A Numerical Ionosphere-Magnetosphere Coupling Model With Variable Conductivities

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We describe a numerical ionosphere–magnetosphere coupling model with a relationship between the field–aligned potential drop and the ionospheric conductivities. A two–dimensional incompressible magnetohydrodynamic (MHD) model for the magnetosphere, field lines with finite resistivity, a "2 1/2" dimensional ionosphere, as well as a phenomenological modeling of the ionization process are integrated in a time–dependent fluid simulation code. We discuss the algorithm and the validity of the results and give an example run with possible application to eastward drifting Ω bands of the morning sector. Our model is valid for time scales longer than the Alfvén bounce time and for spatial scales roughly between 10 and 1000 km. It is found that the inhomogeneous ionospheric conductivities affect the system dynamics substantially. This feature is essential when the model is used to study the development of auroral forms.

Introduction

The problem of ionosphere–magnetosphere interaction has been previously studied from different viewpoints, sometimes emphasizing the ionosphere [*Baumjohann et al.,* 1981; *Opgenoorth et al.,* 1983*a, b; Kan and Kamide,* 1985; *Buchert et al.,* 1990], sometimes the magnetosphere [*Lotko and Schultz,* 1988; *Lotko and Shen,* 1991]. There are a variety of scale sizes involved, ranging from the small–scale discrete arcs (less than 1 km) up to the global convection pattern (several thousand kilometers). We are interested in the intermediate scales, roughly from 10 to 1000 km.

The studies that have concentrated on the role of the ionosphere have modeled, for example, the westward traveling surge (WTS) and the Ω bands. In studies by *Opgenoorth et al.* [1983*a, b*] the ionospheric conductivities were based on riometer observations, and then, by trial and error, such column charges were placed in the ionosphere that the resulting electric field was consistent with STARE observations (Scandinavian Twin Auroral Radar Experiment) and calculated currents agreed with ground magnetic observations. The field–aligned current (FAC) was estimated from the divergence of the horizontal current.

Buchert et al. [1990] presented a stationary model where the FAC was prescribed, the conductivity distribution was calculated from the FAC pattern and the ionospheric electric field and horizontal currents were obtained by solving the current continuity equation. The model was applied to observations of eastward drifting Ω bands. *Kan and Kamide* [1985] presented a stationary numerical model for the WTS where the FAC was calculated from a prescribed potential distribution. The model allowed for varying ionospheric conductivities but, for simplicity, the ratio of integrated Hall and Pedersen conductivities was held constant.

All these auroral substorm modelings have concentrated on the ionosphere. The magnetosphere has been considered as a source of precipitation and FAC but has not been included self–consistently into the model equations. The opposite approach was taken by *Lotko and Schultz* [1988], *Lotko and Shen* [1991], and *Miura and Sato* [1981], who modeled the magnetosphere with two–dimensional incompressible magnetohydrodynamic (MHD) equations. The treatment of the ionosphere, on the other hand, was simplified, as it was assumed horizontally homogeneous. Therefore the models are not suitable for studies of auroral features, which are often characterized by strong horizontal conductivity gradients.

We try to combine these two approaches by developing a time–dependent model with variable ionospheric conductivities, the magnetosphere being included as a two–dimensional MHD layer. The conductivities depend on the field–aligned potential difference. Although this sort of a model has a great degree of self–consistency builtin, we still need to prescribe the magnetospheric driving force to put energy into the system. In this paper, we will present the model equations as well as the solver algorithm, discuss the advantages and limitations of the model and present results from an example run.

Theory and Modeling

Magnetosphere

We model the magnetospheric equatorial plane by two–dimensional incompressible magnetohydrodynamics (MHD). The MHD momentum equation reads

$$\rho(\partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v} - \nu\nabla^2\mathbf{v}) = \mathbf{j}\times\mathbf{B} - \nabla p \qquad (1)$$

where the mass density ρ is assumed constant. This implies $\nabla\cdot\mathbf{v} = 0$. We use the ideal MHD Ohm's law $\mathbf{E}+\mathbf{v}\times\mathbf{B} = 0$ and assume that $\mathbf{B}$ is a constant vector such that $\mathbf{B}\cdot\nabla = 0, \nabla\mathbf{B} = 0$. We assume a small constant viscosity ν. Taking the curl of (1) and defining the vorticity $\omega = \hat{\mathbf{B}}\cdot\nabla\times\mathbf{v}$, (1) yields

$$\rho\left[\partial_t\omega + \hat{\mathbf{B}}\cdot\nabla\times(\mathbf{v}\cdot\nabla\mathbf{v}) - \nu\nabla^2\omega\right] = -B\nabla\cdot\mathbf{j} \qquad (2)$$

We then start using the subscript "m" to denote all magnetospheric variables, as well as ∇_m to denote the gradient operation with respect to two–dimensional magnetospheric coordinates. If the thickness of the magnetospheric layer is H, then because of current continuity $\nabla_m\cdot\mathbf{j}_m = j_{\|m}/H$, where $j_{\|m}$ is the FAC into the magnetosphere. So we have the magnetospheric equation

$$\frac{\partial\omega}{\partial t} = \frac{1}{B_m}\hat{\mathbf{B}}\cdot\nabla_m\omega\times\nabla_m\phi_m + \nu\nabla_m^2\omega - \frac{B_m}{H\rho}j_{\|m} + s. \qquad (3)$$

We have introduced a source term s on the right–hand side of (3) to provide the external energy into the system. We also have the relationship

$$j_{\|i} = \frac{B_i}{B_m}j_{\|m}. \qquad (4)$$

Here $j_{\|i}$ is the FAC out of the ionosphere, and $j_{\|m}$ is the FAC into the MHD layer. The constants B_m and B_i are the equatorial plane and the ionospheric magnetic field magnitudes, respectively. The factor B_i/B_m comes from the flux tube geometry. We use ionospheric coordinates x, y, and z with orientations north, west, and upward, respectively. We still need one equation to close the MHD system, this can be derived as follows:

$$\omega = \hat{\mathbf{B}} \cdot \nabla_m \times \mathbf{v} = \frac{1}{B_m} \nabla_m^2 \phi_m \tag{5}$$

Thus the equatorial potential ϕ_m is obtained from the vorticity ω by solving a Poisson–type equation. Together (3) and (5) are similar to the equations of neutral incompressible two–dimensional fluid, except for the source term s and the ionosphere coupling term.

Field Lines

On the field lines, we assume

$$j_{\|i} = K(\phi_i - \phi_m) \tag{6}$$

where K is the field–line–integrated conductance density, sometimes referred to as the Lyons–Evans–Lundin constant. We let the FAC saturate if the potential difference exceeds ϕ^{thr} [*Goertz et al.,* 1991]:

$$K = K_0 \min(1, \frac{\phi^{thr}}{|\phi_i - \phi_m|}). \tag{7}$$

We have adopted the numerical values $K_0 = 10^{-9}$ mho m^{-2} and $\phi^{thr} = 10$ kV.

Ionosphere

In the ionosphere we have the height–integrated Ohm's law and the current continuity equation,

$$\mathbf{I}_{\perp i} = \Sigma_P \mathbf{E}_i - \Sigma_H \hat{\mathbf{B}} \times \mathbf{E}_i \tag{8}$$

$$\mathbf{j}_{\|i} = -\nabla \cdot \mathbf{I}_{\perp i} \tag{9}$$

Together with the field line relation (6) this yields

$$\begin{aligned} &\Sigma_P \nabla_i^2 \phi_i - K\phi_i + \nabla\Sigma_P \cdot \nabla\phi_i \\ &+\hat{\mathbf{B}} \cdot \nabla\Sigma_H \times \nabla\phi_i = -K\phi_m \end{aligned} \tag{10}$$

Equation (10) is an elliptic boundary value problem for the ionospheric potential ϕ_i once the MHD layer potential ϕ_m and the Pedersen and Hall conductivities $\Sigma_{P,H}$ are known. Because ϕ_m is known at each time level after solving (5), it remains to compute Σ_P and Σ_H.

Although the treatment has thus far been two–dimensional, let us now consider the three–dimensional evolution equation for the ionospheric electron density:

$$\frac{\partial n(x,y,z,t)}{\partial t} = q(x,y,z,t) + q_0(z) - \alpha n(x,y,z,t)^2 \tag{11}$$

Here q_0 is the background production rate profile, assumed independent of time and horizontal coordinates, and q is the precipitation–caused production rate. The recombination coefficient α is assumed to be $3 \cdot 10^{-13}$ m^3 s^{-1} throughout the ionosphere. After $n(x, y, z, t)$ is known, we can numerically integrate $\Sigma_{P,H}$ from the expressions

$$\Sigma_{P,H} = \int dz F_{P,H}(z) n(x, y, z, t) \tag{12}$$

where the quantities $F_{P,H}(z) = \sigma_{P,H}/n$ can be precomputed – they are only functions of z as indicated.

For the background production rate we have used the expression

$$q_0(z) = \alpha \left[3.68 \cdot 10^{10} \mathrm{m}^{-3} (\frac{\Sigma_0}{\mathrm{mho}}) e^{-[(z/\mathrm{km}-126.3)/15.0]^2} \right]^2, \tag{13}$$

where $\Sigma_0 = 3$ mho. This expression yields approximately the background value of Σ_0 for both Σ_P and Σ_H in case $q(x, y, z, t)$ vanishes. The details of the height profile (equation (13)) are irrelevant as far as physical conclusions are concerned. In any case, some kind of background production rate profile is needed because otherwise the electron densities can become zero in regions of downward FAC, which is a numerically very unstable and physically unrealistic situation. Physically, the background ionization production is due to solar UV radiation and suprathermal charged particles of magnetospheric and solar origin.

The expressions given by *Evans* [1974] and *Rees* [1963] allow one to compute the height profile of the ionization rate q. In this paper we are neglecting the effect or secondary electrons [*Kirkwood,* 1992]. To compute the precipitating electron flux energy spectrum one has to know the accelerating potential and the distribution function of the source plasma. The accelerating potential is known at each time level, it is just taken to be $\phi_i - \phi_m$. We assume that the source plasma is Maxwellian with a temperature that depends on the x, y coordinates. The temperature as a function of spatial coordinates is prescribed, not derived. By choosing it appropriately, we can model the diffuse precipitation of the auroral oval. Thus we have the general relationship

$$q(x, y, z, t) = Q\left[\phi_i(x, y, t) - \phi_m(x, y, t), T_m(x, y), z\right] \tag{14}$$

where T_m is the above mentioned magnetospheric temperature and $Q(.,.,.)$ is a known function of three variables. In a practical simulation calculation, Q can be precomputed and tabulated. The accelerated differential flux energy spectrum of the source plasma is given by *Evans* [1974]:

$$\Phi(E, \Delta\phi, T_m) = \frac{n_0}{\sqrt{2\pi m_e}} \frac{E - \Delta\phi}{T_m^{3/2}} e^{-\frac{E-\Delta\phi}{T_m}} \tag{15}$$

where n_0 is the source plasma density, m_e is the electron mass and $\Delta\phi$ is the acceleration voltage, $\Delta\phi = \phi_i - \phi_m$. Equation (15) holds for $E > \max(0, \Delta\phi)$, otherwise Φ vanishes. The dimensionality of Φ is m^{-2} s^{-1} J^{-1}. In terms of

Φ, the production rate Q is given by

$$Q(\Delta\phi, T_m, z) = \int dE Q_0(E, z)\Phi(E, \Delta\phi, T_m) \tag{16}$$

where $Q_0(.,.)$ is the production rate per unit incident electron flux. The dimensionality of Q_0 is inverse length and it depends only on the atmospheric model used; the MSIS–86 model is adopted for this paper [*Hedin,* 1987].

Numerical Model and Algorithm

To summarize, we have to solve the following set of time–dependent partial differential equations:

$$\begin{aligned} \frac{\partial\omega}{\partial t} &= \frac{1}{B_m}\hat{\mathbf{B}}\cdot\nabla_m\omega\times\nabla_m\phi_m + \nu\nabla_m^2\omega \\ &\quad + \frac{B_m^2}{B_i H\rho}[K(\phi_m - \phi_i) + j_0] \end{aligned} \tag{17}$$

$$\nabla_m^2\phi_m = B_m\omega \tag{18}$$

$$\frac{\partial n}{\partial t} = q_0 + Q(\phi_i - \phi_m, T_m, z) - \alpha n^2 \tag{19}$$

$$\Sigma_{P,H} = \int dz F_{P,H}(z) n \tag{20}$$

$$\begin{aligned} &\Sigma_P\nabla_i^2\phi_i - K\phi_i + \nabla\Sigma_P\cdot\nabla\phi_i \\ &\quad + \hat{\mathbf{B}}\cdot\nabla\Sigma_H\times\nabla\phi_i = -K\phi_m \end{aligned} \tag{21}$$

where K is given by (7). The dynamical variables are $\omega, \phi_m, n,$ and ϕ_i, of which n is three–dimensional. The rest of the parameters appearing in (17) through (21) are listed in Table 1, together with their basic values. We have introduced the function $j_0 = B_i H\rho s/B_m^2$ and used it instead of s. The interpretation of j_0 is explained by *Lotko and Shen* [1991]. The functions $F_{P,H}(.)$ and $Q(.,.,.)$ can be precomputed and they depend on the atmospheric model used. The gradient operator in the MHD layer ∇_m is given by *Lotko and Schultz* [1988]

Table 1

$$\nabla_m = (\frac{\partial}{\partial x_m}, \frac{\partial}{\partial y_m}) = \left[\frac{\cos\Lambda}{L}\frac{\partial}{\partial x}, \frac{B_m}{B_i}\left(\frac{L}{\cos\Lambda}\right)^2\frac{\partial}{\partial y}\right] \tag{22}$$

where $\Lambda = 73^\circ$ is the latitude of the center of the ionospheric simulation box and $L = 12$ is the radial distance in Earth radii to the intersection of the corresponding field line with the equatorial plane. So the ionosphere–magnetosphere mapping is assumed to be linear within the simulation box, but in general not isotropic. By varying, for example, B_m we can obtain different anisotropy factors. The linearity assumption holds good if the simulation box is small enough. The numerical values for Λ and L should not be taken literally because the dipolar field model is only qualitatively valid anyway. The ionosphere–magnetosphere stretch factors (Table 1) are the parameters that matter.

The magnetospheric temperature T_m and the energy input term j_0 are user–specified functions. The boundary conditions are Dirichlet in the north–south direction and periodic in the east–west direction. The boundary conditions for the potentials $\phi_{i,m}$ are inhomogeneous; i.e., $\phi_{i,m} - \phi_0$ satisfies the homogeneous Dirichlet and periodic conditions, where ϕ_0 is an ambient potential profile. We have used an expression of the form

$$\phi_0(x,y) = E_0 L_0[\frac{\pi}{2} + \arctan(\frac{x - L_x/2}{L_0})]. \qquad (23)$$

Here E_0 is a parameter (e.g., 30 mV m^{-1}) which gives the maximum boundary condition electric field in the ionosphere and L_0 is a parameter that determines the width of the initial potential step. We used $L_0 = L_x/5$ where L_x is the north–south extent of the simulation box at the ionosphere, $L_x = L_y = 1000$ km.

We difference (17) by splitting the right–hand side operator into x and y dependent parts [*Press et al.,* 1988]. The total time step is performed as two half–steps, first taking into account the x dependent part, then the y dependent part. For the advective term (the first term on the right–hand side of (17)) we use the Lax–Wendroff method, the other terms are treated by a simple Forward Time Center Space (FTCS) method.

The constant–coefficient elliptic equation (equation (18)) can be solved "exactly" (only roundoff error remains) using a combination of fast Fourier transform (FFT) and tridiagonal solving. We used vectorized FFT routines [*Nobile and Roberto*, 1986*a, b*] to keep the relative computing time used with this equation small.

The electron density evolution equation (equation (19)) is an easy one; it can be integrated with the forward Euler method because the recombination time is everywhere much longer than the time step. This is crucial for the feasibility of the algorithm because, being three–dimensional, (19) could otherwise dominate the total computing time.

The real challenge is (21). The large conductivity gradients show up as large off–diagonal elements in the linear system to be solved. After having discouraging experience with the successive over–relaxation (SOR) method we found the alternating direction implicit (ADI) algorithm [*Press et al.,* 1988; *Hockney et al.,* 1988] successful. Using the routine given by *Press et al.* [1988] as a starting point, we wrote a vectorized ADI routine that can handle both Dirichlet, Neumann and periodic boundary conditions. ADI converges almost always (for physically sensible input) and reduces the residual norm relatively rapidly. The number of ADI iterations required per time step varies between 5 and 25 depending on the physical conditions. This phase of the algorithm generally takes about 60–70 % of the total computing time. On a 90 $\times$ 90 grid, the Cray X–MP uses about 0.1 CPU seconds with one time step, achieving 60 Mflops computing speed.

We studied the effects of various discretization related parameters such as the spatial grid spacing and the time step. The effects were observable but insignificant. The grid spacing cannot be made much larger than 10 km, otherwise the ADI may stop converging. If it is made smaller, then the time step has to be reduced as well: the Courant condition $\Delta t < \Delta x/(\sqrt{2} v_d)$ must be satisfied, where v_d is the maximum occurring $\mathbf{E}\times\mathbf{B}$ drift velocity.

As said before, we use Dirichlet and periodic conditions in this paper. We made some experiments with fully periodic conditions but it turned out that the ADI routine has difficulties to converge then. This is not a drawback in the current simulation, but it should be borne in mind if the algorithm is applied to problems with different geometries.

Application

This section presents one example run of the model. The input terms and the average upward/downward FACs are shown in Figures 1 and 2, the maximum upward/downward FAC in Figure 3, simulated magnetograms in Figure 4, and snapshots of the conductivity distribution, electric field and currents are displayed in Plates 1 and 2. The simulation parameters are summarized in Table 1.

Figures 1,2,3

Figure 4

Plate 1,2

Our source current term j_0 was of the derivative Gaussian form

$$j_0(x,y,t) = j_{00}(1+\sigma(x,y))\sqrt{2}ue^{\frac{1}{2}-u^2} f(t) \qquad (24)$$

where $u = 6(x - L_x/2)/L_x$, j_{00} is a constant, $\sigma(x,y)$ is a small turbulent contribution and $f(t)$ is the time dependence, with $\max|f(t)| = 1$. The maximum magnitude of $j_0(x,y,t)$ is roughly equal to j_{00}. The time dependence of the quantity $\langle |j_0 - \langle j_0\rangle|\rangle$ (the angular brackets denote spatial averaging) is plotted in Figure 1, thin dotted line. It is almost the same as $f(t)$ except for the small–scale variation which is due to the turbulent component σ. The reason for including $\sigma(x,y)$ is that its contribution can be easily identified in various output time series and thus the sensitivity of each output time series to small changes in input data can be estimated. The expression of $f(t)$ contains trigonometric and exponential functions. The x dependence of the source term is shown by the solid line in Figure 2.

To mimic the diffuse auroral oval, we used a Gaussian magnetospheric temperature profile,

$$T_m(x,y) = T_m^{min} + (T_m^{max} - T_m^{min})e^{-4(x/L_x - 0.38)^2} \qquad (25)$$

with $T_m^{min} = 0.5$ keV and $T_m^{max} = 2.0$ keV. Note that the peak position of the T_m is about 120 km southward from the center of the simulation box, whereas j_0 is symmetric with respect to the central line $x = L_x/2$ (Figure 2).

In Figure 1 we have plotted the average upward and downward FAC together with the source term. We have multiplied the average field-aligned currents by 2 before plotting, to make visual comparison with j_0 easier. The upward and downward currents are staying very close together in magnitude all the time, the only exception being at about 45-60 min. They follow the source term faithfully with a 5-10 min delay. In the limit of a very large simulation box and negligible discretization error the total upward and downward FACs should be equal in magnitude. Because of current continuity, the only sources of difference between upward and downward currents are at the northern and southern boundaries of the simulation box. Because our north–south boundary conditions are such that we fix the ionospheric potential but not its x derivative (Dirichlet condition), slight changes in the boundary electric field and the horizontal current flowing into the simulation box from the x boundaries are expected.

Figure 3 displays the maximum upward and downward FAC. The upward current stays in the saturated value (10 μA m^{-2}) for a longer time than the return current and it is larger almost all the time. This indicates that the return current extends over a wider area than the upward current.

The simulated magnetograms in Figure 4 show an abrupt "breakup" phase occurring at about $t = 40$ min, which follows the source term maximum with roughly 15–min delay. Note that as the source term continuously increases the energy of the system, it is the time integral of the source term rather than j_0 itself that determines the "breakup" epoch. During the following 20 min, there are sudden changes in the Y and Z components of magnetic field below the center of the simulation box (station 2 in Figure 4). These variations imply that the westward electrojet, which has been intensifying from the beginning of the simulation, has developed into a current vortex. Note that Figure 4 has the standard coordinate system used in magnetograms (X positive north, Y east and Z down), which is different from the system employed otherwise in this paper.

The temporal development of the conductivity and currents is seen in Plate 1. At $t = 40$ min the diffuse precipitation belt (auroral oval), which has been intensifying since the beginning, has developed inhomogeneous in the y direction. During the following 15 min, the region of stronger precipitation drifts eastward, gradually fading, with a large counterclockwise current vortex around it. The large–scale structure moves eastward with an average velocity of about 600 m s^{-1}, agreeing with the $\mathbf{E}_0\times\mathbf{B}$ drift. It is evident from (17) that an $\mathbf{E}\times\mathbf{B}$ drift velocity should emerge. For example, if the magnetospheric electric field is x directed and constant, and if terms on the right–hand side of (17) other than the cross product term are neglected, then (17) just becomes a monodirection wave propagation equation of the form $\partial\omega/\partial t = (E/B_m)(\partial\omega/\partial y)$. What is remarkable, though, is that the observed propagation velocity agrees with E_0, or the boundary condition field, not the instantaneous E which is sometimes 8–10 times larger.

In many ways, these auroral features are reminiscent of the eastward drifting Ω bands, observed in the morning sector. Comparison of observations by *Buchert et al.* [1990, Figure 4; 1988, Figure 8] with our model results in Plate 2 shows a considerable similarity in the conductivity patterns as well as in the electric field.

After the passage of the feature, the situation is again characterized by the diffuse auroral oval and westward electrojet, which is now slightly weaker than during the first 40 minutes, since part of the energy has been dissipated. After 90 min, when the source term increases again, the electrojet intensifies as well, and at about 160 min there is new activity associated with the second maximum of the source current. We inserted the new activity to see whether the instability development is significantly different if it is not started from a complete quietness. There were no major qualitative differences. The amplitude of the disturbance was smaller due to smaller energy input, see Figures 1, 3, and 4.

The ionospheric variables are shown in detail at $t = 50$ min in Plate 2. The dominant auroral form is a large counterclockwise current vortex, inside which there is intense (10 $\mu\mathrm{Am}^{-2}$) upward FAC and associated enhanced Hall conductivity (nearly 50 mho). At the northern boundary of the vortex there is mainly westward horizontal electrojet current, with maximum height–integrated magnitude of 2.7 A/m, which is also seen in the magnetograms (Figure 4): for station 2 (just below the vortex), X is negative. On the northern side, there is a clockwise current vortex, within which a relatively intense downward current is flowing.

The current which closes the downward and upward currents through the ionosphere is driven through regions where the integrated conductivity is less than 10 mho. This requires strong electric fields, indeed the maximum ionospheric horizontal electric field magnitude in Plate 2 is 230 mV/m, which is perhaps somewhat too high. *Buchert et al.* [1990], for example, reports the maximum electric field to be 190 mV/m, which they regard as exceptionally high.

Discussion

Our model of ionosphere–magnetosphere interaction is mostly built in a self–consistent way, the main exceptions being that the energy input and the magnetospheric plasma temperature profile must be prescribed. The reason is that at the present time we cannot model the full solar wind–magnetosphere interaction.

The energy input to the system is through the term s in (3). It is a curl of a force density divided by mass density, with dimensionality inverse second squared. Actually, in (17) we have used j_0 which is up to a constant the same as s. We have preferred j_0, which is equivalent to the function j_0 of *Lotko and Shen* [1991], instead of s because it has the same dimensionality ($\mathrm{A\,m}^{-2}$) as the field–aligned current density, and also the numerical values of j_0 are comparable to the real FAC. Indeed, if the product ρH approaches zero, then j_0 just becomes equal to the FAC. If we want to, we can interpret j_0 as a source current arriving into the MHD layer "from the outside," although there is in reality no such "outside" region. In the formalism of *Lotko and Shen* [1991] this is made explicit by labeling the points inside the MHD layer by a z coordinate.

The time derivative of the vorticity ω gets contribution from the source term j_0. So the MHD fluid accelerates into a counterclockwise or clockwise motion around any point where j_0 is positive or negative, respectively. The shear flow field $\mathbf{v}$ introduces a static electric field $\mathbf{E} = \mathbf{B}\times\mathbf{v}$. The field lines and the ionosphere form a system ("DC circuit") where the conductivity is a complicated function of space and time. The magnetospheric $\mathbf{E}$ field acts as a boundary condition for the system, and currents start flowing. The field-aligned part of these currents couples back to the magnetosphere.

The energy spectrum of the precipitating electrons depends on both the magnetospheric plasma temperature T_m (prescribed function) and the local potential difference $\Delta\phi$. Because T_m is nonzero, there is some precipitation even on regions of downward FAC. Furthermore, the lifetime of the ionization produced depends strongly on how intense the precipitation is. Also the Pedersen/Hall conductivity ratio is not constant but follows the changes in the nature of the precipitation spectrum. The use of the electron continuity equation introduces some delay (of the order of the recombination time in the E region) between the precipitation and the electron density enhancement.

The role of the magnetospheric plasma temperature T_m is crucial in our model. If T_m were zero, the conductivity would rapidly drop near to its background value in regions of downward current, and this would hinder the current closure through the ionosphere. From the experience we have had with our simulation code, we can conclude that there has to be some mechanism which enhances the electron density even in areas of downward current, otherwise no realistical auroral developments can be generated. In the example run we chose a T_m which is a Gaussian function of x (the northward coordinate) to simulate the diffuse auroral oval. However, we are not proposing any definite magnetospheric physical mechanism for the generation of such a variable T_m.

At least qualitatively, an effect similar to T_m could be achieved by postulating (see *Janhunen and Huuskonen* [1992], who also describe earlier versions of the present model) that the FAC is filamented at subgrid scales ($<$ 10 km). Because even if the large–scale current (averaged over 30 km, say) is downward in some region, there may be smaller subregions where the FAC still flows upward and enhances the conductivities locally. In this scenario, the "amount of filamentation" would have qualitatively the same role as T_m has in our model. In reality, the filamentation probably occurs to some extent but how effective it is in raising the average conductivity and what determines the magnitude of the filamentation is still not clear.

Ionosphere

The ionosphere is the best modeled region in the simulation. The ion motion term $\nabla\cdot(n\mathbf{v}_i)$ has not been included in the electron density continuity equation (equation (11)). The omission of the ion motion term can introduce errors on spatial scales smaller than the distance an average ion travels during its lifetime. According to an order-of-magnitude estimate, this term might have some influence on the smallest scale ionospheric features. Inclusion of the ion flux term would introduce the possibility of gradient drift instability, which could cause numerical problems at the grid spacing we are currently using. One possibility of future extension would be to try to model the smaller-scale gradient drift waves by some kind of anomalous conductivities.

Another approximation is that although we compute the electron density in a three–dimensional grid, we integrate over the height coordinate before we insert the coefficients into the elliptic continuity equation. This scheme is valid for spatial scales larger than the thickness of the layer where most of the horizontal current flows (a few tens of kilometers). The conclusion is that spatial scales below 40 km, say, are perhaps not accurately modeled, but since the grid spacing is about 10 km, discretization errors prevent an accurate representation of those scales anyway. Furthermore, most of the ionospheric features are above 100 km scale size because of the filtering effect.

A situation otherwise comparable to ours but with constant conductivities has been studied in great detail by *Lotko and Shen* [1991]. There the filtering effect of the ionosphere is also discussed: scale sizes smaller than $\lambda = (\Sigma_P/K)^{1/2}$ are mapped only weakly from the magnetosphere into the ionosphere. This is evident because if the ionospheric resistivity increases, the ionospheric ends of the current wedges become shorter and the FAC becomes more filamented. The numeric value of the resistive scale length λ is of the order of 100 km. Note, however, that an exact λ can be defined only if the conductivities are constant.

To study the effect of variable conductivities directly, we also made some runs with the ionosphere "decoupled," i.e., we used the constant background conductivity values when solving the elliptic equation (equation(21)), but computed and stored the variable conductivities also to see explicitly the differences in the ionosphere. This "decoupled" version should correspond to the model of *Lotko and Shen* [1991], except for some differences in algorithms, boundary conditions etc. This truncated scheme produces results which at least qualitatively look the same as in *Lotko and Shen* [1991, Figures 4–8]. The result of this comparison was that the initial "breakup" phase (when the Kelvin-Helmholtz instability is working) is not very much affected by the variable conductivities, but the later development is. Especially in the FAC there is much more fine scale structure in the variable conductivity case. In the constant conductivity case, only rather symmetric current vortices emerge, at least if the source term j_0 and the temperature distribution T_m are kept simple as we did.

Field Lines

The linear relationship $j_\parallel = K\Delta\phi$ together with the saturation if ϕ exceeds 10 kV was used for both upward and downward FAC regions. Although a theoretical explanation at least for the downward return current is still unclear [*Lu et al.,* 1991], it seems to be in accordance with observations [*Goertz et al.,* 1991; *Weimer et al.,* 1985, 1987]. To test the sensitivity of the results to the field–line relation, we made one run where the saturation potential was 5 kV for the return current while it was still 10 kV for the upward current. The result was a fast instability development and a somewhat richer variety of smaller–scale forms than in the basic model.

Magnetosphere

The two–dimensional incompressible MHD layer with its prescribed plasma temperature profile and energy input is perhaps the weakest point of our model. One deficiency in the MHD model adopted is that the only possible instability mechanism is the Kelvin–Helmholtz instability. This seems to limit the usefulness of the model because, if the true mechanism of a phenomenon (e.g., Ω band) is not Kelvin–Helmholtz, our results are not directly applicable but must be interpreted.

On the other hand, the two–dimensional viscous incompressible MHD model for the magnetosphere should not be taken too literally. The real situation need not necessarily satisfy any of these assumptions. It is sufficient for our purposes that the part of the magnetosphere which affects most the ionosphere–magnetosphere coupling acts effectively as a viscous incompressible two–dimensional fluid layer. The viscosity may be an effective one, possibly stemming from three–dimensional turbulence. Also parameters such as thickness and mass density of the layer must be understood as effective values.

It can be asked what determines the fastest–growing mode in the instability development phase. Figure 2 by *Lotko and Shen* [1991] gives the answer for their model. For our model, this point is more obscure, because of numerical dissipation and variable conductivities. For our purposes, however, this is not any problem. As a consistency check, we made one run in which the system size and the number of grid points in both east–west and north–south directions was doubled, and two almost identical subsystems emerged (Plate 3). This means that the wavelength of the fastest growing mode is not forced by the grid size. Recall that there is no y dependence in any source terms or boundary conditions except for the random turbulent component. A comparison of the basic 90x90 run and the 180x180 run at $t = 50$ min is shown in Plate 3, where the magnetospheric vorticity function ω is plotted as a color chart. The 90x90 box (lower, square panel) is shown as is, the upper panel shows a 180x90 subregion of the 180x180 run, with 45 grid points wide stripes at the northern and southern boundaries not displayed. The subsystems are not completely identical because of the random turbulent component of the source term, but they agree qualitatively. Also it is seen that although the flow "collides" with the Dirichlet boundary condition at the northern boundary in the 90x90 run (lower panel), the flow is not ap-

Plate 3

preciably distorted. The time step was 0.5 s for the 180x180 run instead of the default 1.0 s.

We used a finite difference method to solve the momentum equation, which is somewhat more inaccurate than the spectral method used by *Lotko and Shen* [1991]. The small scale wavefronts seen in Plate 3 can be partly of numerical origin. There are, however, at least two reasons to believe that the physical conclusions are not affected by these inaccuracies. For one thing, the discretization errors occur mostly at the smallest scale lengths, which are not mapped into the ionosphere because of the filtering effect. The other thing is that because the equatorial plane model contains two user specified functions (the energy input and the electron temperature), any numerical errors can be "absorbed" into these functions. Of course this point must be re-examined if our magnetosphere model is refined in the future. The finite difference method is flexible, for example it does not force us to use a linear ionosphere–magnetosphere mapping.

Conclusions

Our model is a combination and extension of the ionosphere–magnetosphere coupling models mentioned in the Introduction. For the first time, a MHD magnetospheric model is combined with a realistic time-dependent ionospheric model, which allows for inhomogeneous conductivities that are characteristic for auroral phenomena. On the other hand, the previous models of Westward Traveling Surge and Ω bands and other auroral features have not explicitly included any magnetospheric equations. However, the usefulness of the model may be limited by the fact that the only instability mechanism is the Kelvin–Helmholtz instability. Moreover, our model is restricted to certain spatial and temporal scales. It is supposed to be valid at about 10–1000 km spatial ionospheric scales. Time scales should be longer than the Alfvén bounce time. The relevant length parameter is the resistive scale length λ, which is about 100 km (an exact value can be given only in case of constant conductivities). At scale sizes below 10 km, three–dimensional Alfvén wave propagation and three–dimensional ionosphere should be taken into account [*Seyler,* 1990; *Hui and Seyler,* 1992]. The ion Larmor radius and the collisionless skin depth c/ω_e are the relevant length parameters at those scale lengths. Above 1000 km, the global geometry of the geomagnetic field starts to distort our results significantly [*Watanabe et al.,* 1986], and the relevant length parameter becomes the Earth radius. Extending our model to either smaller or larger scales would thus probably lead to three–dimensional calculations.

We have implemented our numerical model on a traditional vector supercomputer. However, the model is also well suited for the massively parallel computers of the future. This is because the operations performed at different grid points are either independent or the information is exchanged only between neighboring mesh points. This can open up new opportunities for cheaper computing as well as runs with longer time extents.

The guiding principle in our design was to model the well–known plasma environments (the ionosphere) as accurately, yet efficiently, as possible, while using a reduced or effective description of the more obscure ones (the magnetosphere). Although the plasma properties at a given point in the magnetosphere are more or less known, there is much less agreement on just which part of the magnetosphere is the "counterpart" of the ionosphere in the coupling.

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Fig. 1. Time dependence of the source term j_0 (thin dotted). Average upward (solid) and downward (dashed) field-aligned currents multiplied by 2. The short-periodic variation in j_0 is due to the random turbulent component [Eq. (24)].

Fig. 2. The x dependencies (south to north) of the source term variation j_0/j_{00} (solid) and of the magnetospheric temperature T_m (dashed).

Fig. 3. Maximum upward (solid) and downward (dashed) field-aligned current as a function of time. Saturation (see Eq. 7) is clearly visible.

Fig. 4. Simulated magnetograms. The stations, numbered from 1 to 4, are located at (800,500), (500,500), (200,500), and (−100,500), respectively. Station 2 is thus at the middle of the simulation box. At $t = 0$ the variation field vanishes, these levels are shown by dashed horizontal lines. X, Y and Z are positive to north, east and down, respectively.

Plate 1. The height–integrated Hall conductivity (color chart) and the height–integrated horizontal current (arrows). From left to right and top to bottom: situation at t = 40, 43, 46, 49, 52, 55 min. The color chart for the Hall conductivity is shown below the panels. The longest arrow corresponds to 4.8 A/m current.

Plate 2. The situation at t = 50 min. (Upper panel) height–integrated horizontal current (arrows), field–aligned current (color chart). (Lower panel) horizontal electric field (arrows), height–integrated Hall conductivity (color chart). The longest arrow in the upper panel corresponds to 2.7 A/m horizontal height–integrated current and 230 mV/m horizontal electric field in the lower panel. Positive values (red) indicate upward field–aligned current in the upper panel.

Plate 3. Comparison of the 180x180, $\Delta t = 0.5$ s, and 90x90, $\Delta t =$ 1.0 s runs. The plotted quantity is the magnetospheric vorticity function ω in units of s^{-1}. The upper panel shows the middle 180x90 stripe of the 180x180 run and the lower panel the result of the 90x90 run.

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TABLE 1. Simulation Parameters

Symbol	Value	Explanation
Δt	1.0 s	time step
L_x	1000 km	simulation box size ($= L_y$)
Δx	11.2 km	grid spacing in x ($= \Delta y$)
L_0	200 km	width of the potential step
Σ_0	3 mho	background Σ_H,Σ_P
E_0	30 mV m^{-1}	maximum ambient E (ionospheric)
H	65000 km	thickness of equatorial layer
ν	10^8 m^2 s^{-1}	MHD-viscosity
ρ	$1.7{\cdot}10^{-20}$ kg m^{-3}	magnetospheric mass density
B_i	60000 nT	ionospheric magnetic field
B_m	40 nT	MHD layer magnetic field
K_0	10^{-9} mho m^{-2}	unsaturated K
ϕ^{thr}	10 kV	FAC saturation threshold
α	$3{\cdot}10^{-13}$ m^3 s^{-1}	recombination constant
n_0	$1.3{\cdot}10^6$ m^{-3}	equivalent source plasma dens.
j_{00}	10 μA m^{-2}	source term magnitude
T_m^{min}	0.5 keV	minimum mspheric el. temperature
T_m^{max}	2.0 keV	maximum mspheric el. temperature
dx_m/dx_i	41	x-stretch factor
dy_m/dy_i	36.5	y-stretch factor
λ_i	55 km	background resistive length

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