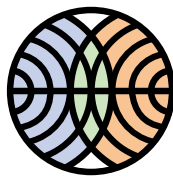


# Determination of ionospheric equivalent currents by the spherical elementary current method

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## Theory - short course

**Basic idea:** construct the equivalent ionospheric current system using a set of *divergence-free elementary current systems* (ECS).

Assume a single ECS in a spherical coordinate system with its pole at  $(r, \theta, \phi) = (R, 0, 0)$ . The current density is

$$\mathbf{J}(\mathbf{r}) = \frac{I_0}{4\pi R} \cot \frac{\theta}{2} \mathbf{e}_\phi \quad (1)$$

Any divergence-free vector field on a sphere can be expressed as a superposition of such ECS. In practice, a discrete set is used.

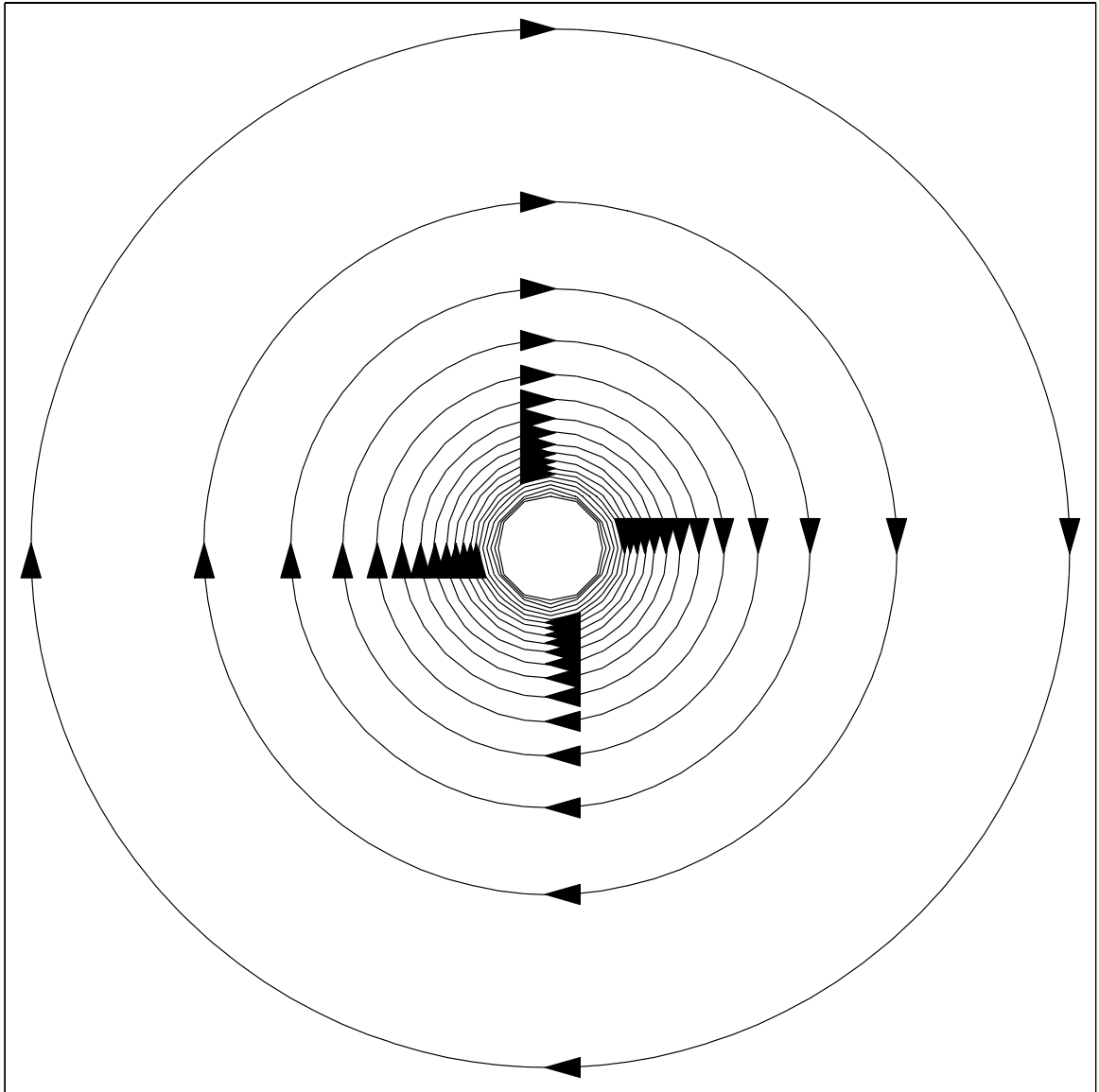


Figure 1: Top view of the current of one divergence-free elementary system (Cartesian case).

## Matrix formulation

Measure the **magnetic field** at

$\mathbf{r}_{m,obs} = (R_g, \theta_{m,obs}, \phi_{m,obs})$  ( $m=1,\dots,M$ ), usually irregularly spaced.

Put a **set of ECS** with poles at

$\mathbf{r}_{n,el} = (R_i, \theta_{n,el}, \phi_{n,el})$  ( $n=1,\dots,N$ ), placed at any points.

Determine the **scaling factors**  $I_{n,el}$  of ECS:

$$\mathbf{T} \cdot \mathbf{I} = \mathbf{B} \quad (2)$$

**T**: geometrical factors (matrix)

**I**: scaling factors (vector)

**B**: observed magnetic field (vector)

The elements of **T** can be calculated in a closed form both in spherical and Cartesian geometries.

The number of observation points ( $M$ ) is usually much smaller than the number of ECS ( $N$ ).

- problem highly underdetermined
- $\mathbf{T}$  badly conditioned

Solve by the [singular value decomposition](#) (SVD):

$$\mathbf{T} = \mathbf{U}\mathbf{w}\mathbf{V}^T \quad (3)$$

where  $\mathbf{w}$  is a diagonal matrix containing the singular values of  $\mathbf{T}$ .

SVD allows to remove badly conditioned parts from  $\mathbf{T}$  by setting small  $w_{mm} = 0$ :

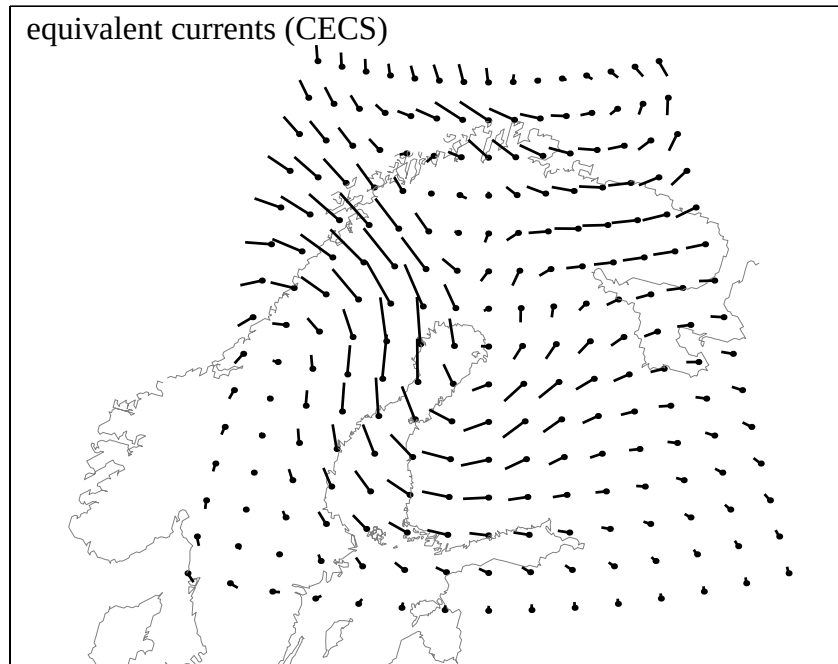
$$\mathbf{I} = \mathbf{V}diag(\tilde{w}_{mm})\mathbf{U}^T\mathbf{B} \quad (4)$$

where  $\tilde{w}_{mm} = 0$  except for

$$\tilde{w}_{mm} = w_{mm}^{-1} , \quad w_{mm} \neq 0 \quad (5)$$

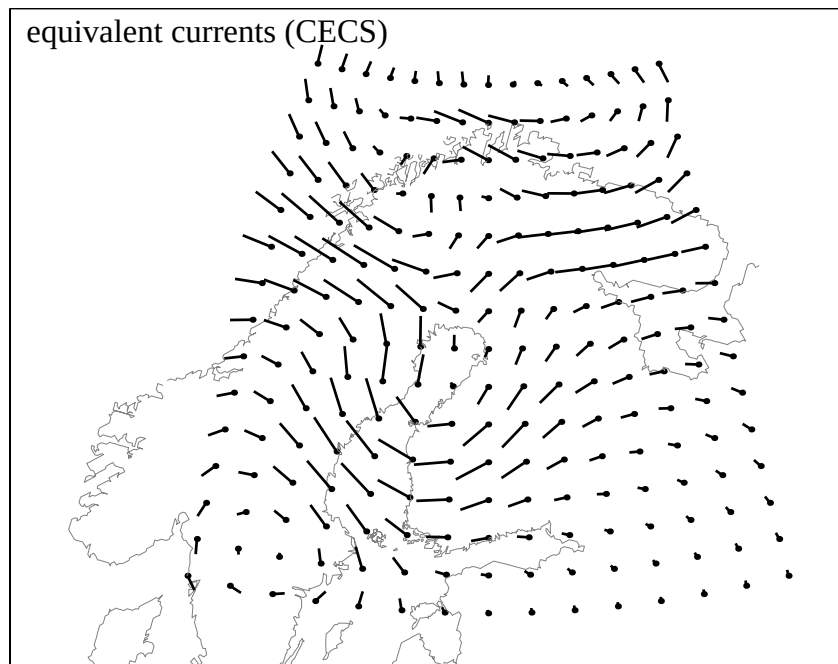
# Example: February 18, 1999

18.02. 1999 (18:35:00 UT)



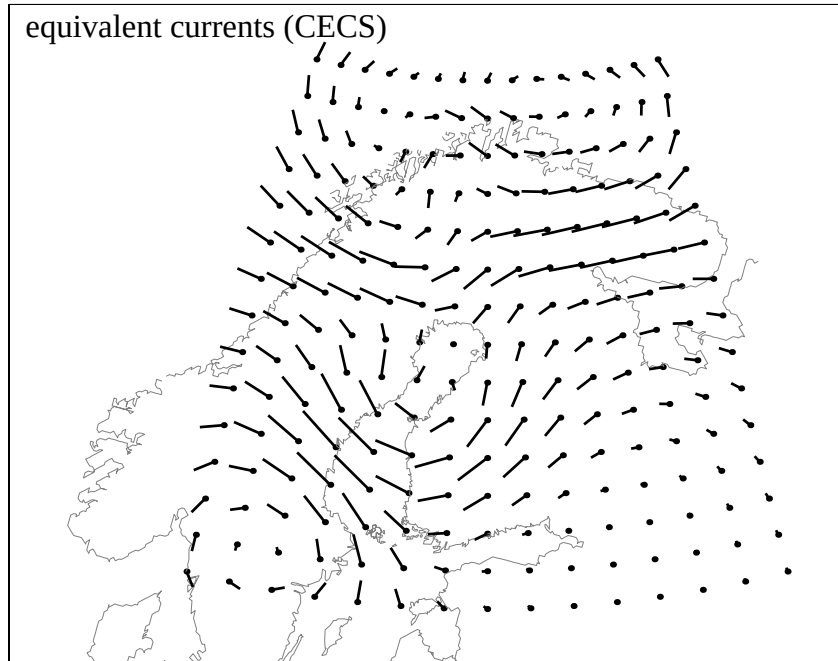
max = 527 A/km

18.02. 1999 (18:36:00 UT)



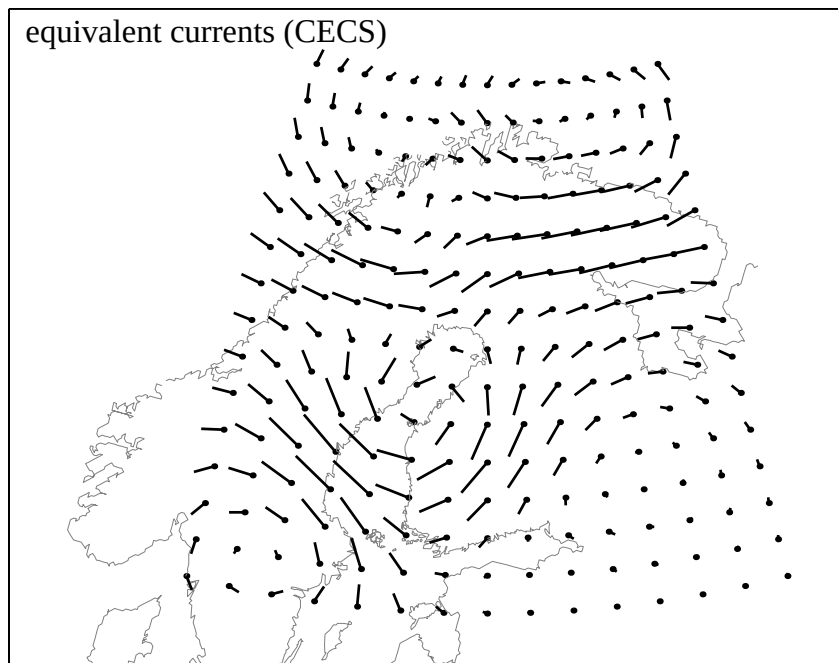
max = 428 A/km

18.02. 1999 (18:37:00 UT)



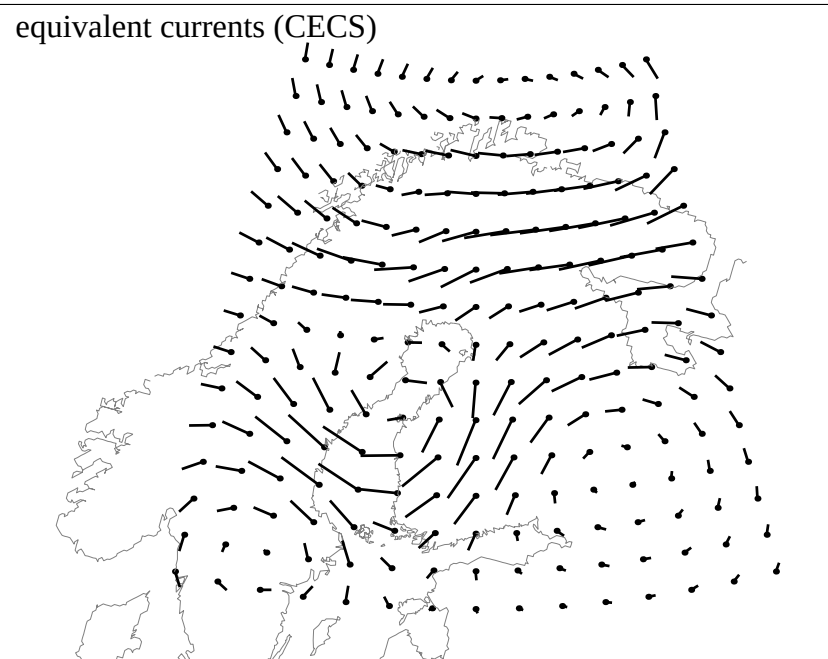
max = 429 A/km

18.02. 1999 (18:38:00 UT)



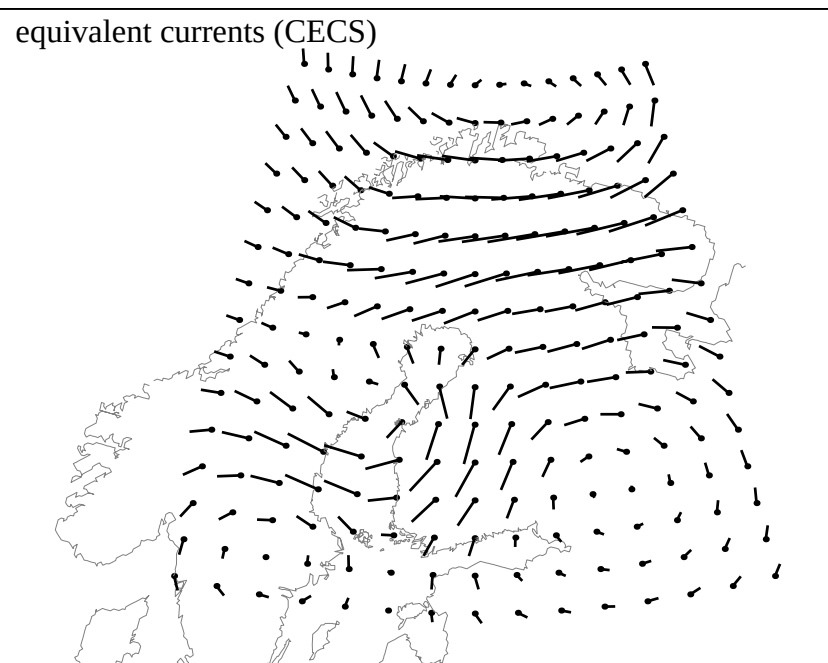
max = 414 A/km

18.02. 1999 (18:39:00 UT)



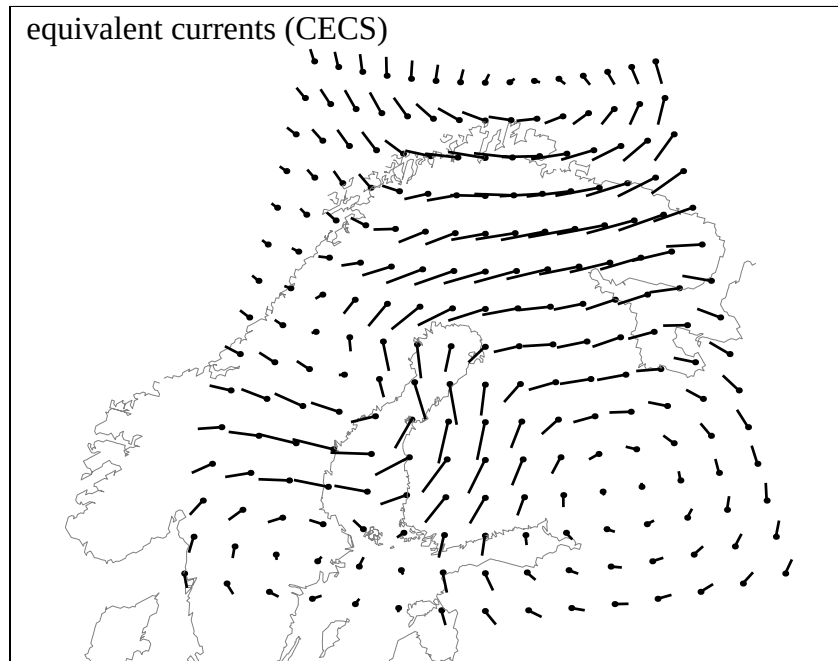
max = 361 A/km

18.02. 1999 (18:40:00 UT)



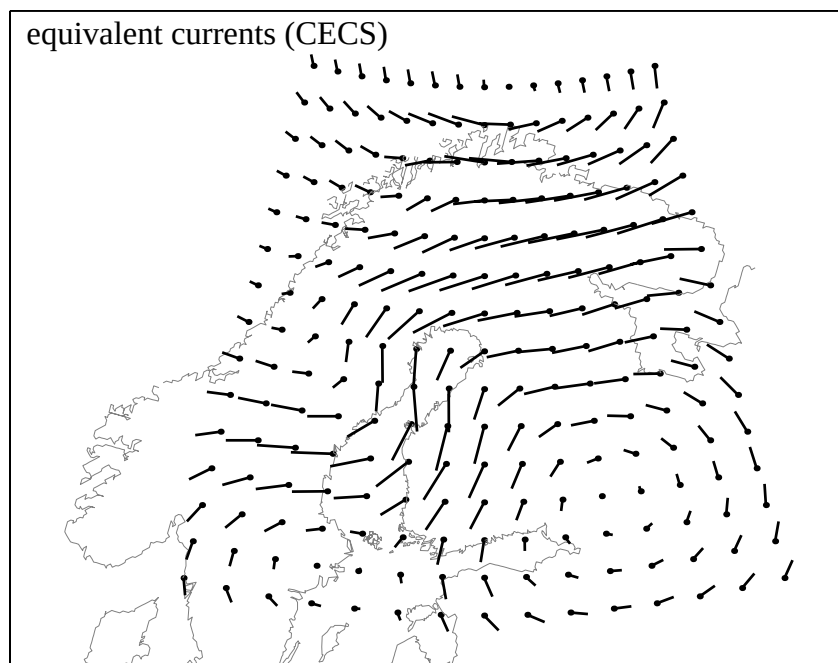
max = 386 A/km

18.02. 1999 (18:41:00 UT)



max = 376 A/km

18.02. 1999 (18:42:00 UT)



max = 371 A/km

## Advantages of the ECS method

(Other methods: Fourier transform, spherical cap harmonic expansion, global spherical harmonic expansion.)

- No fixed limitation of the spectral content in the analysis area.
- Locations of elementary currents can be selected freely.

## Near future tasks

- Derive equivalent currents for (the continental part of) IMAGE since 1994 for each 1-min timestep.
- Integrate the ECS method with the MIRACLE software.

## References

Amm, O., 1997: Ionospheric Elementary Current Systems in Spherical Coordinates and Their Application. *Journal of Geomagnetism and Geoelectricity*, **49**, 947-955.

Amm, O. and A. Viljanen, 1999: Ionospheric disturbance magnetic field continuation from the ground to the ionosphere using spherical elementary current systems. *Earth, Planets and Space*, **51**, 431-440.

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Example animation on WWW:

[http://www.geo.fmi.fi/image/  
cluster/animation.html](http://www.geo.fmi.fi/image/cluster/animation.html)