A statistical study of nightside inverted-V events using Freja electron data: Implications for the current-voltage relationship

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Abstract

We study a set of 47 nightside inverted-V events using the Freja F7 (TESP) electron detector. For each data point, we determine the peak potential, the source plasma density and temperature, and the field-aligned current density. We show that the temperature and peak potential are correlated, which is in line with previous studies: the temperature is on the average 3.3 times smaller than eV, where V is the peak potential. On the contrary, the current density is statistically quite independent of the potential. Thus, based on Freja F7 data the observational current-voltage relationship is that the current is constant with respect to the voltage in an average sense.

1 Introduction

Bright auroras, especially auroral arcs, are caused by field-aligned downward streams of electrons (e.g., [*Bryant,* 1999]). Because of the shape of the electron energy spectrum, one calls such structures inverted-V's [*Frank and Ackerson,* 1971]. The field-aligned electrons, being much more mobile than ions, carry an upward field-aligned current j. The energy spectrum of inverted-V electrons typically has a single peak, which is usually interpreted as being evidence for a field-aligned potential drop [*Evans,* 1974].

The relationship between the current density j and the peak potential V (also called the potential drop or the peak voltage) is called the (observational) current-voltage (CV) relationship. The CV-relationship can be determined by a direct measurement of both quantities in a sample of inverted-V events. The voltage V can be determined from the peak position of a low-orbiting spacecraft electron energy spectrum (see [*Olsson et al.,* 1998] and the references therein), or, more indirectly, inferred from ionospheric radar measurements [*Olsson et al.,* 1996]. The current j can be integrated from satellite electron data or estimated from satellite magnetometer, for instance.

A model for the CV-relationship is needed in many kinds of numerical and analytical investigations of mesoscale ionosphere-magnetosphere coupling processes (e.g., [*Lysak,* 1990]), including global MHD simulations (e.g., [*Raeder et al.,* 1995, *Janhunen and Koskinen,* 1997]). Thus knowing the CV-relationship from both observational and theoretical viewpoints is important for many important areas of auroral and magnetospheric physics.

Already a long time ago, a *theoretical* CV-relationship was derived which gives j as a function of V under the assumptions of isotropic Maxwellian source plasma, adiabatic motion and a potential drop V between the ionosphere and the magnetosphere [*Knight,* 1973]. Under certain assumptions, the full Knight formula can be written in the approximate form $j = KV$, where

$$K = \frac{e^2 n_{\rm src}}{\sqrt{2\pi m_e T}} \tag{1}$$

[*Fridman and Lemaire,* 1980, *Lundin and Sandahl,* 1978, *Janhunen and Olsson,* 1998a]. Here, $n_{\rm src}$ is the source plasma density and T is the source plasma temperature in energy units. Later, rocket measurement suggested that the *observational* CV-relationship is linear, i.e. that j/V is roughly constant within a given event [*Lyons et al.,* 1979].

Sometimes one encounters the point of view that "since both observations and theory support a linear CV-relationship, it is most probably valid". However, there are several possible misconceptions here:

1. If $j = KV$ was a valid theoretical CV-relationship, it would *not necessarily* imply that the observational CV-relationship is linear, i.e., that j/V is constant during one event or arc-crossing. This would be the case only if $K \sim n_{\rm src}/\sqrt{T}$ (Eq. 1) is constant. We know that the source plasma properties may be different at the center of the arc as compared to the edges, thus K could vary as well. More formally, if s denotes a coordinate across an arc, $n_{\rm src} = n_{\rm src}(s)$, $T = T(s)$ and $V = V(s)$ whence $K = K(s)$ and $j = j(s)$. There is no observational way one can separate the V-dependence out, unless one knows each of the functions $n_{\rm src}(s)$, $T(s)$, $V(s)$, which is usually not the case.

2. An accurate approximation for the full Knight formula is $j = K(V + T/e)$ [*Janhunen and Olsson,* 1998a]. Dropping the inhomogeneous term to obtain $j = KV$ is motivated only if $T \ll eV$.

Thus, if one uses the phrase "linear CV-relationship" one must define the meaning carefully: does an inhomogeneous term still count as "linear", and does one mean the observational or theoretical CV-linearity.

In this paper we address the CV-relationship from the observational point of view: we measure V and j independently in many inverted-V events using the Freja satellite. This gives as valuable basic knowledge, but notice that it is *not* a test of the Knight formula: whatever result we obtain for the observational CV-relationship, it can always, in principle, be explained by a relationship of the form $j = K(V + T/e)$ (or even $j = KV$) where K is defined by Eq. 1, if one assumes suitable functional forms for $n_{\rm src}$ and T.

2 Instrumentation

The Swedish-German satellite Freja was launched in 1992 [*Lundin et al.*, 1994]. The inclination of the orbit was 63°, the apogee was 1750 km and the perigee was 600 km. All the events in this study are from northern auroral oval crossings. These crossings were usually rather tangential and took place close to the apogee (1750 km altitude), which is almost always below the acceleration region. The Two-dimensional Electron Spectrometer (TESP) on the Freja F7 experiment [*Boehm et al.,* 1994] measures simultaneously in 32 directions sweeping over 32 discrete energy levels between 20 eV and 25 keV.

Freja is an ideal spacecraft for gathering a lot of data points from inverted-V events since its electron detector F7 (TESP) has a high time resolution (31.5 ms, 32 samples per second) and because the orbital characteristics of Freja are such that the spacecraft often moves tangentially to the auroral oval and thus spends a longer time in the inverted-V region.

It has been shown that different methods of determining the field-aligned current from Freja data generally agree [*Olsson et al.,* 1998]. In this study we choose to determine j from the electron data, because this method is not sensitive to the orientation of the current sheet, which would be an issue if the magnetometer method was used.

3 Data analysis

In this section we explain how the fittings of Maxwellian particle fluxes to Freja electron spectra were carried out in practice.

Fitting Maxwellian and kappa distributions to low-orbiting satellite data is rather complicated because one has to locate the primary peak position from secondary electron background and take into account possible missing and erroneous data points and multiply peaked spectra. For these reasons, in our earlier studies [*Olsson et al.,* 1998, *Olsson and Janhunen,* 1998] the peak position was found manually, the fit was done by a fitting routine and then either accepted or rejected using manual inspection. In [*Olsson et al.,* 1998] we used Maxwellian fitting in logarithmic space, which yields linear fitting routines. In [*Olsson and Janhunen,* 1998] we used both Maxwellian and kappa fits using nonlinear fitting routines. Unlike Maxwellian fitting, kappa fitting can not be linearized by taking the logarithm. When dealing with nonlinear fitting there is the additional complication of finding good enough initial guesses for the parameters. This was again something that had to be done manually.

In [*Olsson and Janhunen,* 1998] it was found that using kappa fitting, the results did not differ too much from Maxwellian fitting results in cases where the spectra were singly peaked. Thus in this paper we are again using Maxwellian fittings and do it in logarithmic space in order to be able to use a linear fitting routine. This saves computer time and frees us from finding initial guesses. We have automated the procedure completely. This enables us to process much larger data sets than before. We have validated the automatic procedure by doing one event manually and comparing the results with the automatically generated ones. The agreement was good, which also shows that the choice of the initial value which was required in the manual fittings did not affect the results. One event typically contains about 1500 data points, for which fitting will be attempted (about 60 s of data with 31.5 ms resolution).

The automatic linear Maxwellian fitting for Freja TESP data works as follows. We neglect the two highest energy channels (20 and 25 keV) because they have turned out to be unreliable. The highest energy level then becomes 16 keV. Then the peak position has to be found. We find it by starting from the highest energy and going down until the flux values stop growing. In order to get a reliable estimate for E_c we require that there are at least 3 energy levels above the peak position. This limits the highest possible peak energy to 8.7 keV. This is no problem in the present study as we limit ourselves to studying stable discrete arcs which do not have very high peak energies.

The peak must be strong enough, meaning that the flux must be at least 10 times higher than the minimum flux above it. Furthermore, the peak position must not be too small (not less than 200 eV). If these criteria are satisfied, then a fit is carried out. The fitting routine produces estimates for the standard deviations for

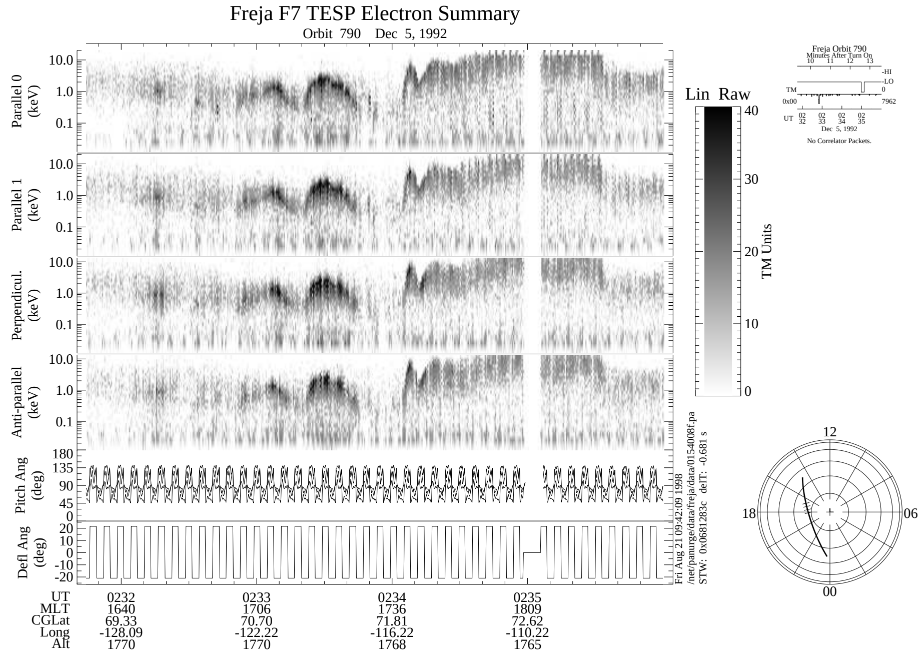

Figure 1: Freja TESP data corresponding to Figure 2. Two inverted-V structures to which the fitting is done are shown in the interval 02:33:00-02:33:50 UT.

the fitted parameters. If one of the standard deviations is larger than 50 %, the fit is rejected. Finally, a fit is rejected if the source plasma density $n^U_{\rm src}$ becomes smaller than 0.01 cm $^{-3}$. Each data point presented below corresponds to one successful fit.

4 Observations

We selected 47 nightside inverted-V events from Freja TESP (F7) data. The events were selected from the whole available F7 data, which extends from November 30, 1992 to September 6, 1994. Events for which the peak energy was higher than 8.7 kV were excluded, for reasons explained in the previous section. This restriction implies that substorm-related events were mostly excluded, because they typically have peak energies larger than 10 kV [*Olsson et al.,* 1998].

Fig. 1 shows TESP data for an event containing two typical inverted-V traversals. Using the fitting routines for the time interval 02:33:00-02:33:50 UT, we obtain the parameters shown in Fig. 2. Besides the potential V, temperature T and the source plasma density $n^U_{\rm src}$, Fig. 2 shows the ratio $a = eV/T$. These kinds of fits were performed for all 47 events.

Each successful fit corresponds to three parameters $(n^U_{\rm src}, V, T)$ where V is the peak position, $n^U_{\rm src}$ is the source plasma density, which is equal to the true magnetospheric plasma density within the loss cone if there is a U-shaped potential drop between the ionosphere and the magnetosphere, and the temperature T depends on how steeply the spectrum falls beyond the peak V.

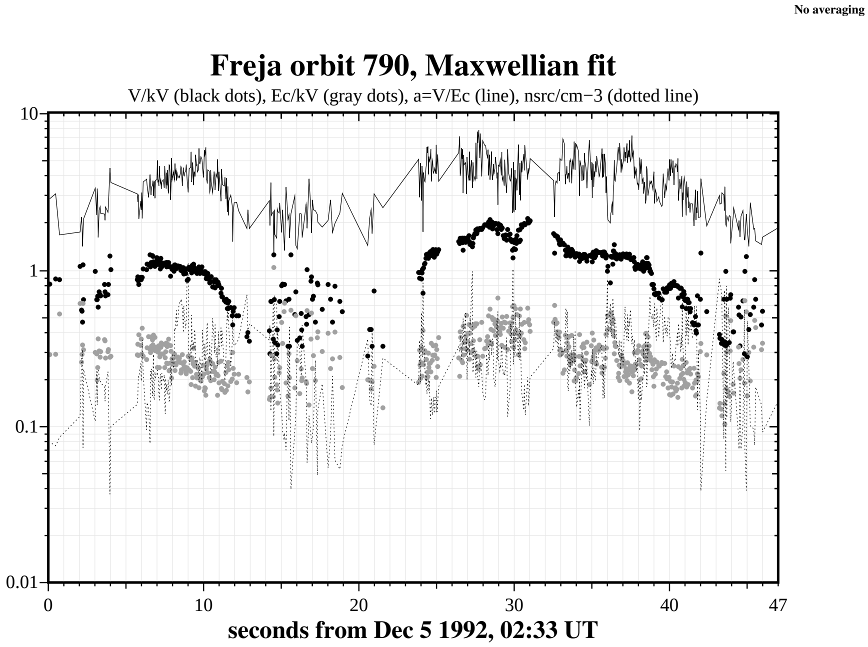

Figure 2: Peak position V (kV, black circles), thermal energy T (kV, open circles), $a = eV/T$ (solid line) and $n_{\rm src}^{U}$ (cm $^{-3}$, dotted line) for Freja orbit 790 during 02:33:00-02:33:47 UT. This event was also analyzed in [*Olsson et al.,* 1998] using manual fitting.

4.1 TV-relationship

Fig. 3 shows the temperature-voltage (TV) relationship for all events. We see that T and V are clearly correlated, which is in line with previous results [*Lin and Hoffman,* 1979].

The clustering of data points in Fig. 3 is an instrumental effect related to the discreteness of the energy levels of the TESP instrument. To verify that this is indeed the source of the clustering, we show in Fig. 4 a histogram of the peak position when the flux is a Maxwellian whose accelerating voltage parameter varies continuously from 1 kV to 6 kV. The histogram shows maxima and minima corresponding to the TESP energy levels. We determine the peak by fitting a parabola about the maximum data point. If we simply picked the maximum data point, all the points in a cluster would be equal and the histogram would be composed of discrete lines. Using a higher-order polynomial fit might reduce the clustering, but it would also increase the possibility of generating spurious maxima, a problem that a second order polynomial avoids.

The low-T cutoff which is also evident from Fig. 3 may be an instrumental effect, because the smallest detectable temperature depends on the spacing between adjacent energy channels, which is roughly proportional to $\log(E)$ for TESP. Most data points are concentrated along the fitted line so the error introduced by this is likely to be small; it might yield to a slight underestimation of a.

We used five different methods for estimating $a = eV/T$. A least-squares linear fit to the (V, T) data points gives $T = (0.100\text{keV}) + eV/3.67$ (Fig. 3). Thus, the offset term is quite small, and neglecting it is justifiable. Computing a as an arithmetic average of eV/T gives $a = 3.58$. This method of estimating a might suffer from the drawback that weak inverted-V events (for which the source plasma density is small) have equal weight as strong events. Computing a as an arithmetic average

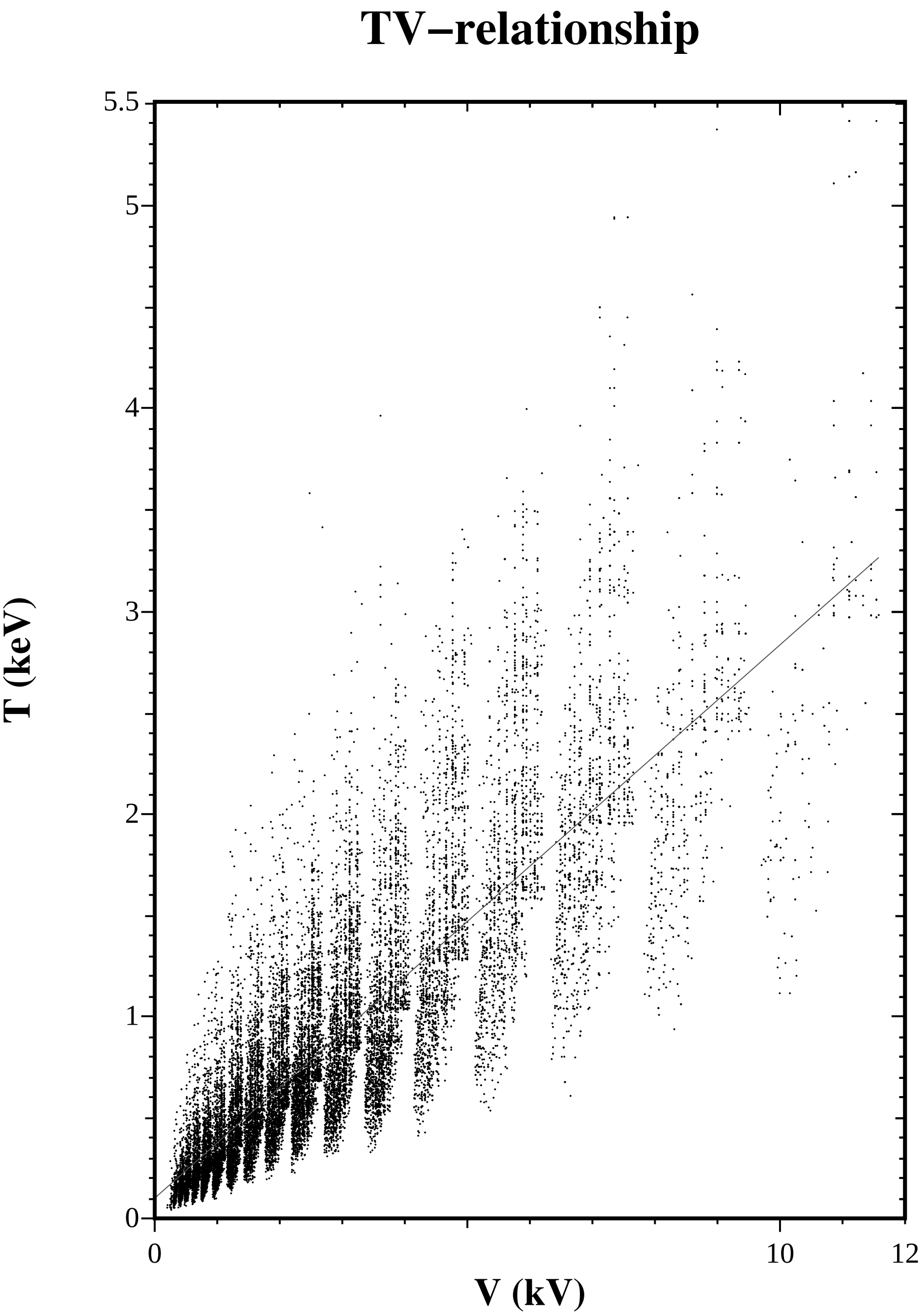

Figure 3: Thermal electron energy T versus peak voltage V for all events (about 28000 data points). A least-squares linear fit to the data points is also shown, the equation of the line is $T = (0.100keV) + eV/3.67$. The clustering of data points in the V axis is due to the discreteness of the energy levels in the TESP instrument. The low-T cutoff may be an instrumental effect, because the smallest detectable temperature depends on the spacing between adjacent energy channels, which is roughly proportional to $\log(E)$ for TESP. Most data points are concentrated along the fitted line so the error introduced by this is likely to be small.

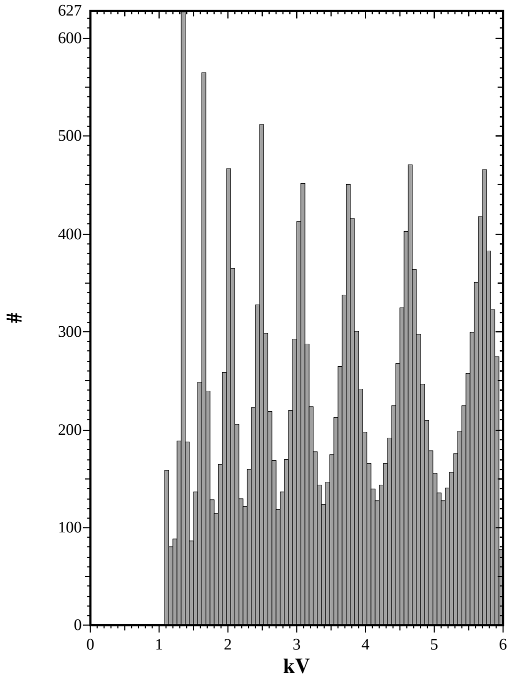

Figure 4: Histogram of the determined peak position by the parabolic fitting method, when the distribution function is a Maxwellian with continuously-varying V-parameter. This explains the origin of the clustering seen in the other figures.

but weighted by $n^U_{\rm src}$ indeed gives a smaller value, $a = 3.35$. One could think that a geometric average would be better than an arithmetic one. A geometric average over eV/T data points gives $a = 3.33$ without weighting and $a = 3.08$ if one weights by $n^U_{\rm src}$. These numbers are in reasonable agreement with earlier studies [*Lin and Hoffman,* 1979].

4.2 CV-relationship

The observational current-voltage relationship is shown in Fig. 5. The current density has been computed from the TESP electron data by integrating the field-aligned electron flux over the peak energy. No statistical dependence can be seen between the voltage and the current: the current varies from case to case, but is on the average independent of the voltage. We will return to the implications of this in the Discussion section below.

4.3 $n^U_{\rm src}$-V relationship

In Fig. 6 we show the source plasma density $n^U_{\rm src}$ versus the voltage V. Since the theoretical CV-relationship is

$$j = \frac{e^2 n^U_{\rm src}}{\sqrt{2\pi m_e T}} \left(V + \frac{T}{e} \right) \tag{2}$$

we obtain, after substituting $T = eV/a$ and solving for $n^U_{\rm src}$

$$n^U_{\rm src} = \frac{j\sqrt{2\pi m_e}}{e^{3/2}} \frac{\sqrt{a}}{1+a} \frac{1}{\sqrt{V}}. \tag{3}$$

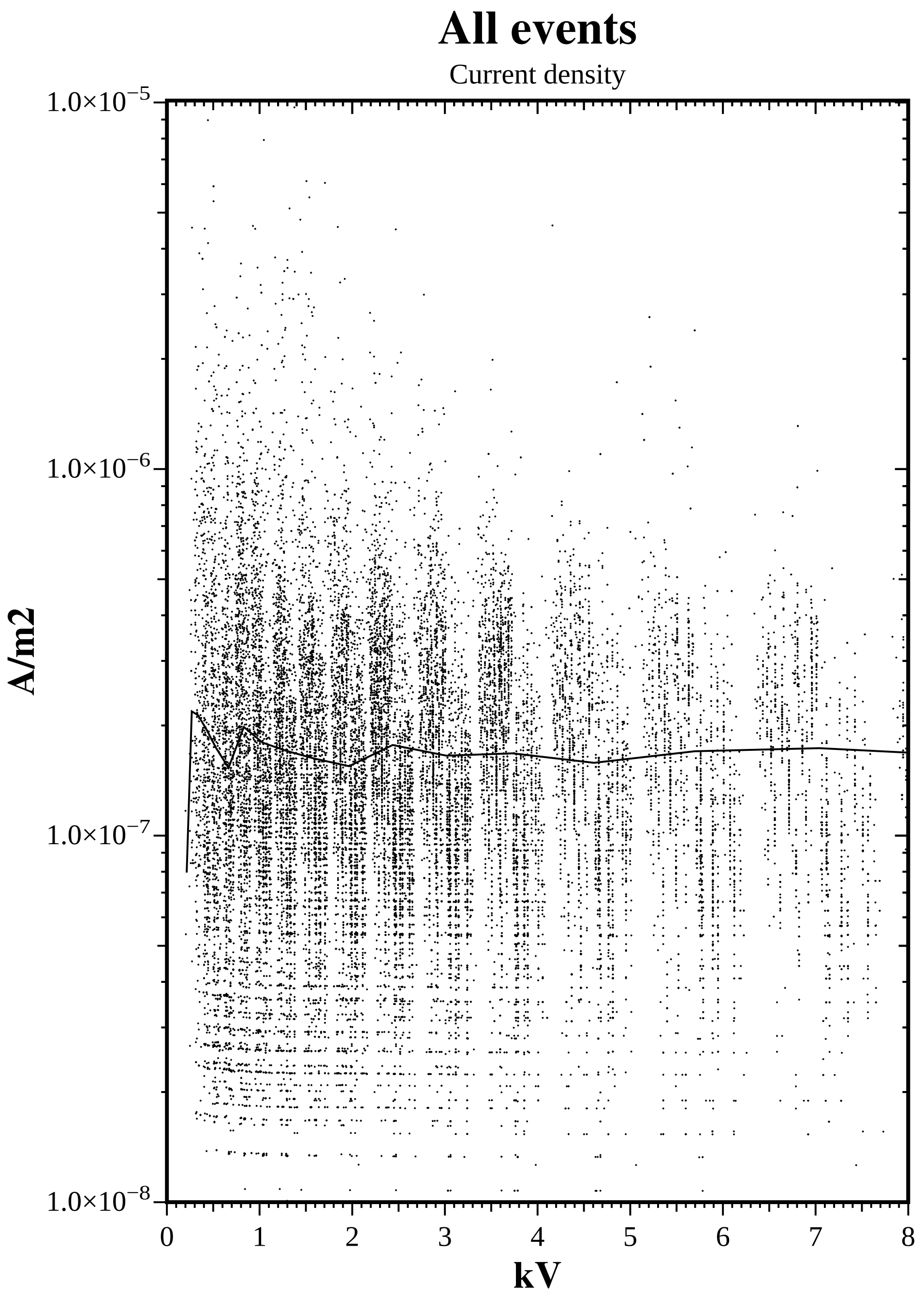

Figure 5: The observational CV-relationship. The horizontal axis is the peak position V and the vertical axis is the current density as obtained by numerical integration of the particle flux above V. The solid line is a geometric average of the data points. Statistically, the current density does not depend on the potential V.

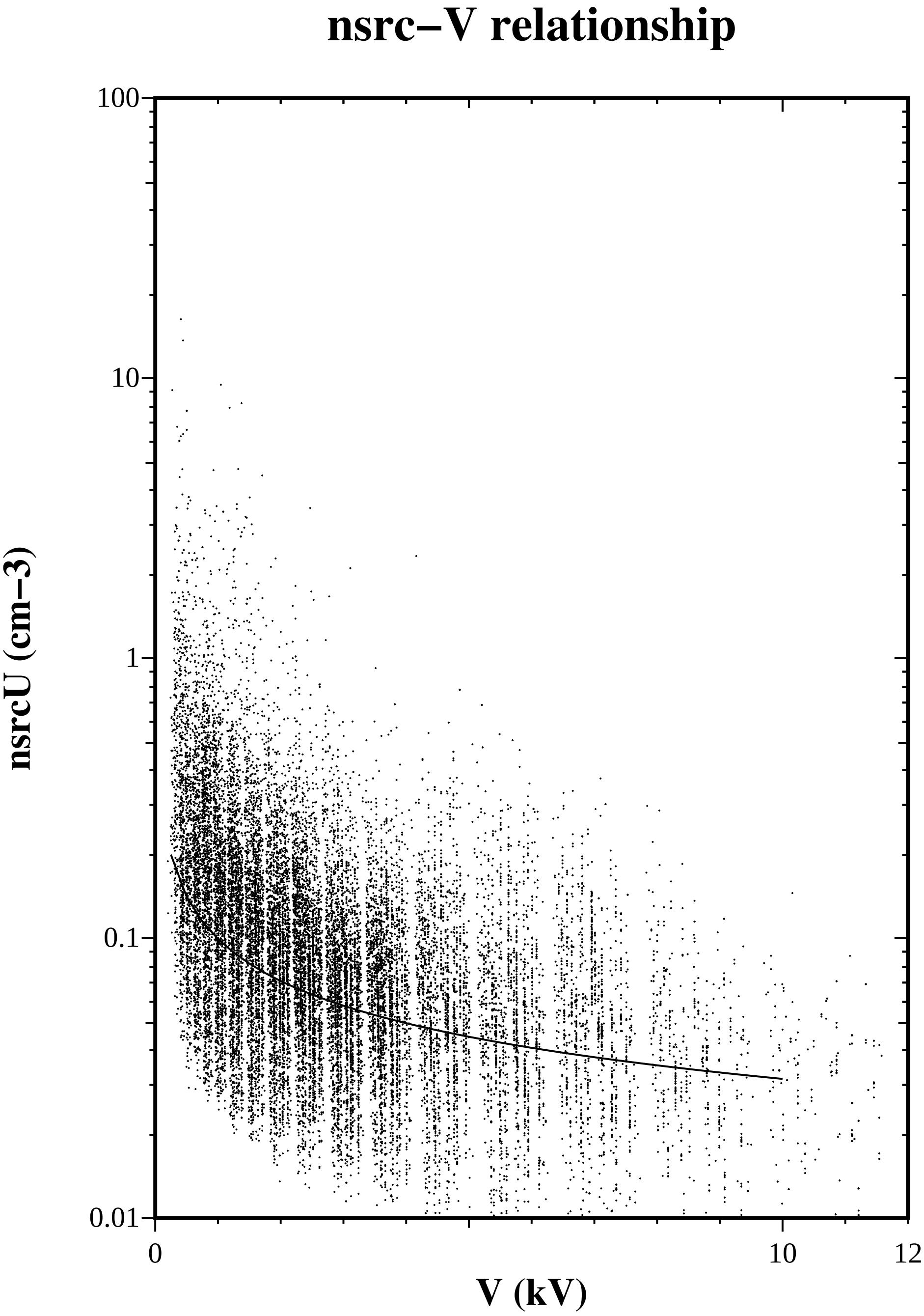

Figure 6: The "U-shaped" plasma density n_{src}^{U} is anticorrelated with the peak energy V. The clustering of points is an instrument effect as in Fig. 3. The line corresponds to a current of 0.2μ A m^{-2} ($\sim$ average from Fig. 5).

The solid line displayed together with the data points in Fig. 6 is Eq. 3 with $j = 0.2\mu$ A m^{-2}, the geometric average from Fig. 5. Thus the downward trend shown by the data points in Fig. 6 is due to the relative constancy of the current density: when V increases, the velocity v of the current carriers increases as $v \sim \sqrt{V}$, thus in order to keep the current density $j = env$ constant, we must have $n \sim V^{-1/2}$. The $n^U_{\rm src}$-V relationship can be thought of as another formulation of the CV-relationship and not an independent result, and plotting it is a useful consistency check.

5 Discussion

5.1 Implications

Thus we have shown that the current density j is statistically independent of the voltage V, while at the same time there is a clear correlation between V and T. This suggests that while the current must play an important role in the formation of arcs, the formation of a parallel electric field (represented by V) is more related to the plasma density $n^U_{\rm src}$ and temperature T than the current. Thus, the current density seems to behave more like an imposed boundary condition than a resultant quantity. As an alternative to using the concept of "CV-relationship" one might instead think of formulas such as $j = K(V + T/e)$ as an "equation of state", which binds the variables $n^U_{\rm src}$, V and T (or j, V and T) together.

Does the behavior of $n^U_{\rm src}$ with respect to V (Fig. 6) represent real source plasma density variations in the magnetosphere? We would need simultaneous measurement in the source region in order to answer this question, but logically, there are at least two possibilities: If the potential geometry above the inverted-V region is U-shaped all the way to the equatorial plane, then the variation shown in Fig. 6 is due to real magnetospheric density variation. If the potential geometry is O-shaped (i.e., geometry is U-shaped at low altitude but the contours close below a certain altitude, e.g. 20000 km) as suggested by recent Polar electric field observations [*Janhunen et al.,* 1999], the quantity $n^U_{\rm src}$ would not correspond to the density deep in the magnetosphere, but rather at the altitude where the center of the O-shaped potential resides.

5.2 Comparisons with previous work

The pioneering observational work on the current-voltage relationship [*Lyons et al.,* 1979] reported a linear current-voltage relationship to be approximately valid in 3 sounding rocket flights, i.e., that the observed precipitating current j depends on the accelerating peak potential V so that j/V is most often constant during each event. In *Lyons et al* (1979) study, the energy flux ε was estimated by integrating over a fixed energy range, and the result was divided by V^2, to get the effective K ($\varepsilon = KV^2$). Since the whole energy range was included, secondary electrons were not explicitly removed. According to the authors, the secondaries do not make a large contribution to the energy flux ε, whereas they do contribute to the current density j significantly.

To compare our Freja events with the results of *Lyons et al* (1979), we plot in Fig. 7 a typical event, which shows a non-constancy of $K_{\rm eff} \equiv j/V$ and actually suggests that the current density is constant. Similar non-constancy of $K_{\rm eff}$ was found in about 75 % of the cases. In the remaining cases (about 25%), $K_{\rm eff}$ was found to be constant within some subregion of the observable energy range. Typically this subregion started from 1.5 or 2 keV and extended up to 5 or 8 keV. An example event is shown in Figure 8.

When all events are put together (Figure 9), the non-constancy of $K_{\rm eff}$ wins.

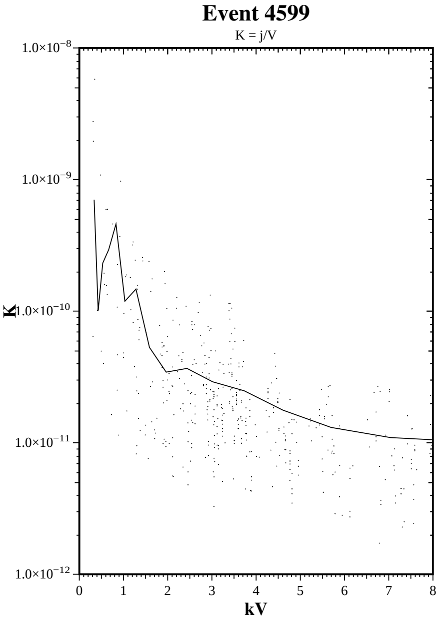

Figure 7: The effective $K_{\text{eff}} = j/V$ parameter for a typical inverted-V event (from Freja orbit 4599). The current density was computed in the same way as in Fig. (5). Between 2 and 8 keV, K_{eff} drops roughly by a factor of 4 which means that the current density j is constant rather than K_{eff}. The line shown is a geometric average.

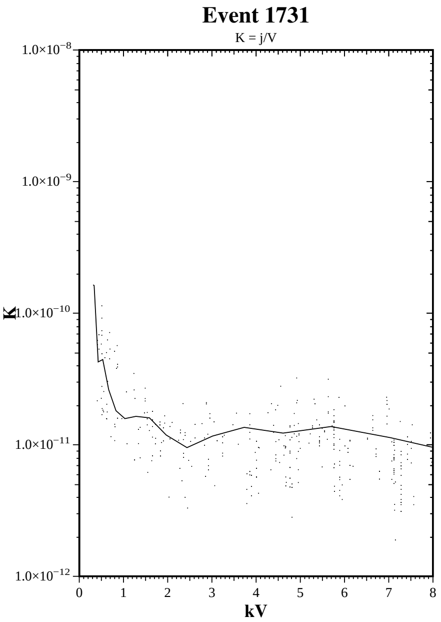

Figure 8: Similar to Fig. 7, the effective $K_{\text{eff}} = j/V$ parameter for the minority type of inverted-V events (orbit 1731), where K_{eff} is roughly a constant over some subregion of the energy interval. In this case the subregion starts from roughly 2 keV.

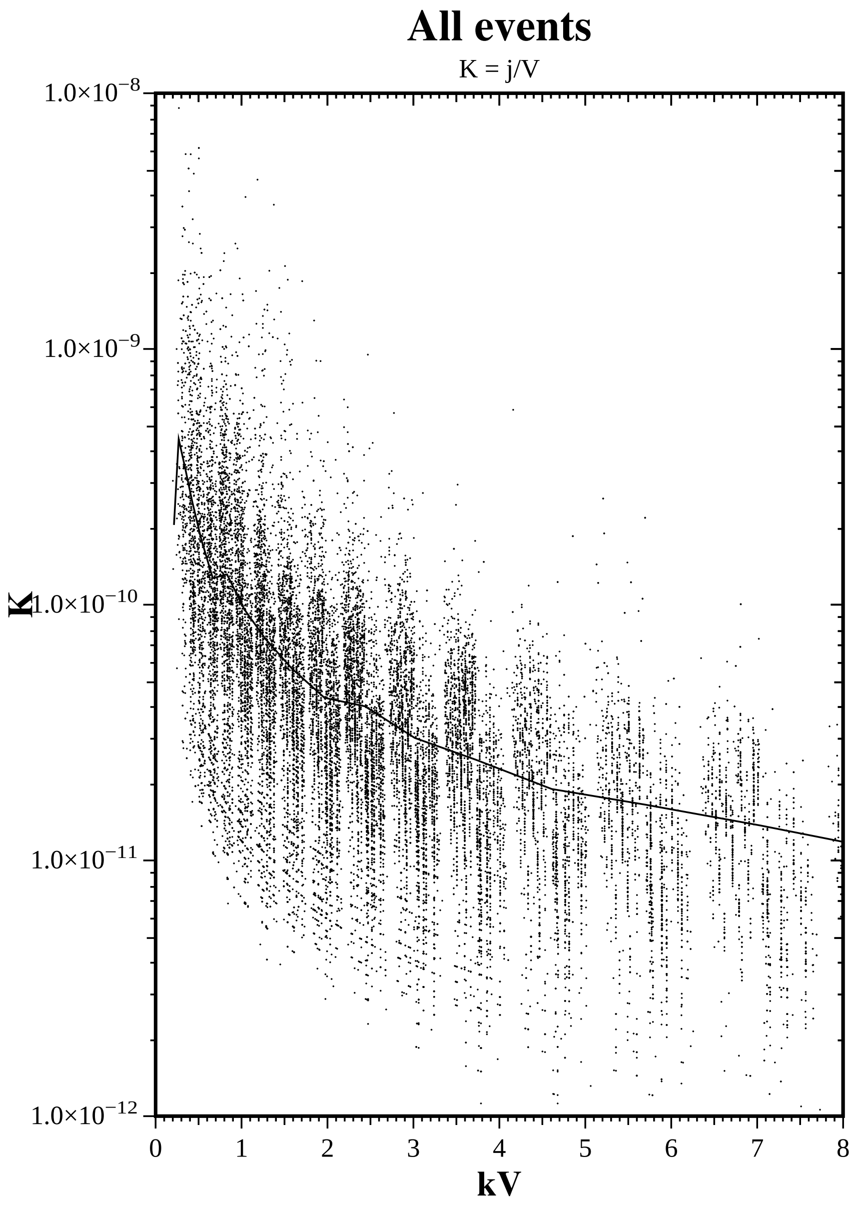

Figure 9: Similar to Fig. 7, but containing all events. K falls down approximately like V^{-1}.

Looking at the geometric average curve in the figure, K_{eff} drops by factor of 8 when V changes from 1 keV to 8 keV. This means that on the average, $K_{\mathrm{eff}} \sim V^{-1}$ so that $j = K_{\mathrm{eff}} V$ is independent of V.

Doing the same as *Lyons et al* (1979) did (including secondaries and using the energy flux) yields results which do not differ very much. The K_{eff} values drop down less steeply than using our standard method, which is to compute the current density rather than the energy flux and include only those particles with energy above V. Between 2.5 and 8 keV, K_{eff} drops by the factor 2.9 using Lyons et al method, while it drops by 3.3 by using our method.

To summarize, the reason why the non-constancy of K_{eff} was not observed by *Lyons et al* (1979) could be due to three factors: (1) Secondaries were not removed by Lyons *et al.* The effect of this is not very large as shown above, but is in the right direction. (2) Lyons *et al* most data points are for energies larger than 1.5 keV. Our results show that K_{eff} falls down more steeply for smaller energies. (3) Lyons *et al* looked at only three events. Since we saw a rather constant K_{eff} above 1.5 keV in about 25 % of the cases, this might also explain the discrepancy.

6 Conclusions

1. The fitting procedure has been automated for Freja F7 data, which enables us to perform a large statistical study. Modifying the procedure for other satellites is not difficult.

2. The peak voltage V and the source plasma temperature T are found to be linearly correlated. Different methods of estimating $a = eV/T$ in our 47 inverted-V events give an average value which varies between 3.1 and 3.6, depending on what type of averaging one uses. The offset in the linear fit is small (100 eV). These are in line with previous findings [*Lin and Hoffman,* 1979].

3. The observational CV-relationship is such that in about 75 % of the cases the current density is independent of the potential. In the remaining 25 % of the cases one can find a subrange of the energy interval (typically this subrange starts from 1.5 or 2 keV), where K_{eff} is roughly constant). One can summarize the result by saying that the current density is statistically independent of the potential, or that the average CV-relationship is that there is "no relationship".

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