A scale-based distortion metric for mesoscale weather verification

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Acknowledgements to: Beth Ebert, Alan Seed, and John Bally.

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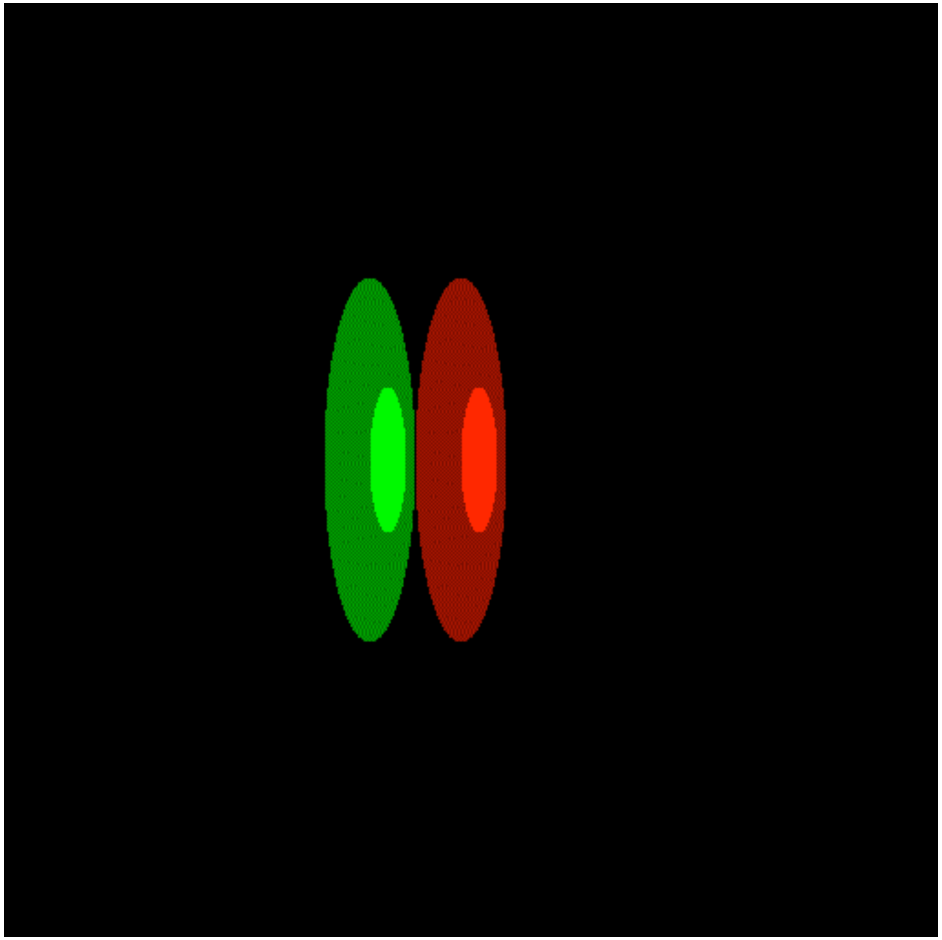
Talk Outline

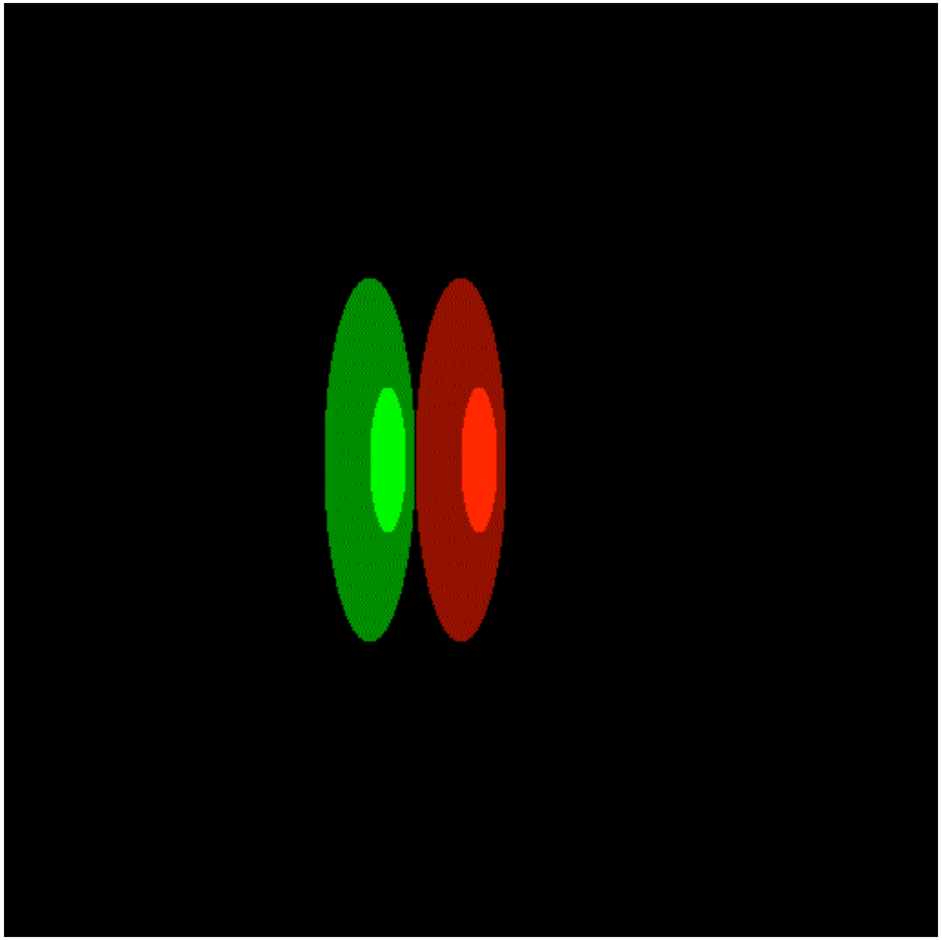
1) Set the scene for distortion metrics
2) Provide some background
3) Highlight assumptions
4) Describe new methodology
5) Show examples

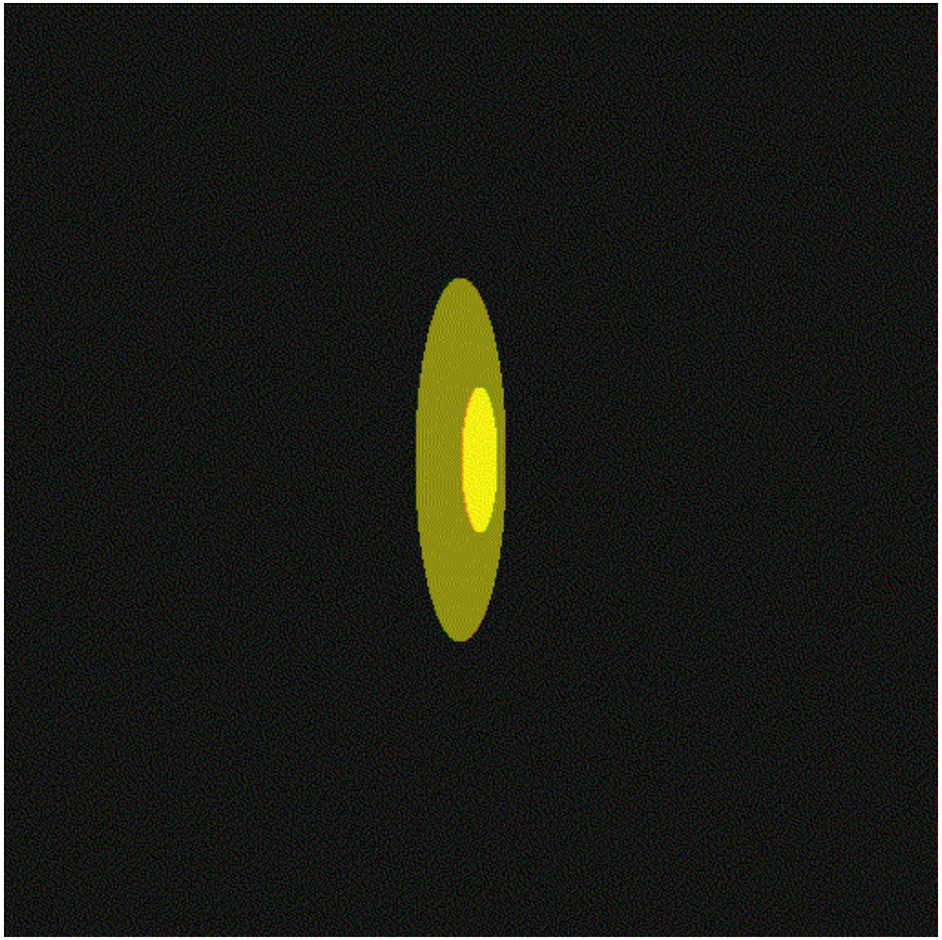
Red – reference image   Green – Image to be warped



Red – reference image Green – Image to be warped

Yellow – Images Superimposed





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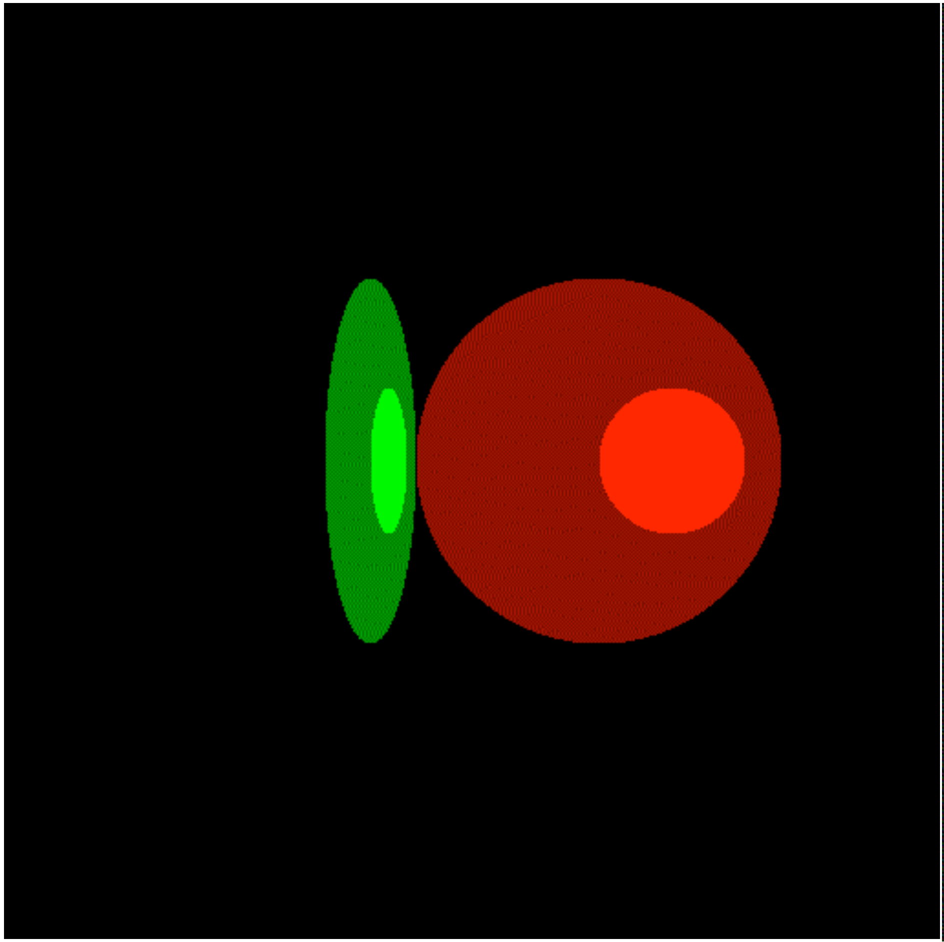
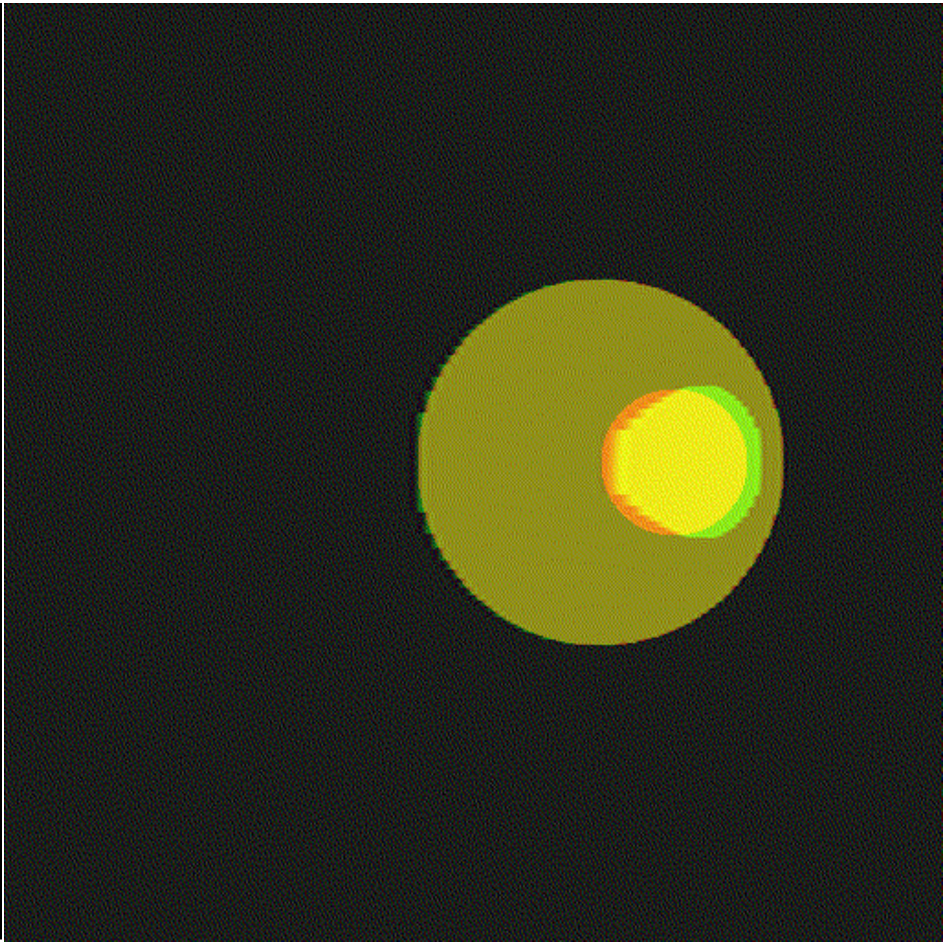
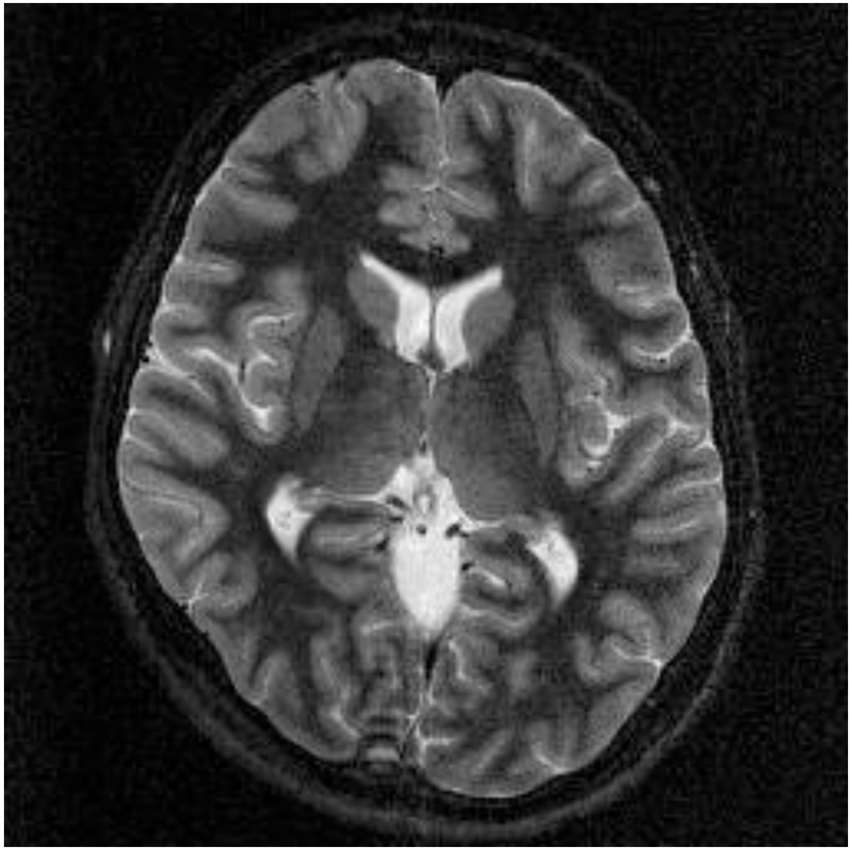




Jan Kybic and Michael Unser, “Fast multidimensional elastic image registration”, IEEE Transactions on Image Processing, 2003.



Reference

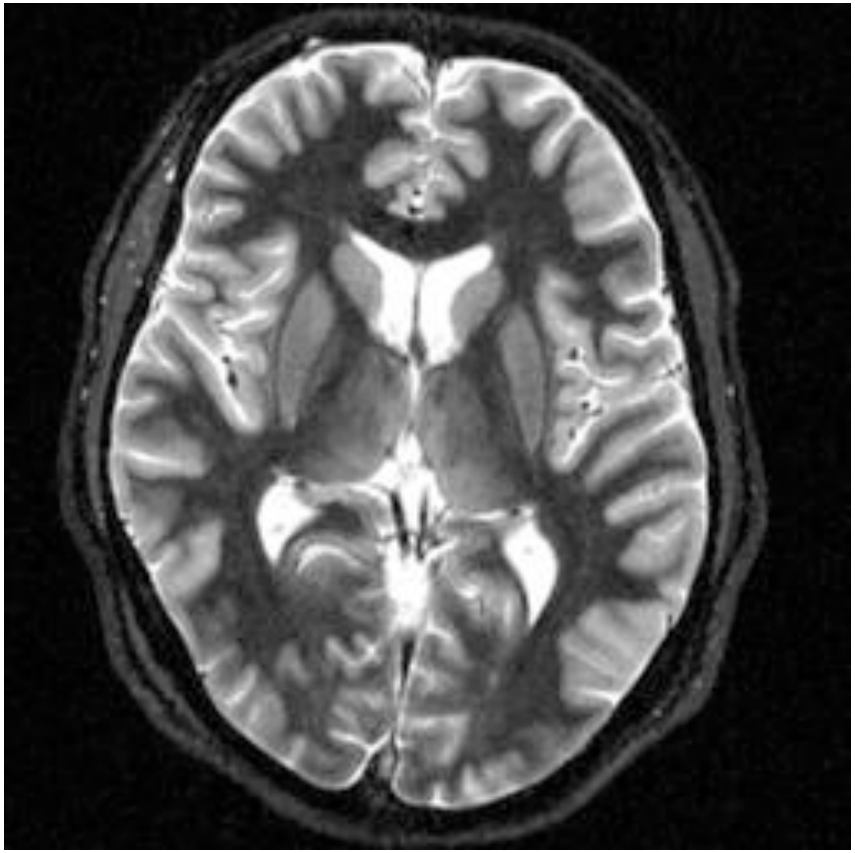


Image to be warped

(Kybic and Unser, 2003)

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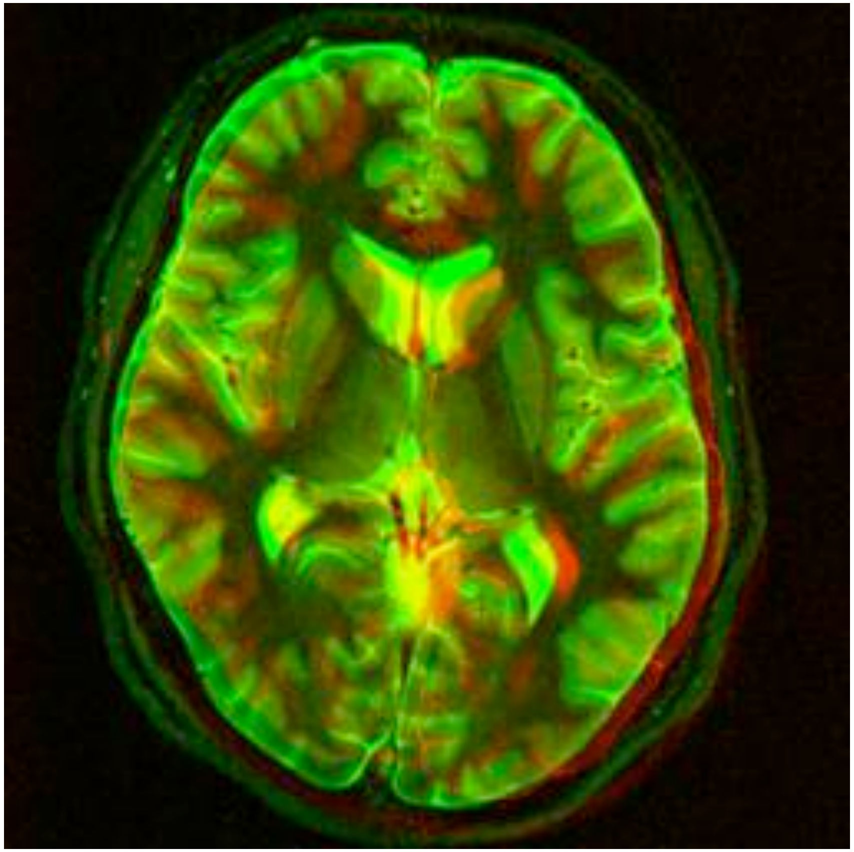
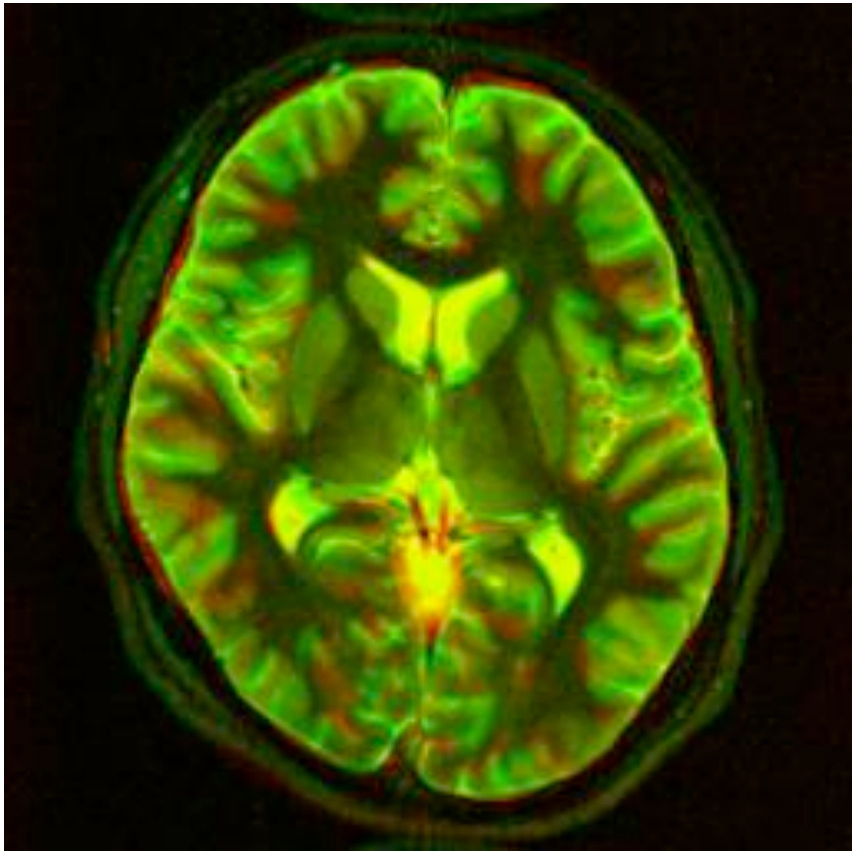
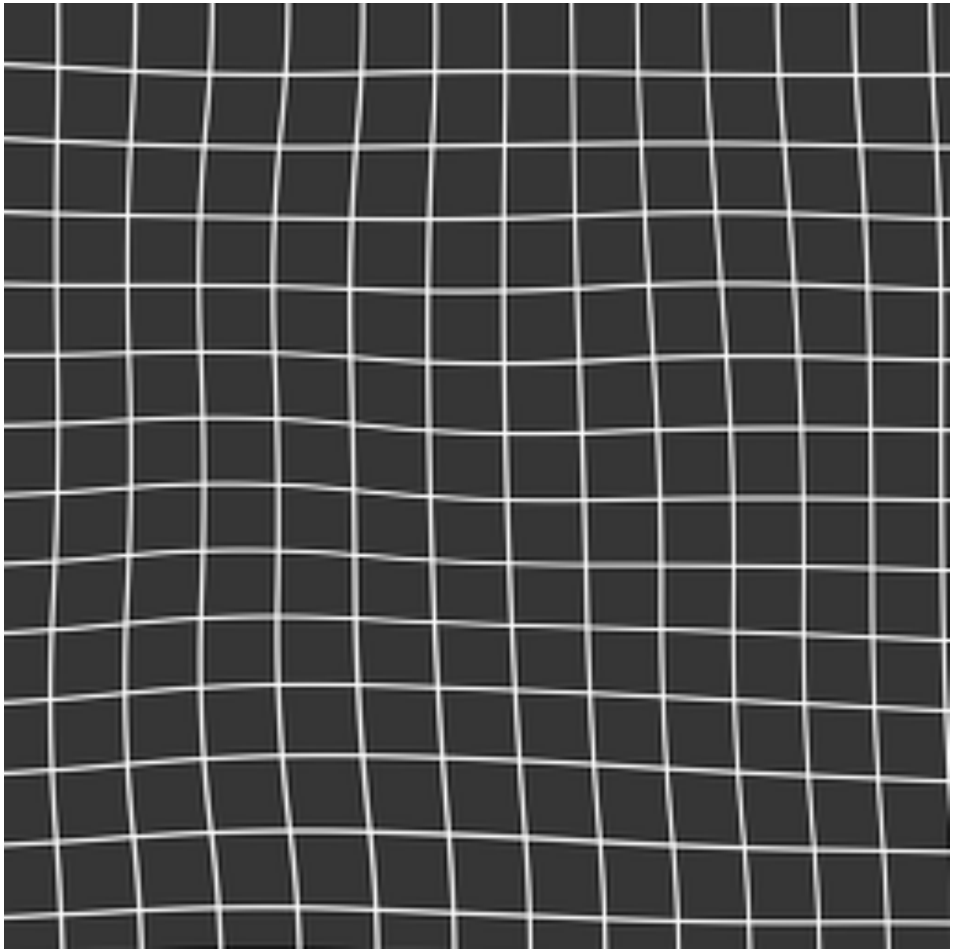


Superposition before Registration

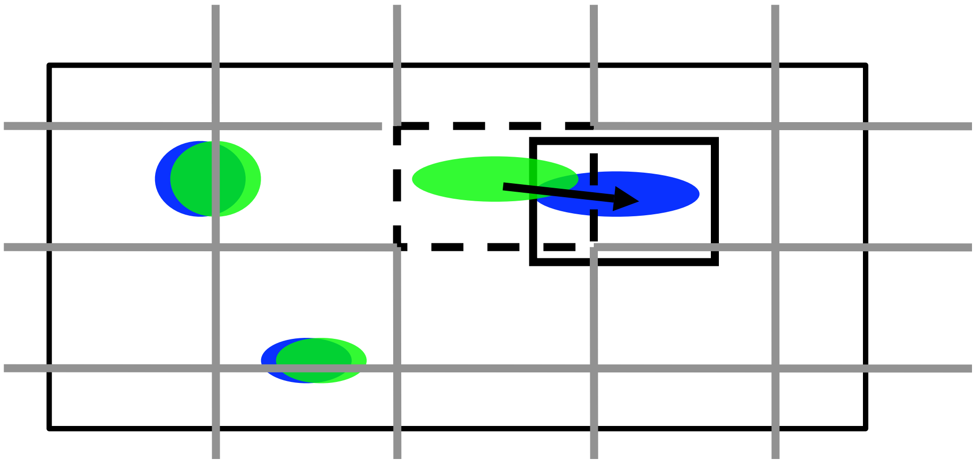


Superposition after

Deformation Matrix



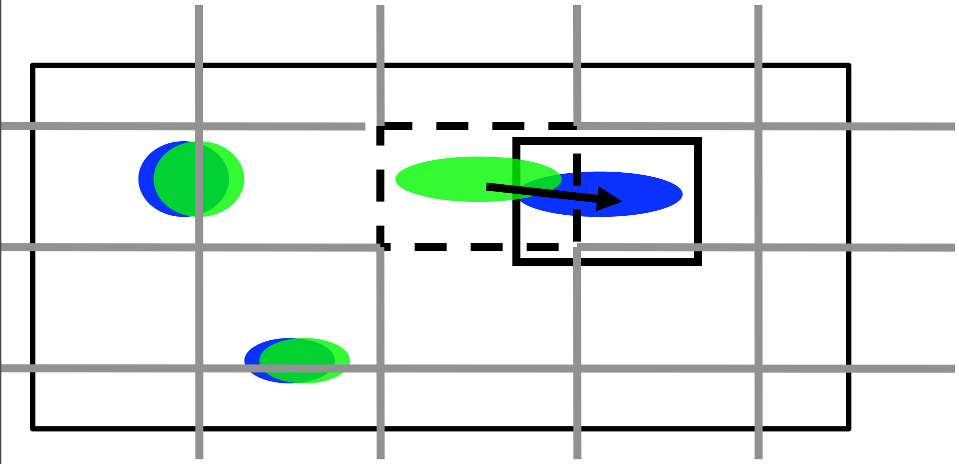
Compare image before and after deformation, see if the difference has reached a minimum. If not, take another step

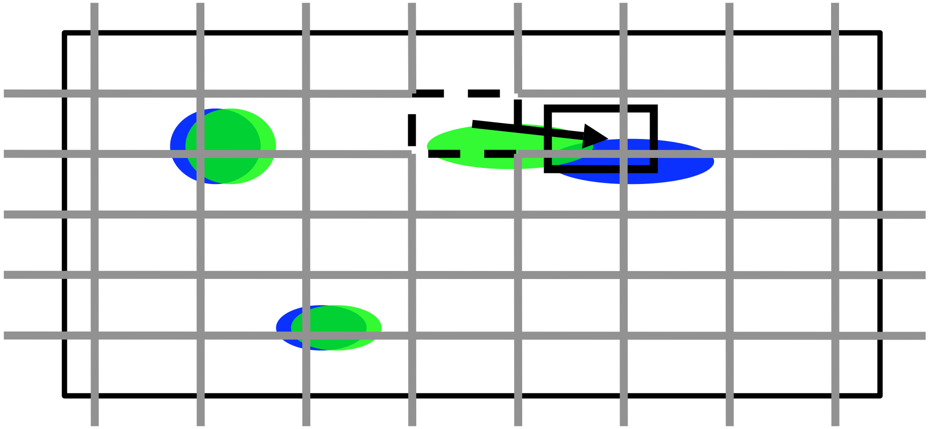


- Use displacement vectors to construct deformed grid locations
- Interpolate deformed image from original image using deformed grid locations

Instead of deforming the grid locations directly, approximate them using B-spline basis functions

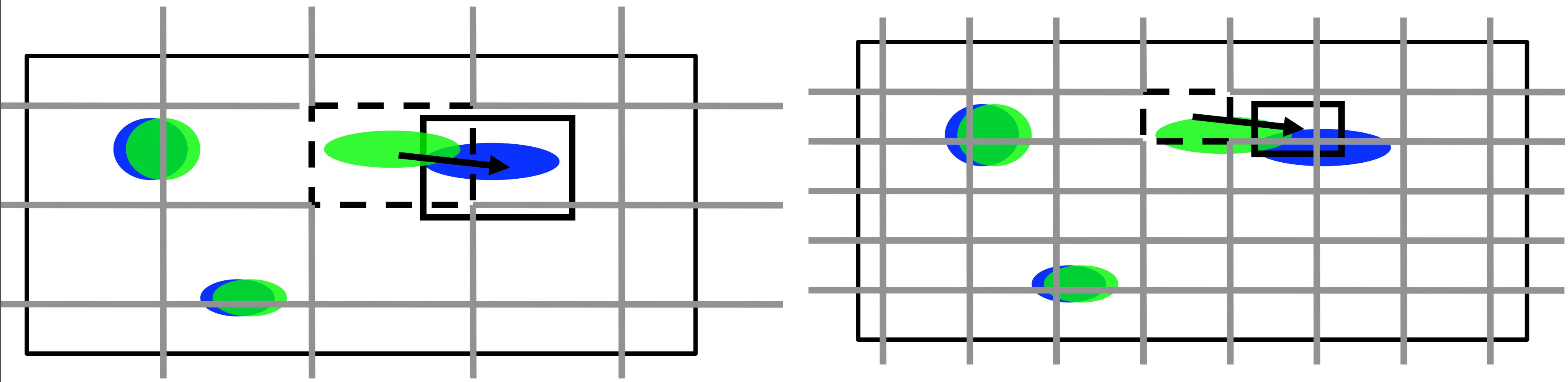
Localization





Localization

- The size of the area (or scale of B-spline basis) is related to the scale of the displacements or deformations & localization



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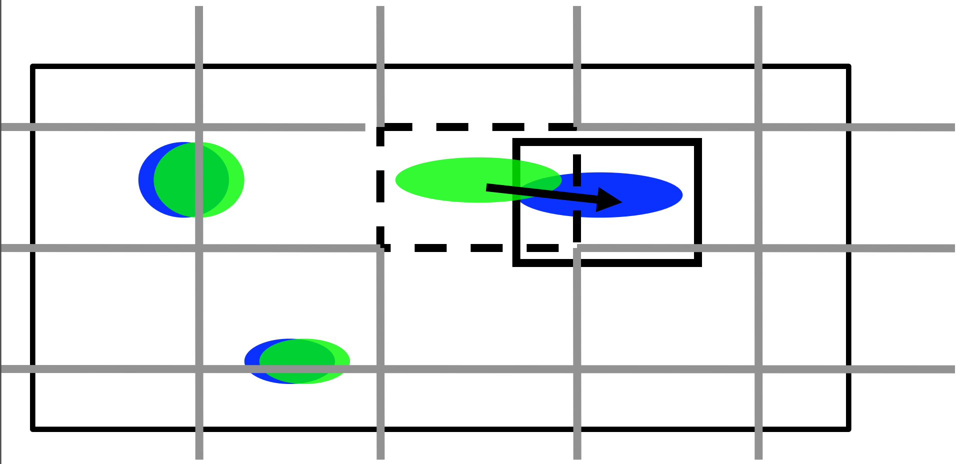

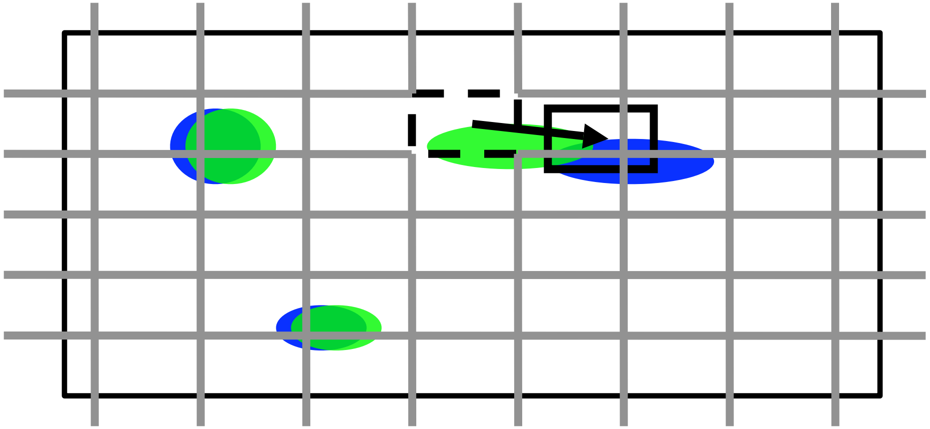

• Can use a single area (or scale) size or a cascade of sizes, ranging from large-scale to smaller-scale.

Kybic-Unser Cost Function:

$$\mathrm{E} = \frac{1}{\|I\|} \sum_{i \in I} (f_t^c(g(i)) - f_r(i))^2$$

where

$$\mathrm{g}(\mathrm{x}) = \mathrm{x} + \sum\nolimits_{j \in J} c_j \phi_j(x)$$

and

$$g(x) = x + \sum_{j \in J \subset (Z)^N} c_j \beta_{n_m}(x/h - j)$$

where

$$\beta_{n_m}(x)$$

defines the B-spline, with n_m defining the degree of the spline used, and h is the knot spacing.

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minimization control variables
i.e. instead of x parameters, only need J.

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Multi-resolutional (scaled) version of B-spline

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$$g(x) = x + \sum_{j \in J \subset (Z)^N} c_j \beta_{n_m}(x/h - j)$$

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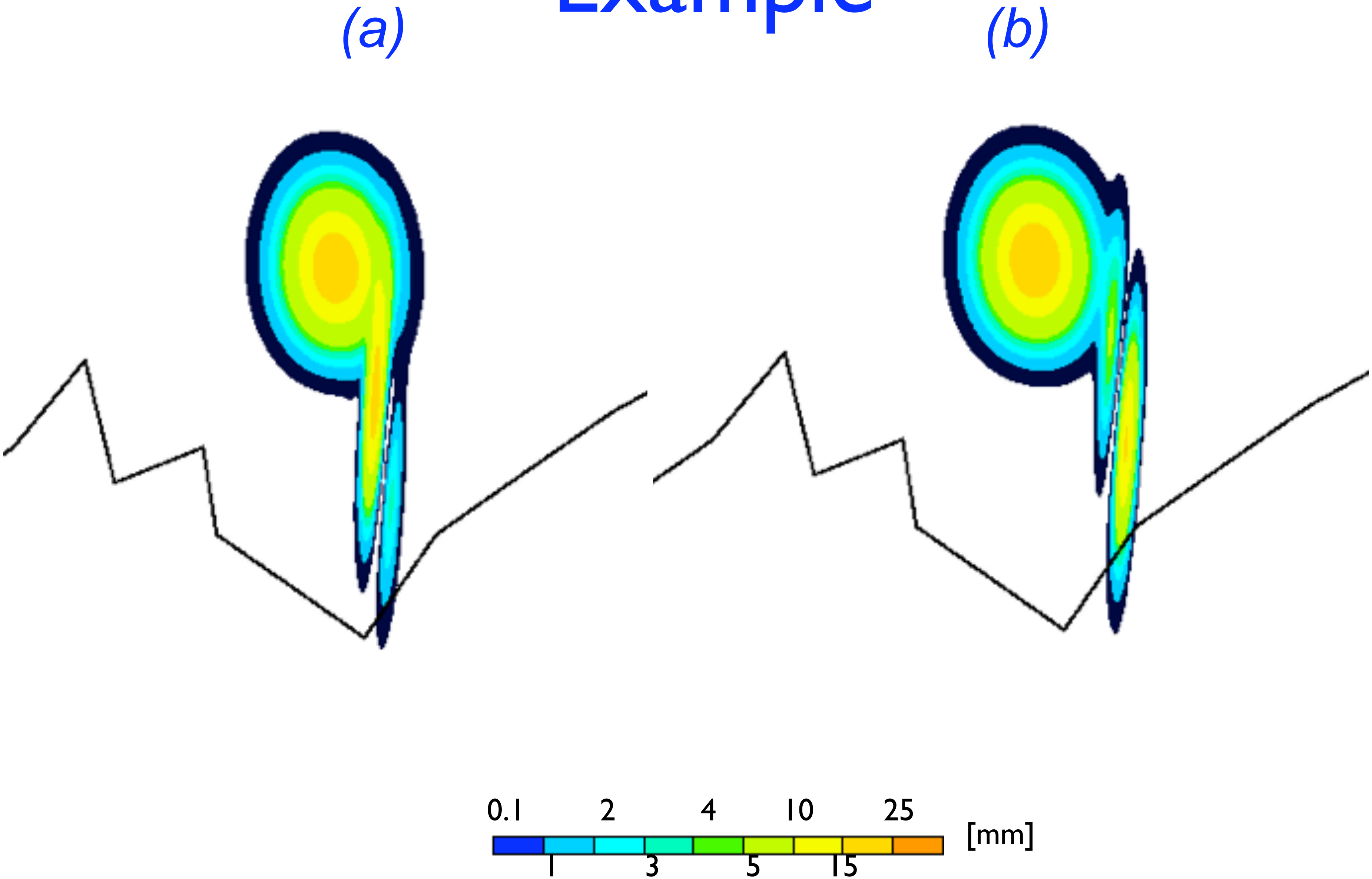
Assumptions in scheme

1. Amplitudes constant (***Intensity constraint***).
2. Phase correction (distortion) field smooth (implicit ***smoothness constraint***)

Algorithm designed for one field of movement - cannot handle superimposed independent movement

Example



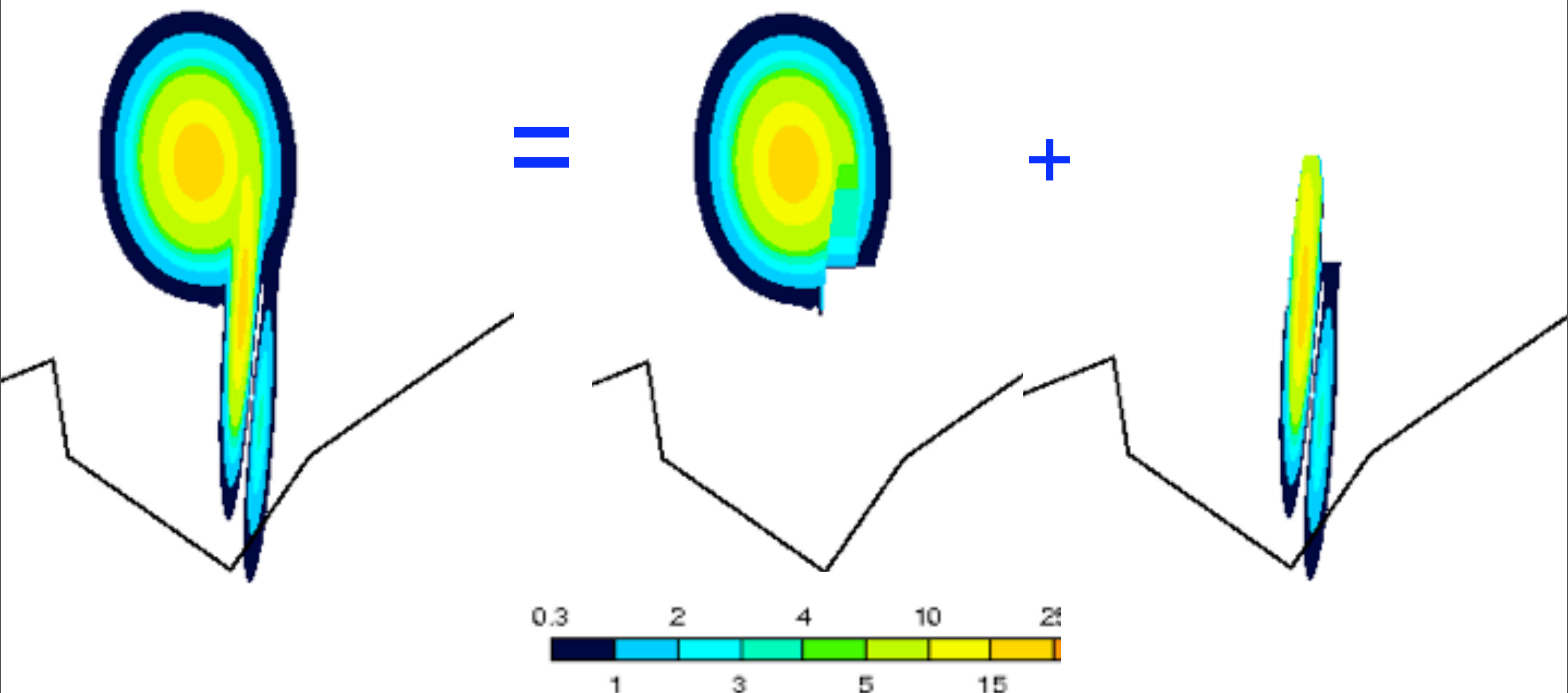
How can we get around this?

Scale separation using characteristic scales

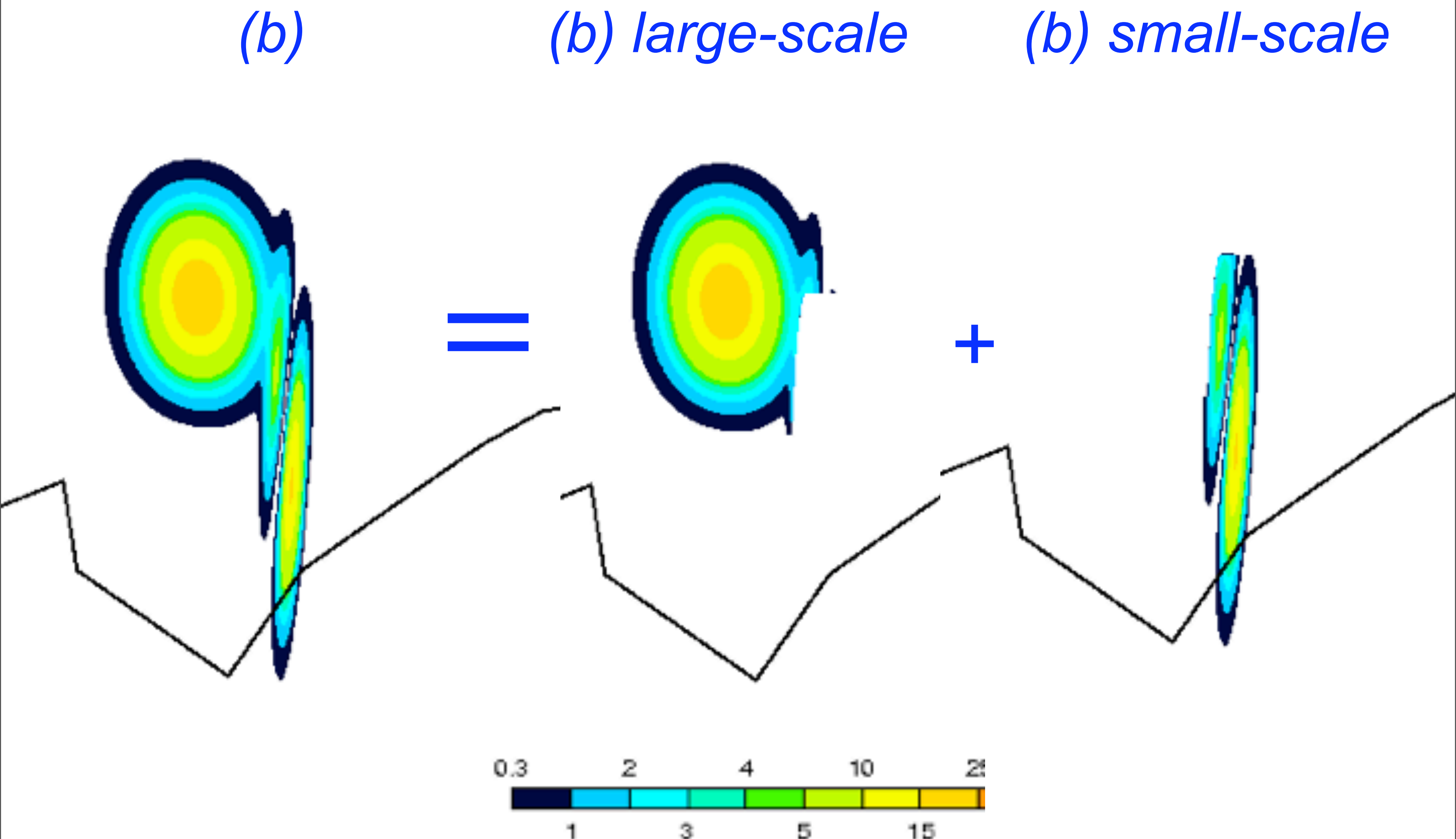
Assume that features will be relatively consistent with scale if not with placement.

Scale separation

(a) *(a) large-scale* *(a) small-scale*



Scale separation



Limitations of standard verification scores

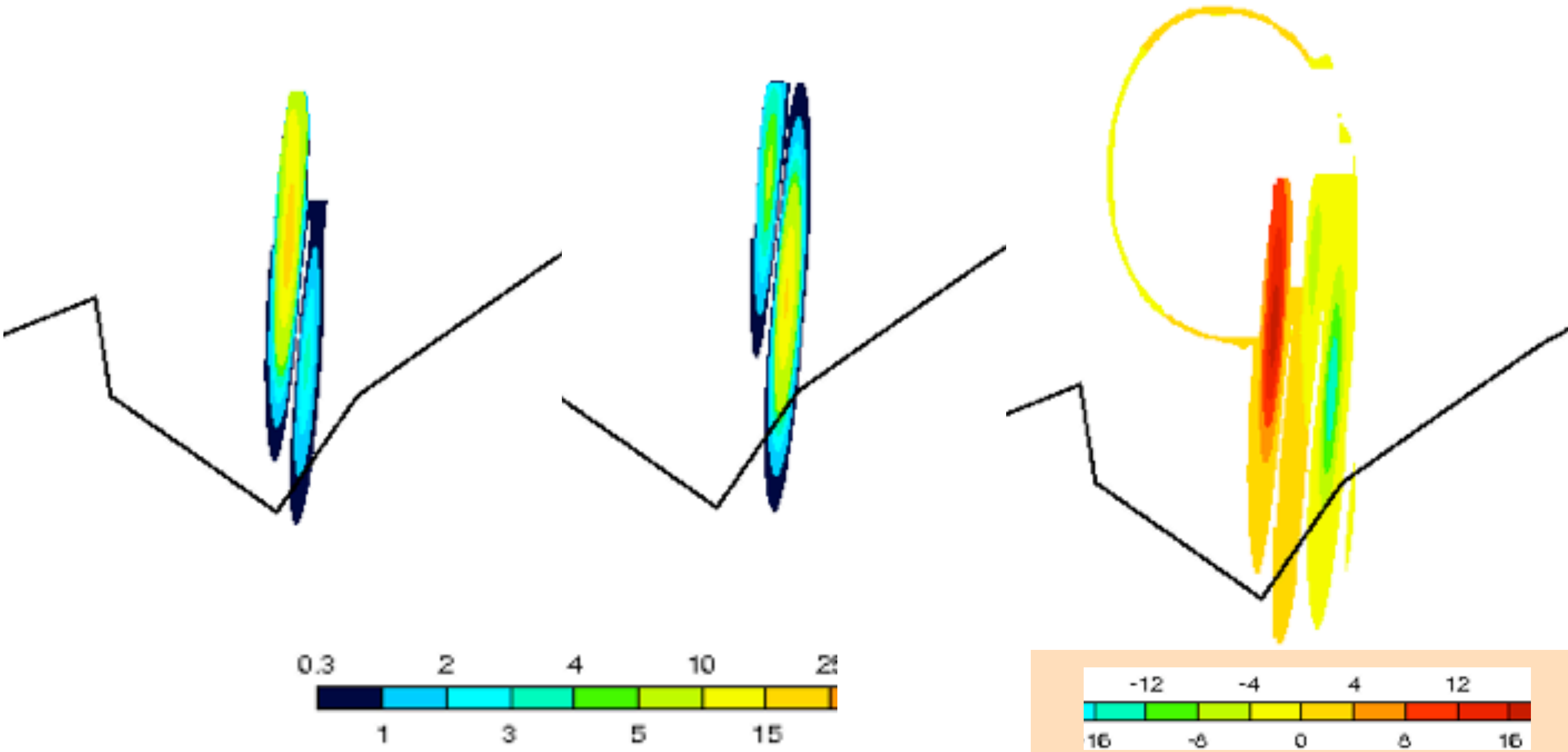
forecast (raw)

analysis

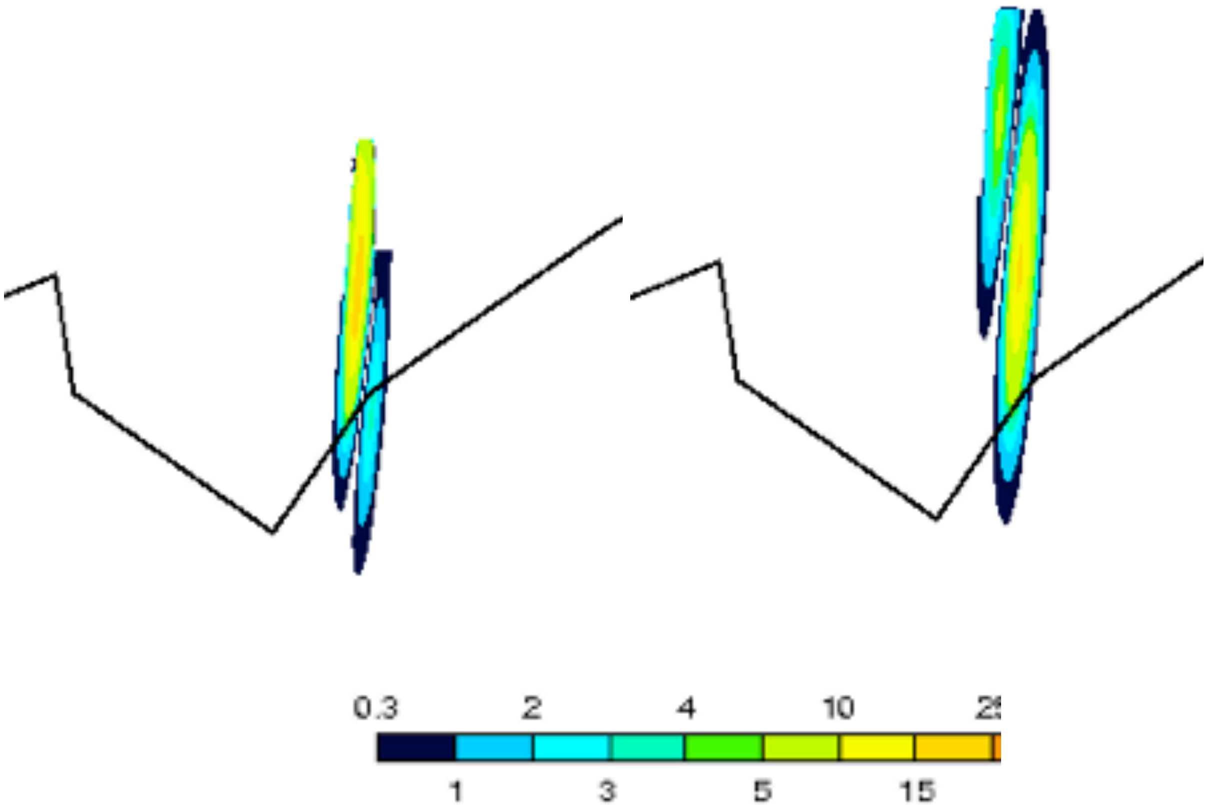
FORECAST (RAW) - ANALYSIS

MSE (all) 1.37

MSE (where rain) 25.8



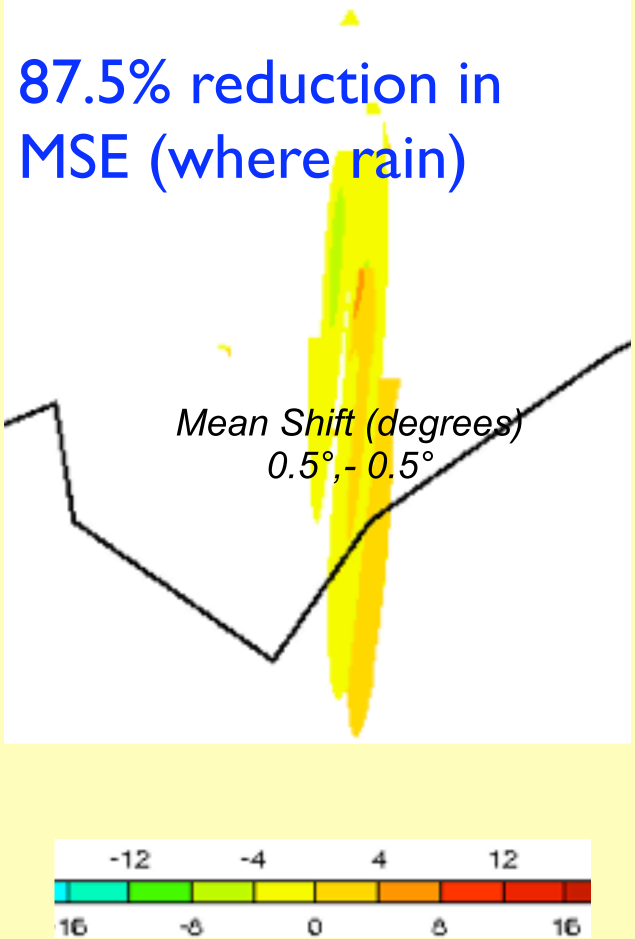
forecast (moved) analysis



FORECAST (moved) - ANALYSIS

MSE (all) 0.11

MSE (where rain) 3.23



87.5% reduction in MSE (where rain)

Note: experiencing problems due to large displacement, and due to discontinuity in the field

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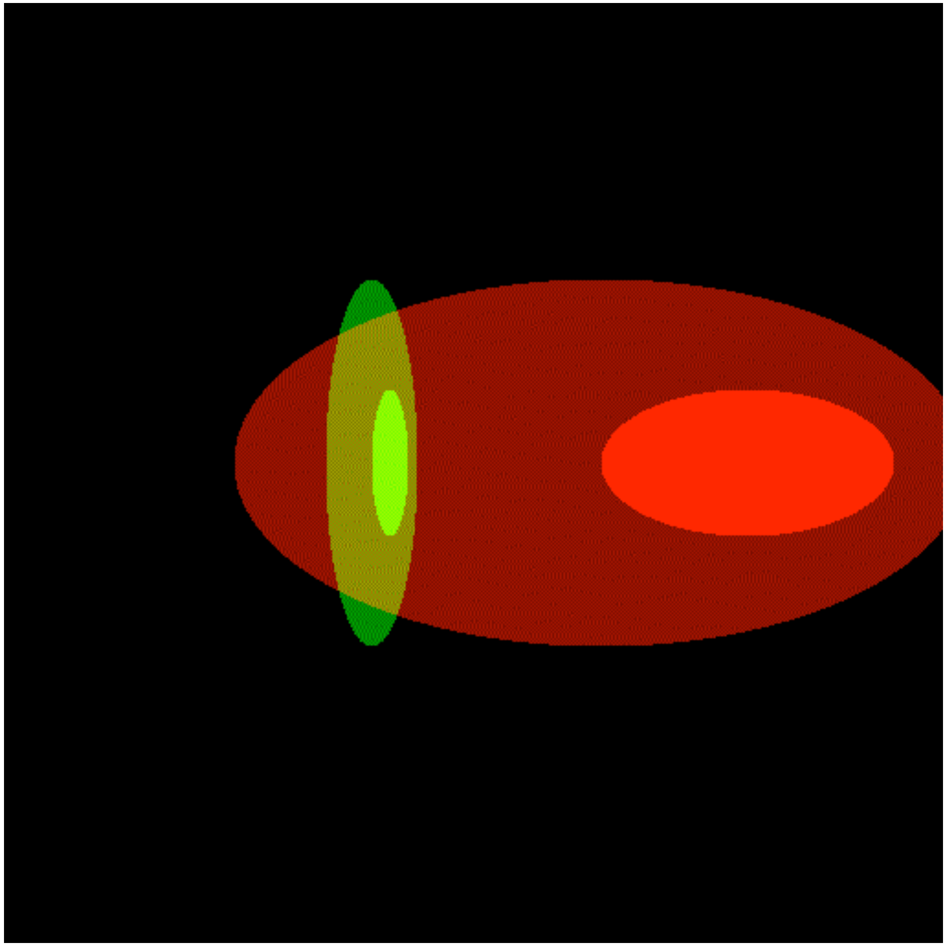
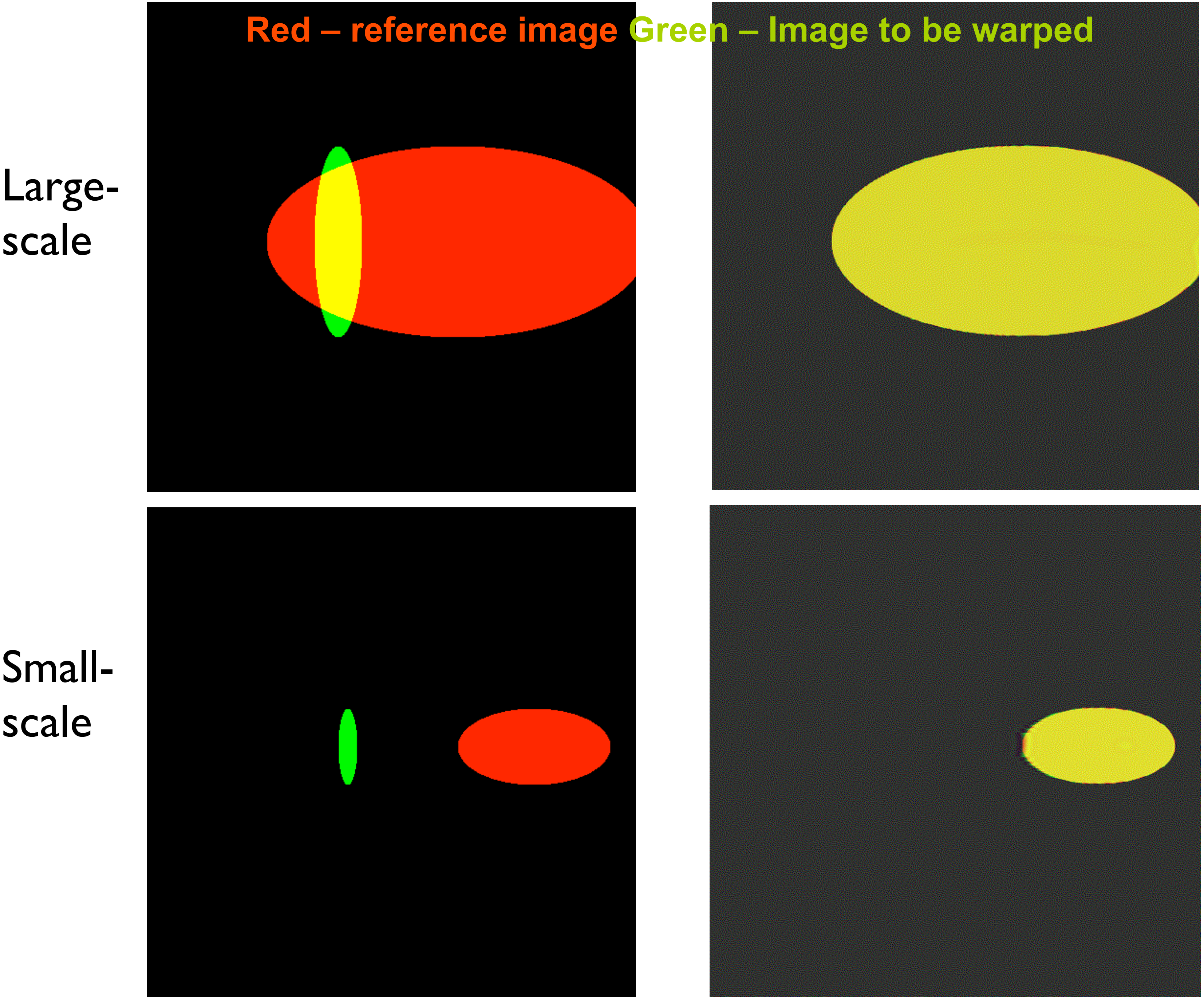
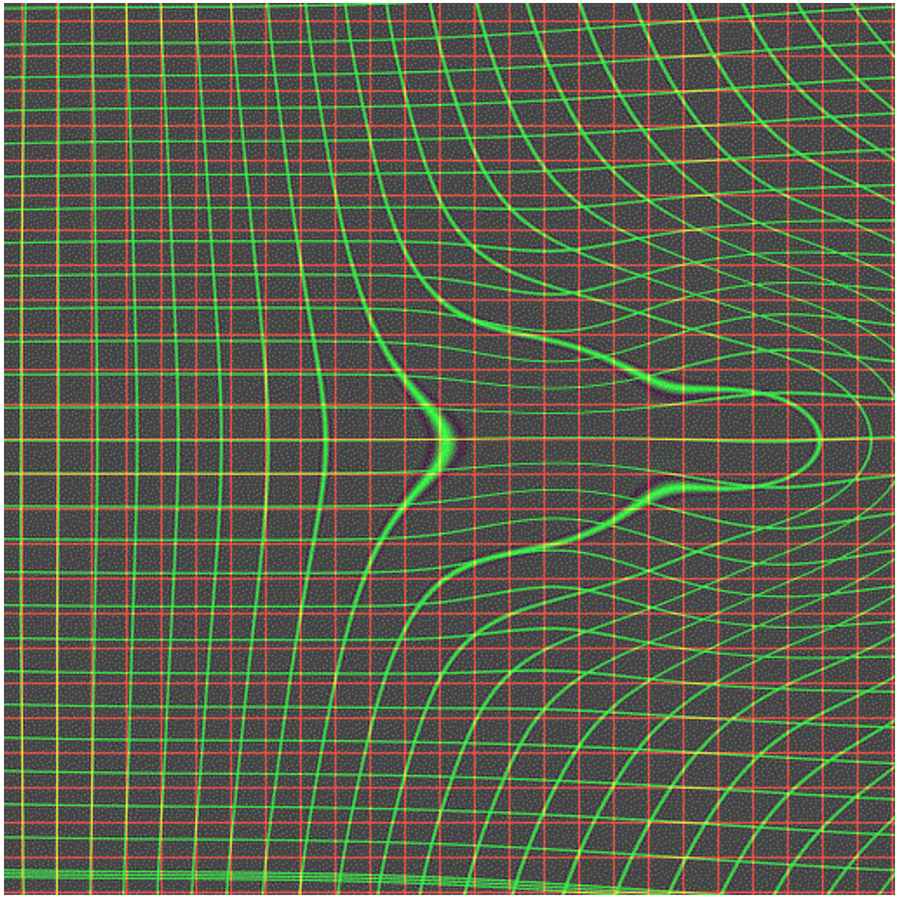



Red – reference image Green – Image to be warped
Large-scale
Small-scale

Ultimate Aim

- Verification metric with three components:

1. Amplitude
2. Shift
3. Distortion



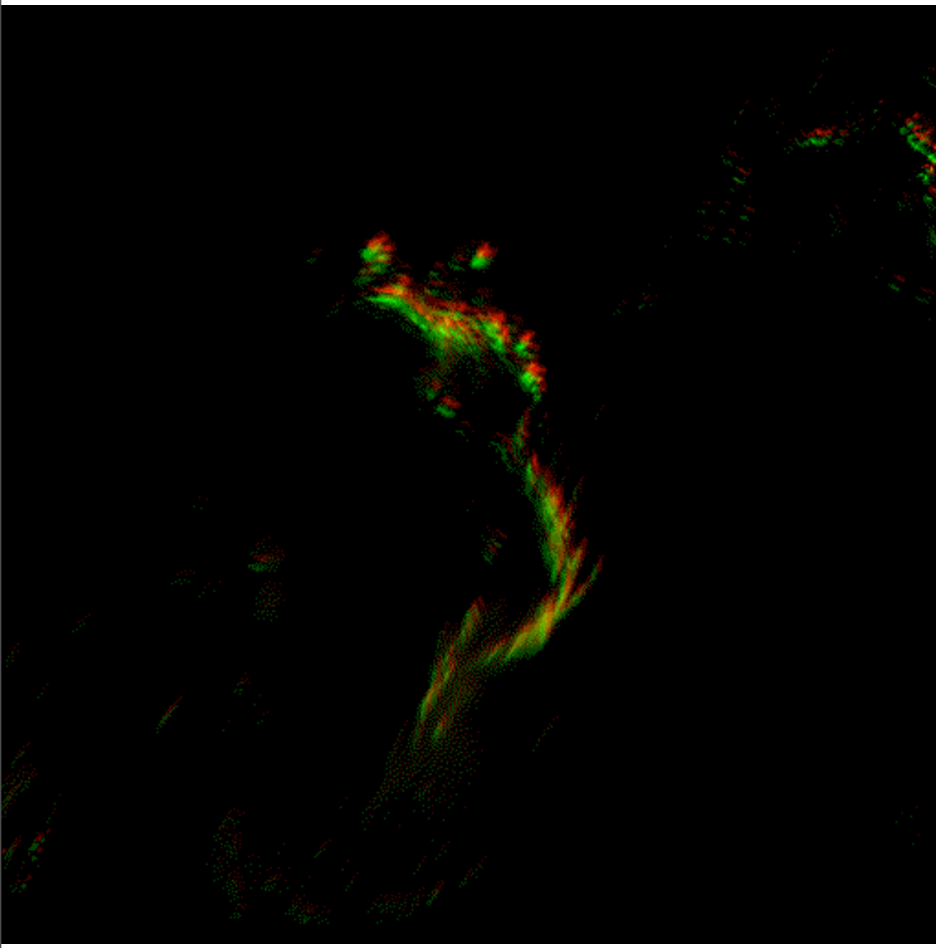
Red – reference grid

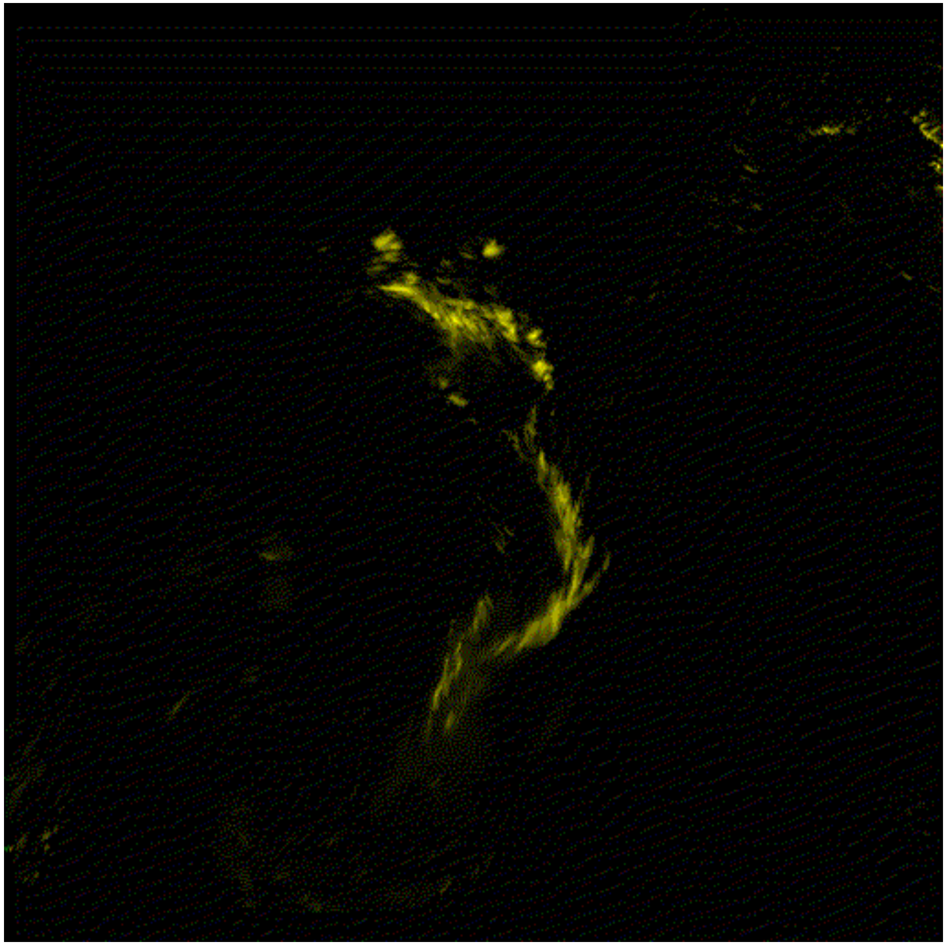
Green – Warped grid

Further work

- Design metric based on deformation
- Deal with Unwanted Ripping/Folding of field
- Deal with Gibbs phenomena in cubic B-spline interpolation.
- Calibrate warping algorithm
- Scale separation:
  - Deal with Gibbs phenomena in wavelet decomposition
  - Define solution dependent upon input field

Red – reference image Green – Image to be warped





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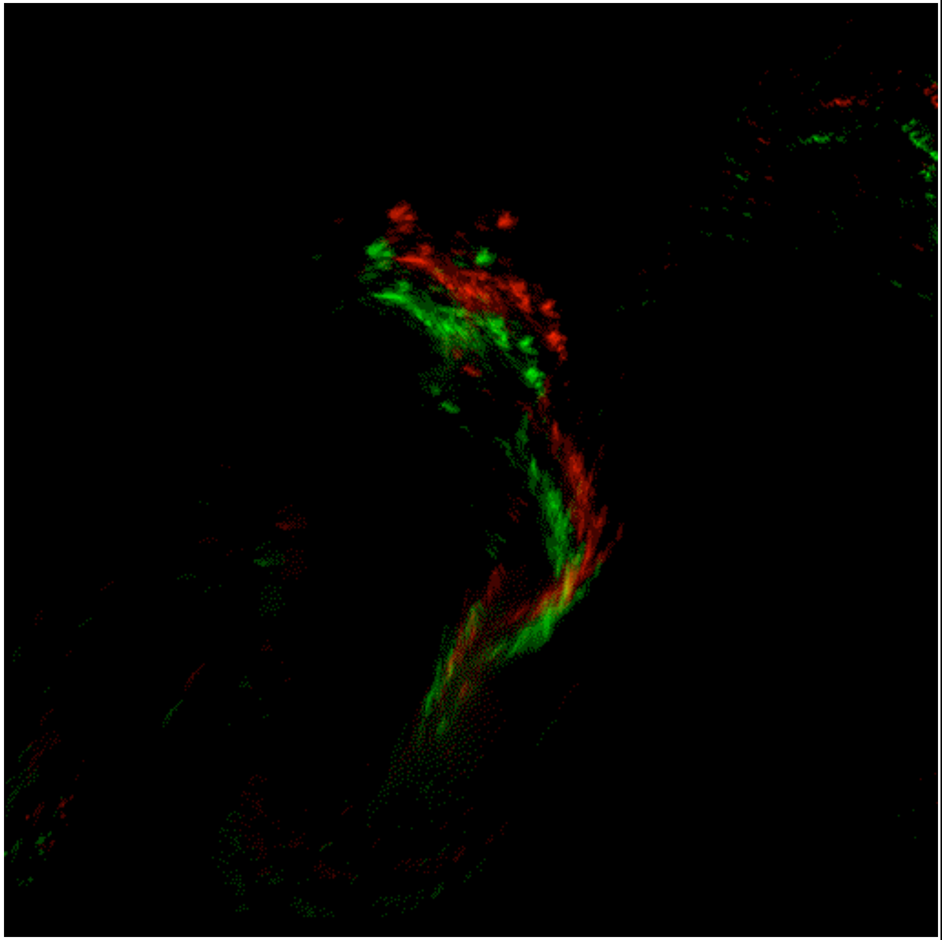

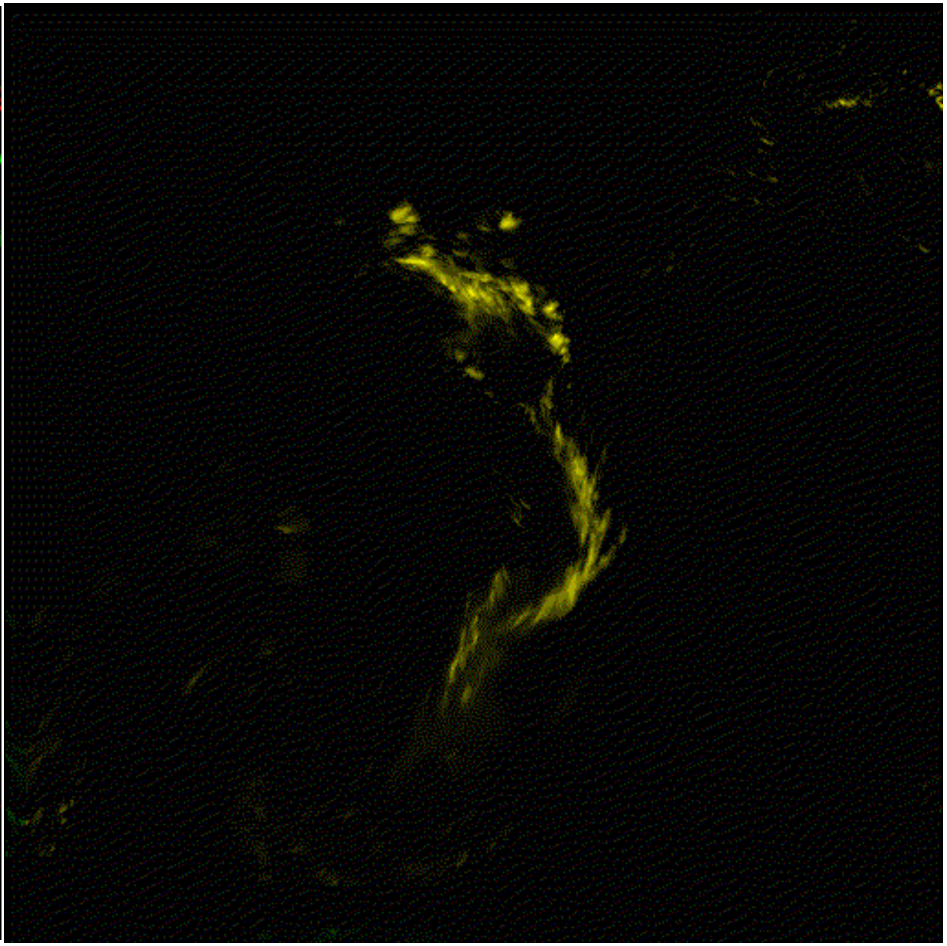

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