path algorithm (B) */
InelasticProbability= 1−exp[−(Δt/l_F^inel) u_thermal];
v_0 = l_F/Δt;
for (p = 1; p < N_e; p++) {
   v_n =GaussianRandom();
   if (Random(0,1) <InelasticProbability)
      v_p = v_n;
   else if (Random(0,1) > exp(−|v_p − v_n|/v_0)) {
      b =Random(−1,1);
      b =sign(b)√|b|;
      u_⊥ = 2b√(1 − b²)/(1 + m/m_n);
      u_∥ = (m/m_n + 2b² − 1)/(1 + m/m_n);
      w = v_p − v_n;
      v_p = v_n + u_∥ w + u_⊥ ẑ×w;
   }
}
```

As with algorithm A, the above definition is slow. We have used a variant in which only a fraction of particles is tested for collision. The error introduced by this speed–up will be negligible if the collision probability at each particle remains much smaller than unity. In other words, we should test for collision at a much higher rate than the collision frequency. It was found that the neutral collision phase using algorithm B can be arranged so that it takes only 10-15 % of the total computing time, while still keeping the collision probability small.

On physical grounds, algorithm A should be used if the collision cross section is known to be inversely proportional to the relative speed. On the other hand, algorithm B simulates classical hard balls where the cross section is independent of the velocity. *Machida and Goertz* [1988a,b] used algorithm A, whereas we ended up using B.

Acknowledgments. The author thanks Asko Huuskonen, Anssi Mälkki, Kristian Schlegel, and Mihail Uspensky for many useful discussions and comments. K. Schlegel also let me use his two–dimensional program for comparison purposes, for which I am especially grateful. The main computations were carried out on a Cray X–MP located at Centre of Scientific Computing, Espoo, Finland.

The Editor thanks M.J. Keskinen and a second referee for their assistance in evaluating this paper.

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(Received December 29, 1992; revised October 27, 1993; accepted January 12, 1994.)

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Figure 1. The plasma temperatures and drifts as a function of time for $E = 2.5$. The temperatures are in units of neutral temperature; a unit velocity corresponds to the $\mathbf{E}\times\mathbf{B}$ drift velocity.

Figure 2. The electron temperature T_e (spots), ion temperature T_i (open circles), and threshold–relative electric field (solid line) as a function of the electric field. Temperatures are relative to neutral temperature T_n. For $E = 3$ there are two T_e data points from runs with different initial conditions.

Figure 3. Neutral heating rate in the simulation box as a function of time in percentages of initial thermal electron energy per unit time ω_{pe}^{-1}, divided by number of electrons $N_e = 380\ 000$. The stepwise lowered driving electric field is also shown. This is a scan over all six runs.

Figure 4. The mean neutral heating rate as a function of electric field, together with the fitted form const$\times E^2$. The unit of the heating is explained in Figure 3.

Figure 5. Turbulence level $< (\Delta n/n)^2 >^{1/2}$ as a function of time. Only eight modes (0,1), (1,0), (-1,0), (2,0), (-2,0), (0,2), (1,1), (-1,1) are included in the mode sum to reduce thermal noise. For 20 000$< t <$30 000 there is a second $E = 2$ phase (cf. Figure 3). In addition, for 10 000 $< t <$ 20 000 there is a second branch with finite $L = 50\ 000\ \lambda_{De}^{(0)}$.

Figure 6. The self–normalized temporal power spectrum for $E = 2$, mode (0,1), which corresponds to wavelength 128 $\lambda_{De}^{(0)}$ and flow angle $\theta = 0$. The dotted line is the heated ion acoustic velocity C_s, the dashed line is the fluid theory prediction for the phase velocity. The scale is linear. Kinetic theory would give in this case about 8% higher phase velocity than fluid theory; the difference in insignificant.

Figure 7. (a) Heated ion acoustic velocity C_s as a function of the electric field. Unit electric field would give an $\mathbf{E}\times\mathbf{B}$ drift velocity equal to $C_s^{(0)}$ the non-heated ion acoustic velocity. It has the expression $C_s^{(0)} = (2k_BT_n/m_i)^{1/2}$. (b) The simulation–produced phase velocity (stars), the heated ion acoustic velocity (open circles), the full fluid linear theory (solid line), and the cosine law (equation (12), dashed line), for mode (0,1) (wavelength 128 $\lambda_{De}^{(0)}$, $\theta = 0$). (c) and (d) Same as (b) but for modes (0,3) and (-1,2), respectively.

Figure 8. Same as Figure 7 but plotted as a function of the Farley–Buneman threshold relative electric field.

Figure 9. Solid contours: simulation produced phase velocity as a function of k_x and k_y, for $E = 2$. Unit k corresponds to wavelength 128 $\lambda_{De}^{(0)}$. Phase velocities in meters per second. Dashed contours: the linear cosine law result.

Figure 10. Phase velocity as a function of flow angle θ for $k = 2$ (solid), $k = 3.5$ (dashed), and $k = 5$ (dotted).

Figure 11. The spatial Fourier spectrum for $E = 2$. Logarithmic contour spacing. The vertical and horizontal axes are similar to Figure 9. This is the nonsymmetric spectrum where only $\omega > 0$ power is included; the "backstreaming" power on negative k_y is very small. For visual convenience the (0,0) mode power has been interpolated from its nearest neighbors.

Figure 12. Spectral power as a function of flow angle θ for $k = 2$ (solid), $k = 3.5$ (dashed), and $k = 5$ (dotted). Note logarithmic scale in vertical axis.

Figure 13. (Left) The phase velocity of most excited mode as a function of electric field. Obtained by bilinear interpolation at the "center of mass" of the spatial power spectrum. (Right) The same plot as a function of the relative electric field. See Figure 7 for an explanation of plot symbols.

Figure 14. The "center of mass" of the spatial spectral power for different electric fields. Electric field in positive x direction, $\mathbf{E}\times\mathbf{B}$ in positive y direction.

Table 1. First–Principle E region Studies

3D	Kin.	NL	Nature	Sample References
yes	no	no	analytic	*Buneman* [1963]
yes	yes	no	analytic	*Farley* [1963]
no	no	yes	2D fluid sim.	*Keskinen et al.* [1979]; *McDonald et al.* [1974]; *Ferch and Sudan* [1977]
yes	yes	no	analytic/numerical	*Ossakow et al.* [1975]; *Schlegel* [1983]
yes	no	yes	3D fluid simulation	*Janhunen* [1992]
no	yes	yes	2D particle sim.	*Machida and Goertz* [1988a]; present paper
yes	yes	yes	3D particle sim.	In the future ...

Here 3D=three–dimensional; Kin.=kinetic; NL=nonlinear.

Table 2. Simulation Parameters

Parameter	Value	Explanation
Δt	$0.2\ \omega_{pe}^{-1}$	time step
Ω_e	$1\ \omega_{pe}$	electron gyrofrequency
Ω_i	$0.01\ \omega_{pe}$	ion gyrofrequency
m_i/m_e	100	mass ratio
m_i/m_n	1	ion/neutral mass ratio
ν_{en}/Ω_e	0.02	electron elastic coll. parameter
ν_{in}/Ω_i	8.8	ion elastic coll. parameter
Δx	$1\ \lambda_{De}^{(0)}$	grid spacing in x
Δy	$1\ \lambda_{De}^{(0)}$	grid spacing in y
N_x,N_y	128	number of grid cells in x,y
N_e,N_i	380 000	number of electrons/ions
E	$(0.5\ ..\ 3)\ B\sqrt{2/3}u_n$	electric field
V_d	$(0.3\ ..\ 3)\ \sqrt{2/3}u_n$	drift velocity
u_n	$500\ \mathrm{m\ s}^{-1}$	neutral thermal velocity
$\lambda_{De}^{(0)}$	0.01 m	Debye length