SOME OBSERVATIONAL PHENOMENA ARE WELL REPRODUCED BY OUR GLOBAL MHD WHILE OTHERS ARE NOT: REMARKS ON WHAT, WHY AND HOW

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ABSTRACT

We address the question of what features our recent global magnetohydrodynamic (MHD) simulations of the ionosphere-magnetosphere system reproduces well, what not so well, and where the problem areas generally lie in the MHD framework. We show that features which depend on the conservation of mass, momentum or energy, or which involve a balance of forces, are generally reproduced well. The magnetopause position is a typical example. On the other hand, features which depend on different parallel/perpendicular physics, or on the existence of multiple temperatures, are badly reproduced. Unfortunately, many important near-Earth phenomena like auroral acceleration fall in this category. We also discuss some numerical method specific problems: In the Yee-mesh scheme, the upstream leakage of the magnetic field may become a problem, while the volume average schemes usually require an elliptic solver to reduce the divergence of the magnetic field.

INTRODUCTION

Numerical simulations of the whole ionosphere-magnetosphere-solar wind system are most often based on solving the magnetohydrodynamic (MHD) equations. In this paper we address the question of what kind of phenomena are typically well reproduced by such simulations and where, on the other hand, the problem areas lie. Our results are mostly based on our own simulation program (Janhunen, 1996), but the conclusions are most probably applicable to other global MHD codes as well. Our treatment is not comprehensive or complete, but on the other hand we include some material that is difficult to find in the literature. Thus we hope to give the reader some idea of the level of usefulness of MHD simulation results for studying various problems in near-Earth space plasma physics.

WHAT MHD REPRODUCES WELL

MHD simulations generally reproduce well the overall shape of the magnetosphere, together with its dynamical behavior in a variable solar wind. This includes the magnetopause location, the dayside Chapman-Ferraro current system, the cross-tail current sheet, and the magnetopause current. The various plasma regions bounded by these current sheets are also reproduced: for example the plasma sheet and the tail lobes. As an example of a good correspondence, Figure 1 shows a comparison of the simulated magnetopause location with an empirical model of (Shue et al., 1997).

The Region-1 current system is also well reproduced; its closure path was also studied and it was found that it bends more towards the dayside than what was thought earlier (Janhunen and Koskinen, 1997). During northward IMF the MHD simulations develop a “NBZ” (or Region-0) current system together with a 3- or 4-cell convection pattern, as opposed to the 2-cell pattern prevailing for IMF $B_z \leq 0$ (Janhunen et al., 1998).

Also more complicated features such as the behaviour of the ionospheric projection of the open/closed field line boundary (the polar cap boundary) as a function of the interplanetary magnetic field (IMF) and the ionospheric convection pattern (Janhunen et al., 1998) have been studied and found to correspond to

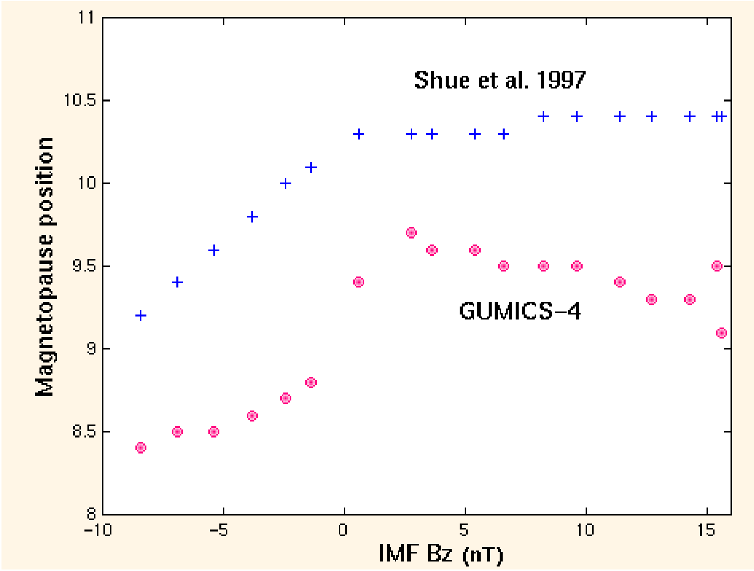

Fig. 1. Comparison of the distance of the subsolar magnetopause from the Earth (in Earth radii) from simulation (filled circles), and from an empirical model (crosses) (Shue et al., 1997), as a function of IMF B_z. Other IMF components are assumed zero, the solar wind velocity 400 km/s and density 7.3 cm^{-3}. Apart from a systematic offset, which may simply come from different definitions of the magnetopause, the agreement is good.

the general picture that we have from measurements. Figure 2 shows four examples of the ionospheric situation together with the polar cap shape for varying IMF. For negative IMF B_z the polar cap is enlarged and round, while for positive B_z it is small especially in the east-west direction. If the IMF vector turns rapidly enough, one may obtain a split polar cap (Figure 2d), which may be related to a transpolar arc event (Kullen, 2000).

For southward IMF, MHD simulations are also able to produce energy release events that have many features common with magnetospheric substorms. The timing of substorms with respect to data is also relatively good, even though it is typical that the substorm in the simulation may occur some minutes too early. The position of the substorm is what one expects: close to the midnight meridian, but slightly on the eveningside. Figure 3 show a 3D view of the 6 keV temperature isosurface during a substorm, while Figure 4 shows the height-integrated Pedersen conductivity during the same substorm. The substorm is seen to be fairly well localized in magnetic local time (Figure 3). Its ionospheric footprint is close to the midnight but slightly on the eveningside (Figure 4).

SCHEME-SPECIFIC PROBLEMS

We now describe some problem areas of MHD that are specific to some numerical solution method. A good MHD solution scheme should be conservative, positive, and preserve $\nabla \cdot \mathbf{B} = 0$ (Janhunen, 2000). By conservation we mean that mass, momentum, and energy should be conserved exactly (i.e., within roundoff error), but not necessarily the volume integrals of the Cartesian magnetic field components. A conservative scheme is very desirable e.g. because it guarantees correct shock speeds (LeVeque, 1992). We say that a scheme is positive if it preserves $P > 0$, $\rho > 0$ for all initial data that satisfy these inequalities. The monopole-free condition $\nabla \cdot \mathbf{B} = 0$ should ideally be exactly preserved in some (one) numerical sense.

A finite-volume method (FVM) discretization of the conservative form of the MHD equation system leads automatically in a conservative numerical scheme. Such a scheme is not usually positive, however. (By saying that a scheme is not positive we mean that there exists some positive initial data that yields a non-positive solution appearing at some point later on. Such a non-positive scheme may still produce a positive result in a concrete problem.) It has been shown recently that by explicitly allowing magnetic monopoles, a conservative and positive MHD scheme can be constructed (Janhunen, 2000).

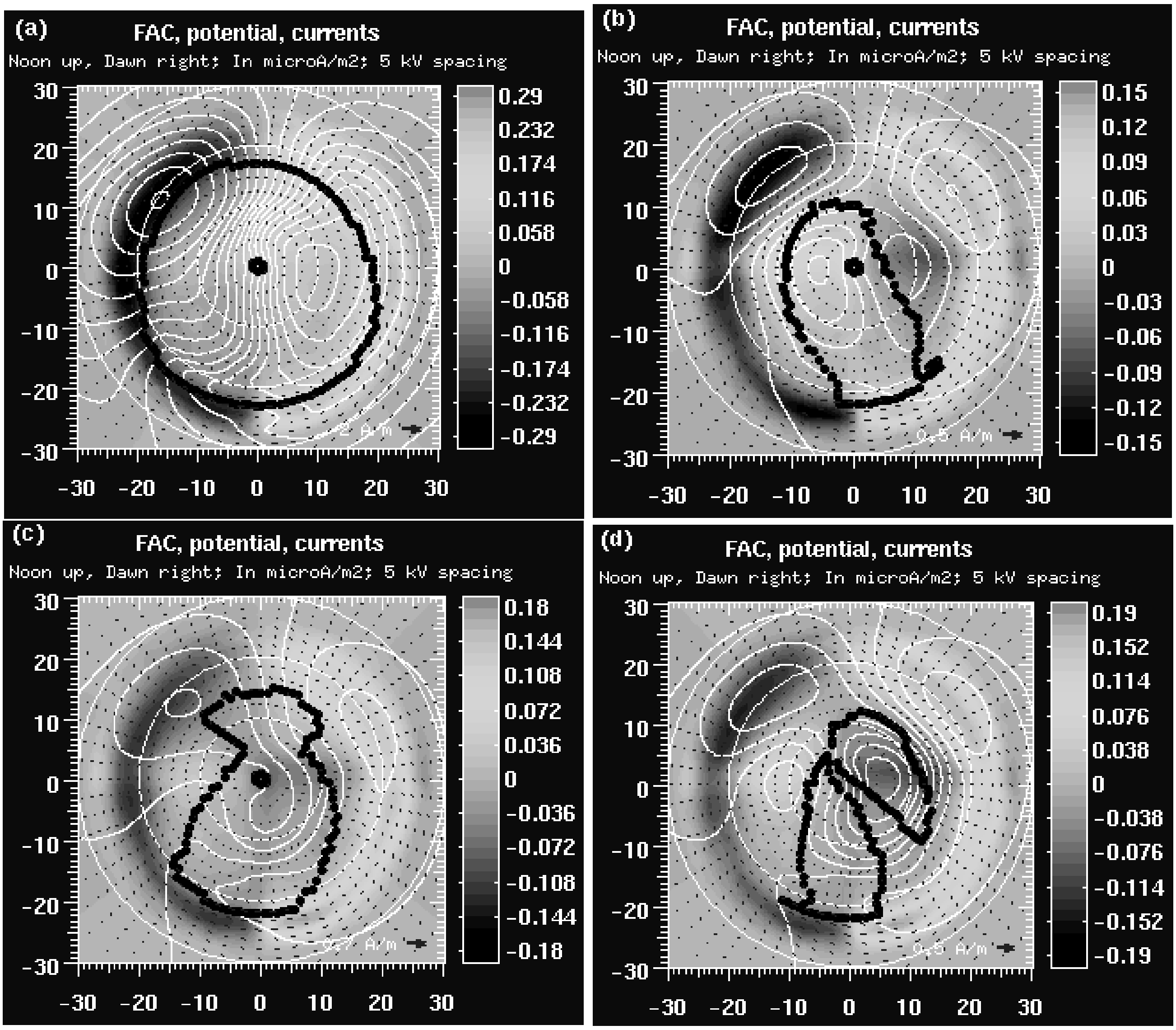

Fig. 2. Polar cap (PC) shape (black curve) for different IMF conditions: (a) large PC during southward IMF, (b) narrow PC during northward IMF, (c) "hourglass"-shaped PC after northward excursion, (d) split PC during IMF rotating in YZ plane (varying B_y). Case (d) may be related to a transpolar arc. Potential contours with 5 kV spacing are shown by white lines, the field-aligned current intensity by dark (upward) and light (downward) shading with scale bar shown on the right at each panel. Horizontal ionospheric current intensities are indicated as small black lines. The magnetic pole is marked with a black dot in cases a,b,c.

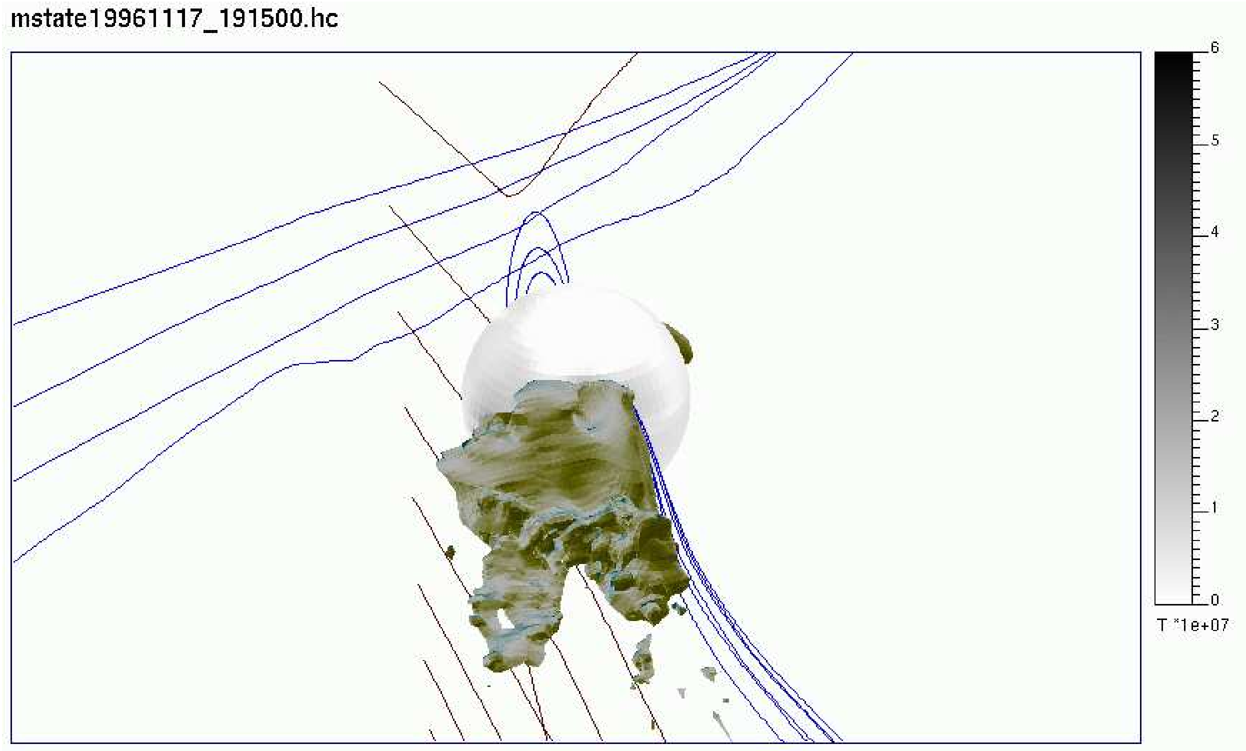

Fig. 3. 6 keV temperature isosurface during a substorm. Some magnetic field lines and plasma streamlines are also shown.

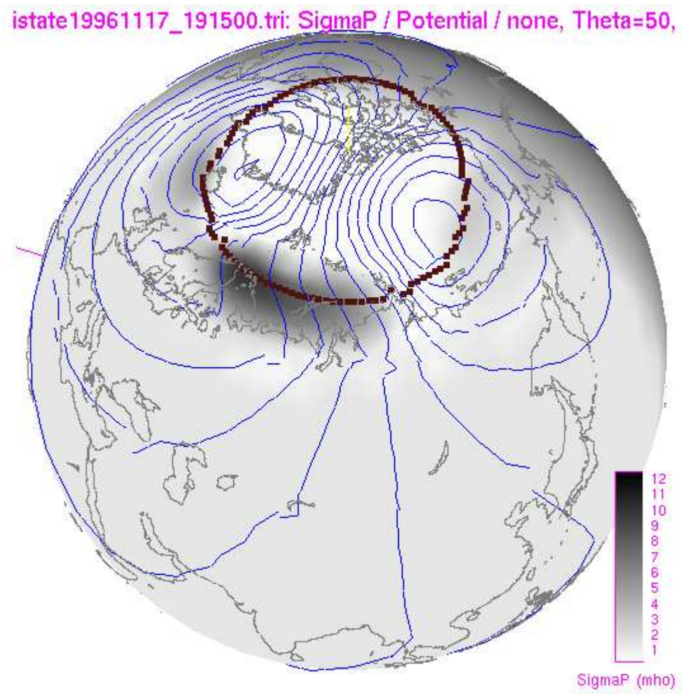

Fig. 4. Height-integrated ionospheric Pedersen conductivity and potential contours during a substorm (simultaneous with Fig. 3). Polar cap boundary shown by grey dots. The auroral bulge over northern Scandinavia is clearly seen.

If positivity of pressure breaks down in a practical calculation at some grid cell, it usually leads into program termination sooner or later, unless some artificial fixes are used to restore positivity. To fix a negative pressure, one has to increase the thermal energy locally. One can do it e.g. by "borrowing" kinetic and/or magnetic energy from the same or a neighbouring cell. Such a fix breaks momentum conservation, but preserves mass and energy conservation. If such an approach is not enough to make the pressure positive, one can abandon energy conservation locally and simply increase the local total energy density U. In many cases these fixes are able to save the simulation from crashing, and often also without a visible local artefact in the results.

Currently there is no known way to maintain $\nabla \cdot \mathbf{B} = 0$ exactly in an FVM scheme on an unstructured grid. In these methods, therefore, one has to periodically call an elliptic solver to clean the curl-free part of $\mathbf{B}$ (Brackbill and Barnes, 1980, Ramshaw, 1983). This is our method of choice for now. Another possibility is to use Powell's method (Powell et al., 1999), which means that one adds source terms proportional to $\nabla \cdot \mathbf{B}$ to the equations whose presence is expected to keep $\nabla \cdot \mathbf{B}$ small without a need for elliptic cleaning. If one calls the elliptic cleaner after every timestep, the results are good, but the elliptic solver phase very probably then takes the dominant fraction of computing time. If the cleaner is called less frequently, its contribution to the computing time goes down, but as the cleaner modifies $\mathbf{B}$ suddenly, it acts like a "hammer" hitting the magnetosphere, causing e.g. spurious Alfvén-waves. Fortunately, in practice a usable compromise can be found between these two problems.

If one abandons the FVM grid for the magnetic field and instead stores the Cartesian components of $\mathbf{B}$ as cell face averages (which we will call the Yee-mesh approach), $\nabla \cdot \mathbf{B} = 0$ can be maintained exactly. The problem which arises then is that the connection between total energy density U and pressure P, $P = (\gamma - 1)(U - (1/2)\rho v^2 - B^2/(2\mu_0))$, involves the components of $\mathbf{B}$. As U and P are cell-averaged quantities but $\mathbf{B}$ is stored on cell faces, one must interpolate $\mathbf{B}$ to cell centers. Guaranteeing that the subtraction of the kinetic and magnetic energy densities from U produces a positive result is difficult enough already when all quantities are cell-averages (cf. Janhunen, 2000). It looks completely hopeless in the case when $\mathbf{B}$ has to be interpolated. Thus a Yee-mesh scheme must monitor the positivity, and possibly make a compromise between conservation and positivity whenever the latter one is violated.

Our experimentation with the Yee-mesh scheme for MHD indicates a new type of problem: the magnetic field tends to leak upstream e.g. at the bow shock. The reason is probably that the face-based discretisation of Faraday's law is inherently spatially symmetric and thus has no upwind detection. Introducing an upwind weighting in the interpolation of $\mathbf{E} = \mathbf{B} \times \mathbf{v}$ at the nodes cures the problem only partly.

An interesting mathematical connection between positivity and conservation is that a positive and conservative scheme is numerically stable in the sense that the quadratic kinetic and magnetic energy terms remain bounded from above.

PHYSICAL PROBLEMS

We will now discuss some of the problems of MHD at the continuum level, independent of the numerical scheme used.

Field-Aligned Density Gradients

Since particles are free to move along magnetic field lines, non-isotropic particle distributions, or parallel electric fields, are required to produce field-aligned density gradients (Janhunen, 1999). Since neither mechanism is explicitly included in MHD, any field-aligned density gradient produced by MHD is potentially unphysical. There is nothing in MHD that would prohibit the formation of such gradients, and indeed we sometimes see them in the simulations, especially in the near-Earth region, as soon as the grid is fine enough to have enough "room" for them to form. Figure 5 shows a rather extreme case of problems that may occur: an "island" of enhanced density has formed, centered at about $X = -6R_E$, $Z = 3R_E$. It is a bit reassuring, however, that even in this case, other variables such as pressure and current distribution still look normal.

Multi-Temperature Auroral Plasma

The plasma on field lines threading the auroral oval is typically composed of overlapping populations having widely different temperatures. The populations themselves may still be close to Maxwellian. For example, in one of the cases studied (Janhunen and Olsson, 2000) the plasma over auroral arcs consists of

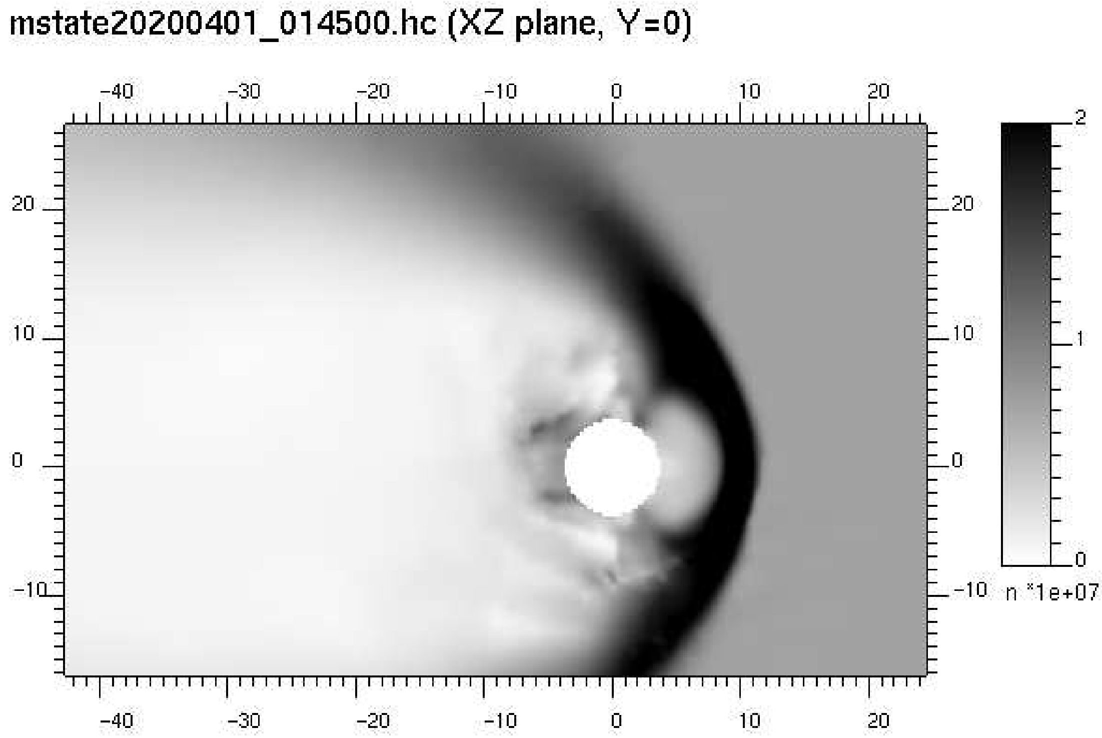

Fig. 5. Extreme case of an MHD problem area: Electron density distribution in the midnight meridian plane for a case where there is a probably unphysical density "island" on the nightside, centered at about $X = -6R_E$, $Z = 3R_E$.

cold electrons (< 50 eV), medium-temperature anisotropic electrons (50-200 eV), and hot electrons (4 kV), whereas for ions there are two populations: cold (< 50 eV) and hot (> 30 keV). The medium-temperature electron population was in this case also highly anisotropic. Any phenomena that depend on the multi-temperature nature of the plasma are not included in the MHD description. For example, the Region-2 current system comes out as too weak from the simulation, the reason being probably that it should originate from the ring current region, and the ring current is carried by non-thermal high-energy particles which are not properly included in MHD (if the hot particles are in reality heated adiabatically, their pressure should eventually be given correctly by MHD, but the time to achieve this may exceed the timescale of the MHD simulation.)

Weakness Of Substorm Current Wedge

Our simulations do produce a substorm current wedge (SCW), but its magnitude is quite weak: in one case we estimated the current to be 30 kA. Recent satellite studies have indicated that most of the upward field-aligned current associated with a westward traveling surge is fed by downward currents near the surge rather than by a more remote eastern downward return current area (Marklund et al, 1998). This means that the net current flowing in the horizontal westward current feeding the surge (which is essentially the SCW) can be correspondingly weaker, which is qualitatively in agreement with our result of a weak SCW. Thus, the weakness of the SCW in the simulation is not necessarily a problem at all.

CONCLUSIONS

We can summarize the findings by saying that features which come from conservation (of mass, momentum, or energy), or which involve a force balance, are generally well reproduced. Examples are the bow shock and the magnetopause. On the other hand, features that depend on different parallel/perpendicular physics, or on the existence of multiple temperatures, are reproduced badly, if at all. Examples are many of the auroral phenomena (such as auroral acceleration), and the near-Earth region in general.

We also found that the Yee-mesh (staggered mesh) and volume average based schemes have different weaknesses and strengths. In the Yee-mesh, the upstream leakage of the magnetic field is a problem, and in the volume schemes it is the need for elliptic cleaning. At the moment it shows to us that the elliptic cleaning, although not very pleasing theoretically, works rather well in practice and produces satisfactory results.

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