Time delays from blended data

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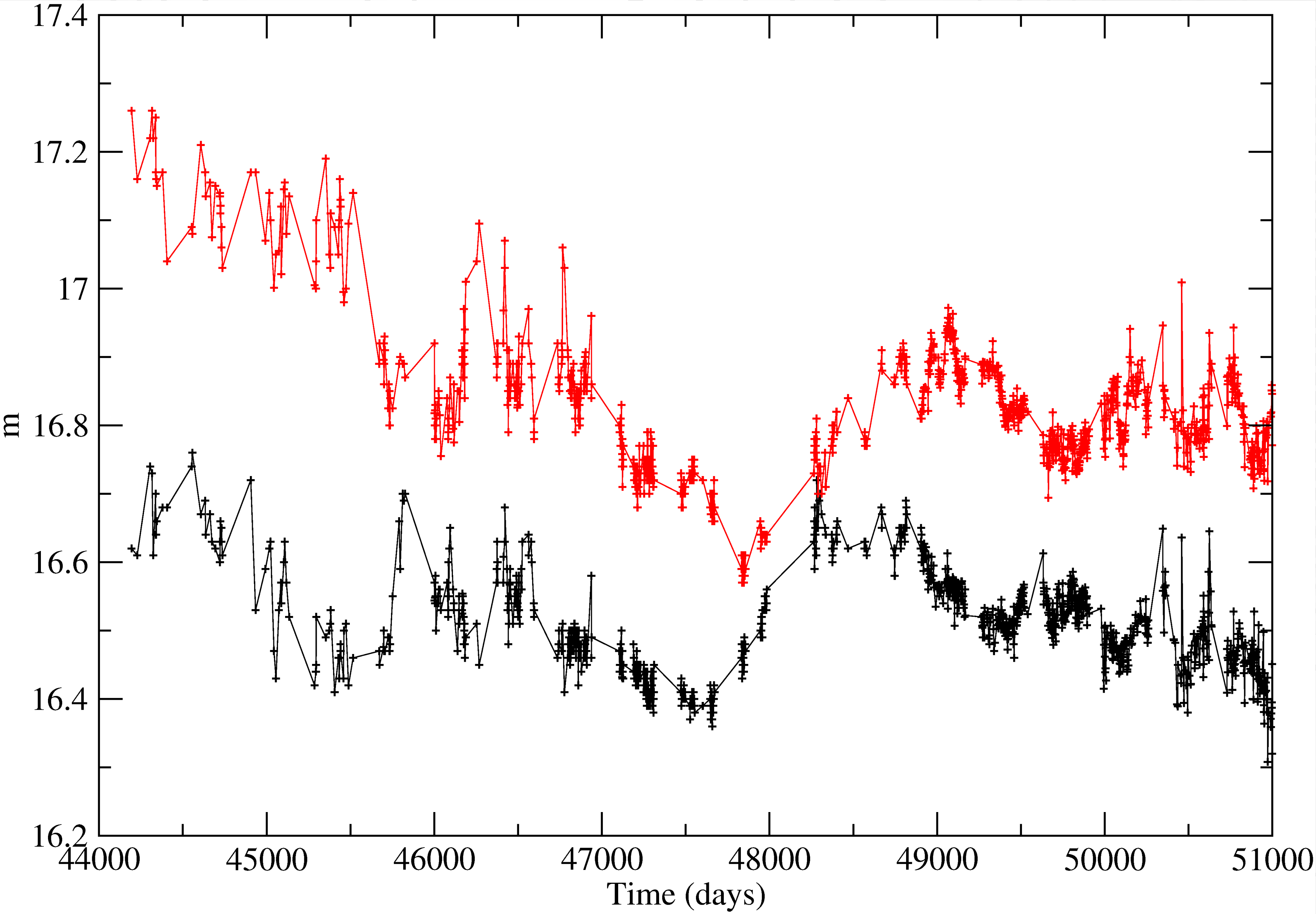
Introduction

- Gravitationally lensed quasar images
  - different light paths and flight times -> time delays
- Why are the time delays of lensed quasar images important for us?
- Measuring time delays
  - longtime monitoring -> smaller telescopes -> blended images and blended time series

Time series of QSO 0957+561



Two cases of blended light curves

- A clean image and a blend
- Two blended images

Over simplified models:

- continuous curves
- no noise

Idea of the method for recovering time delays developed by Jaan Pelt et al.

- Compose artificial blend(s) from observed data, using trial delays
- Calculate difference of the curves
- Minimise the difference by varying trial delays

A clean image and a blend
(continuous case)

Pure curve: $f_1(t) = a_1 q(t - t_1)$

Blended curve: $f(t) = f_2(t) + f_3(t) =$

$$= a_2 q(t - t_2) + a_3 q(t - t_3)$$

Artificial blend ->

$$g(t) = f_1(t - \Delta_l) + \alpha f_1(t - \Delta_l - \Delta_s) =$$

$$= a_1 q(t - t_1 - \Delta_l) + \alpha a_1 q(t - t_1 - \Delta_l - \Delta_s)$$

Observed blend $f(t)$ and artificial blend $g(t)$ are magnified copies of each other if

$\Delta_l = t_2 - t_1$ $\Delta_s = t_3 - t_2$ $\alpha = a_3 / a_2$

Two blended images
(continuous case)

Observed blends:

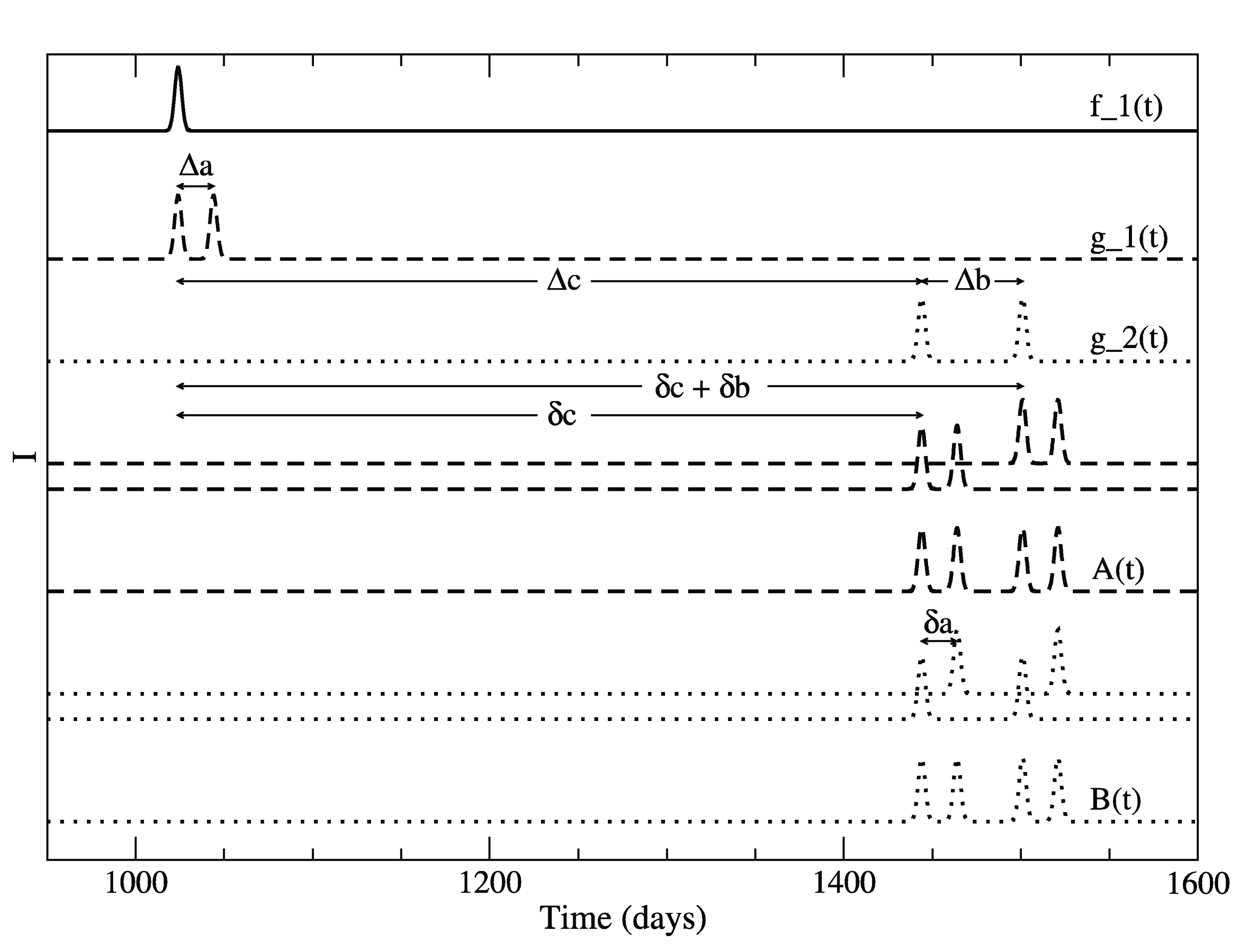
$$g_1(t) = a_1 q(t - t_1) + a_2 q(t - t_2),$$
$$g_2(t) = a_3 q(t - t_3) + a_4 q(t - t_4)$$

Assuming $a_1 \approx a_2, \quad a_3 \approx a_4,$ we get:

$$g_1(t) = f_1(t) + f_1(t - \Delta a),$$
$$g_2(t) = f_1(t - \Delta c) + f_1(t - \Delta c - \Delta b),$$

where $\Delta a = t_2 - t_1, \quad \Delta b = t_4 - t_3, \quad \Delta c = t_3 - t_1$

Two blended images
(graphical explanation)



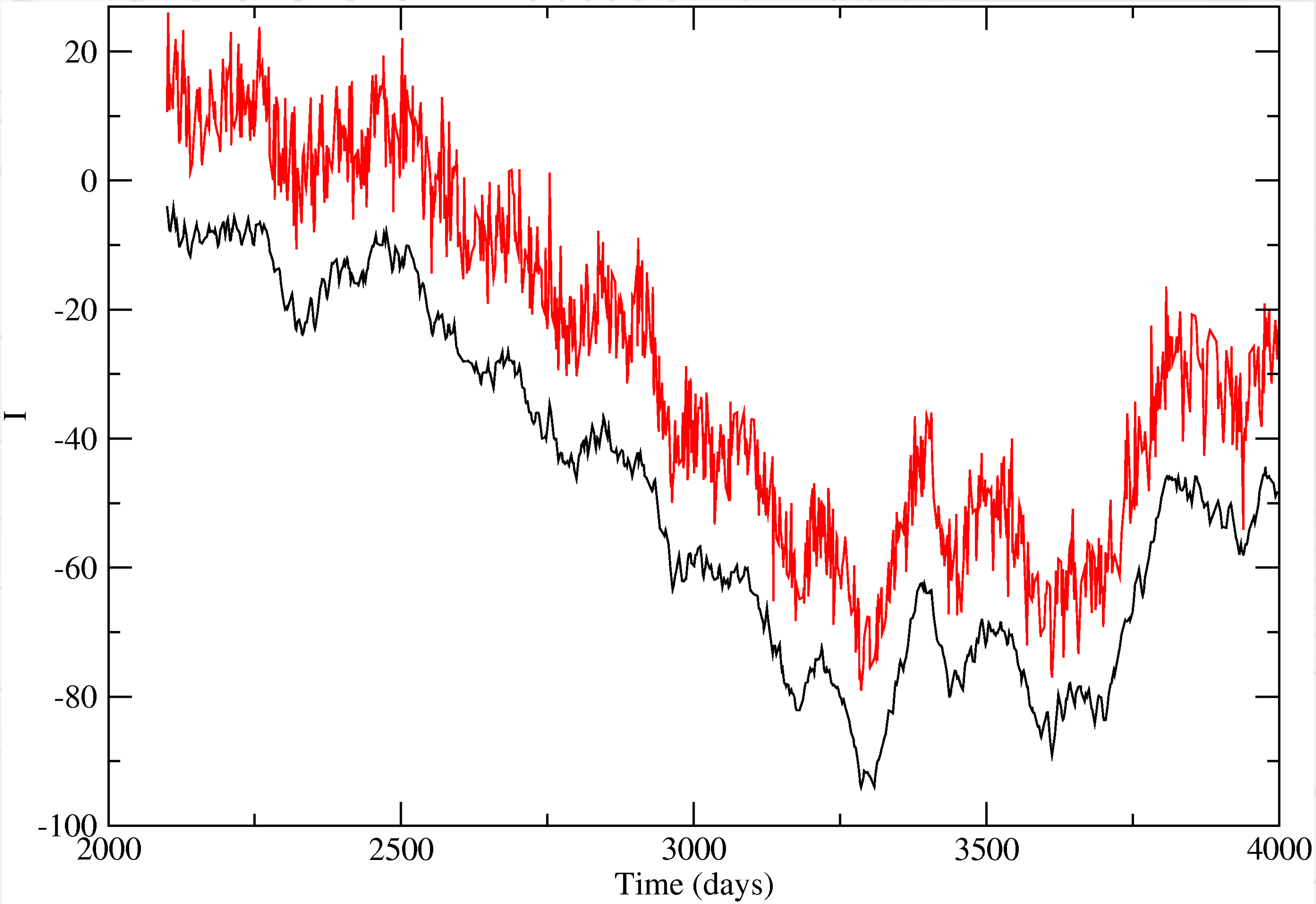
Two blended images -
matching artificial blends

Artificial blends from observed data using trial delays $\delta c, \delta a, \delta b$:

$$A(t) = g_1(t - \delta c) + g_1(t - \delta c - \delta b),$$
$$B(t) = g_2(t) + g_2(t - \delta a)$$

Moving to more realistic picture

- Sampled curves
- Noise



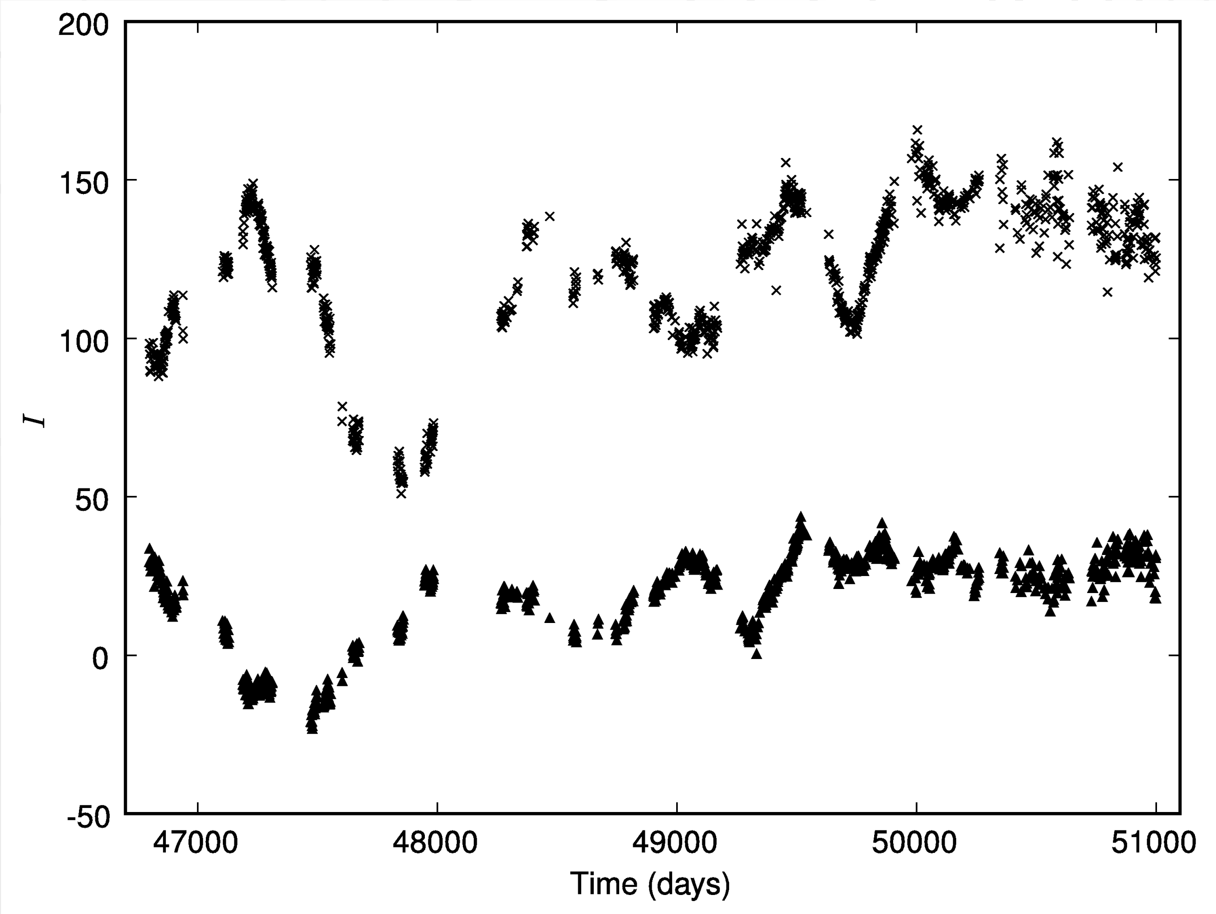
Details of the method

- Simulating quasar variability to get test data
- Adding and subtracting sampled curves
- Statistic for measuring the difference of sampled curves
- Visual representation of the results

Simulating quasar variability and blended data

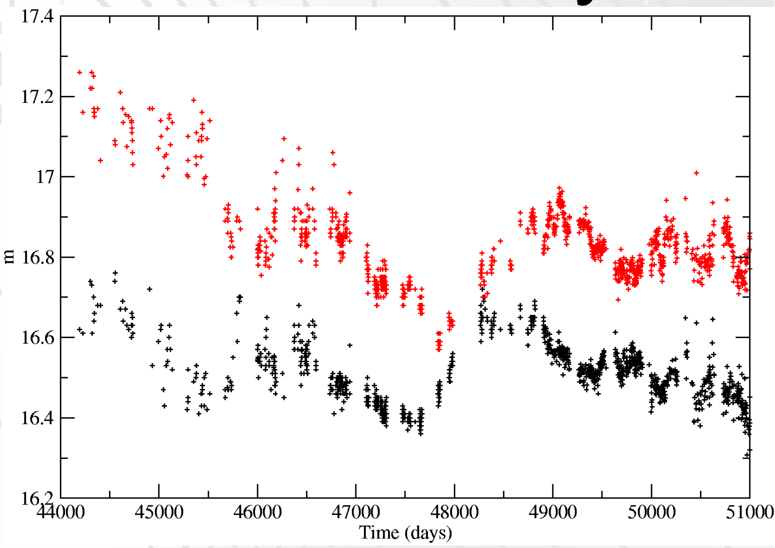
- Random walk: $q(t_i) = q(t_{i-1}) \pm 1$
- Shifting in time
- Combining the blend
- Sampling

A simulated time series (a clean image and a blend)

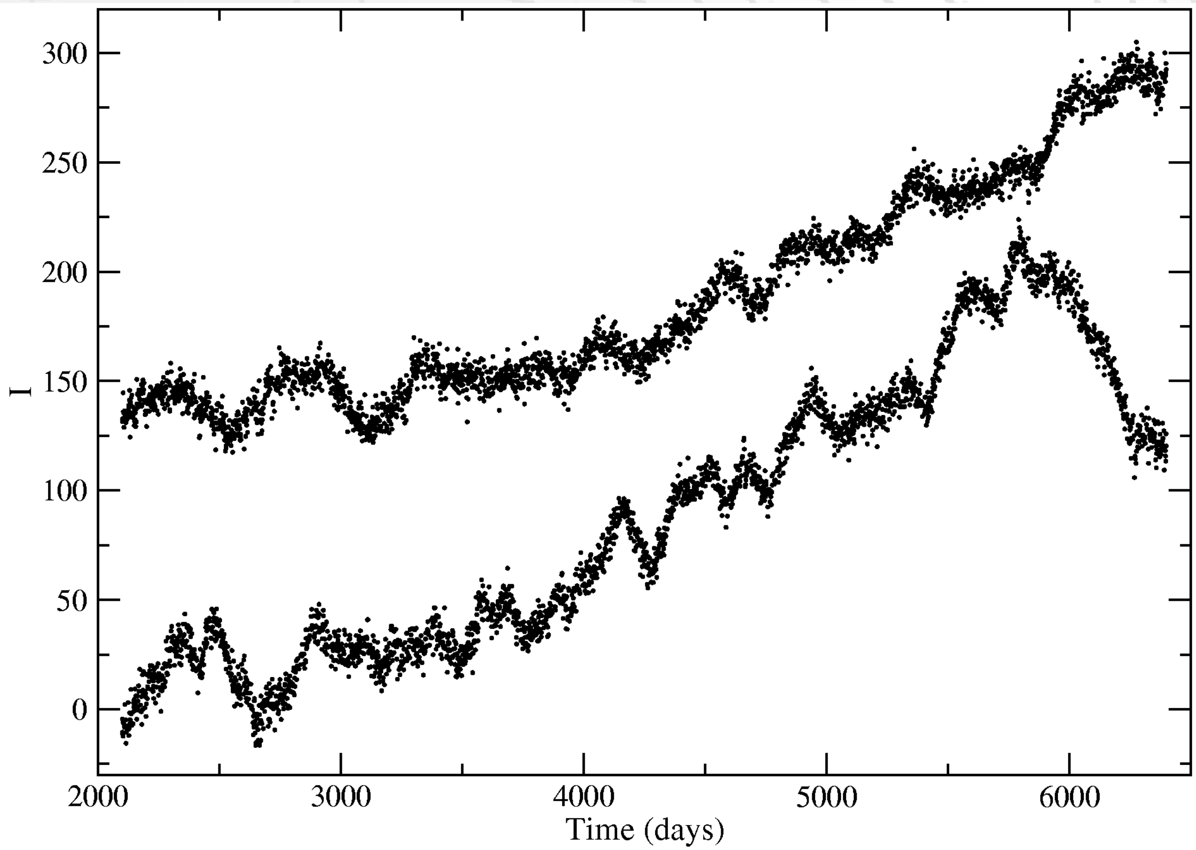


A random walk (lower curve) and a blend with Schild's sampling.

420.15, 20.21days



A simulated time series (two blended images)



Good sampling.

420.2, 20.2 (lower), 56.5 (upper) days

Adding sampled curves to form the artificial blend from observed data

$$t_n, f_n, W_n, n = 1, 2, \ldots, N$$

$$t_m, g_m, W_m, m = 1, 2, \ldots, M$$

$$\frac{t_n + t_m}{2}, f_n + \alpha g_m, W_{n,m},$$

where the combined weights

$$W_{n,m} = S_{n,m} \frac{W_n W_m}{\alpha^2 W_n + W_m},$$

$$S_{n,m} = \begin{cases} 1 - \frac{|t_n - t_m|}{\sigma}, & \text{if } |t_n - t_m| \le \sigma, \\ 0, & \text{if } |t_n - t_m| > \sigma. \end{cases}$$

Adding sampled curves to form the artificial blend from observed data

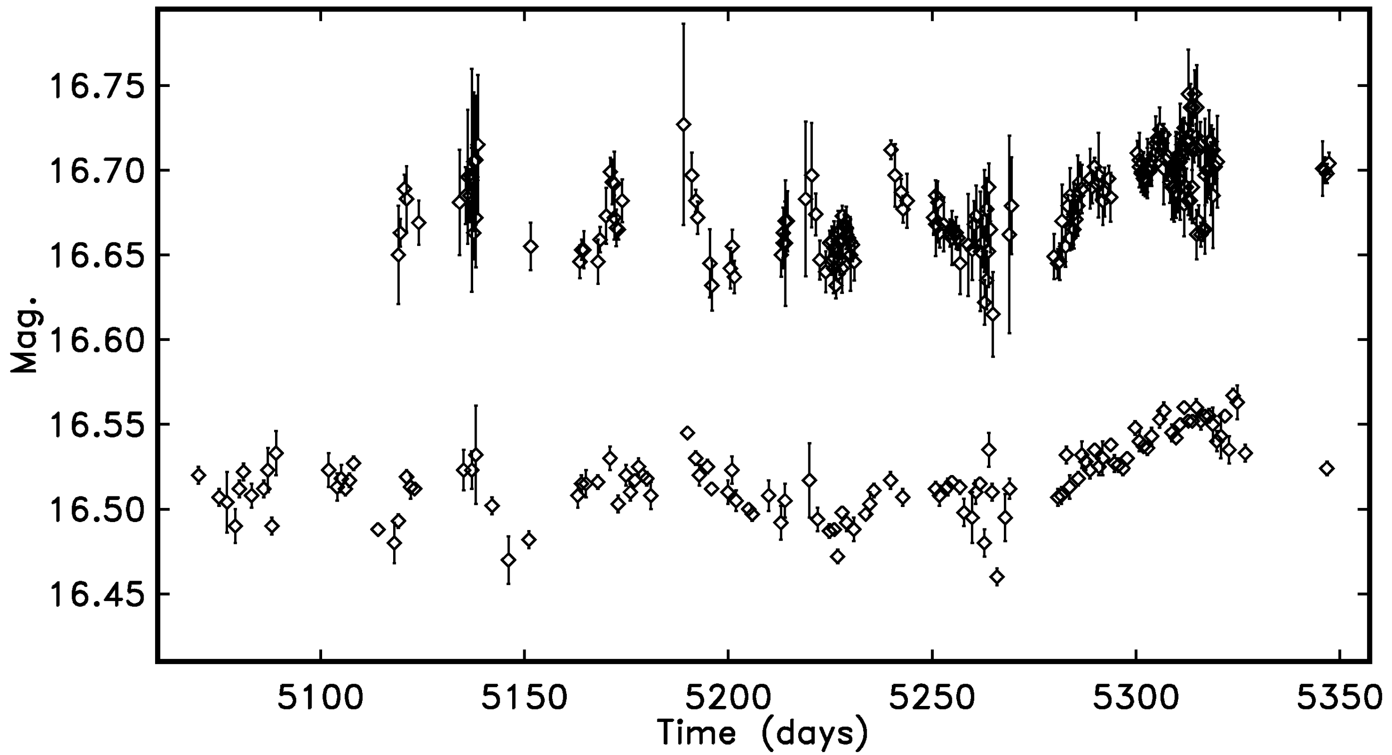


Fig. 1. Combining an original time series with its shifted version. Depending on the spacing of time points and the downweighting parameter σ, the resulting series can be sparser (middle part of the series) or denser (right part). The combined error estimates are larger than the original (given) values.

Subtracting sampled curves

$$t_n, f_n, W_n, n = 1,2,\ldots, N$$

$$t_m, g_m, W_m, m = 1,2,\ldots, M$$

$$\frac{t_n + t_m}{2}, (af_n + b - g_m)^2, W_{n,m},$$

$$W_{n,m} = S_{n,m} \frac{W_n W_m}{W_n + a^2 W_m}$$

Statistic for measuring the difference of sampled curves

$$D^2 = \min_{a,b} \frac{\sum_{n,m}(af_n + b - g_m)^2 W_{n,m}}{\sum_{n,m} W_{n,m}}$$

We are going to search for the minimum of the statistic by varying trial delays (and magnification).

Visual representation of the results (one blend)

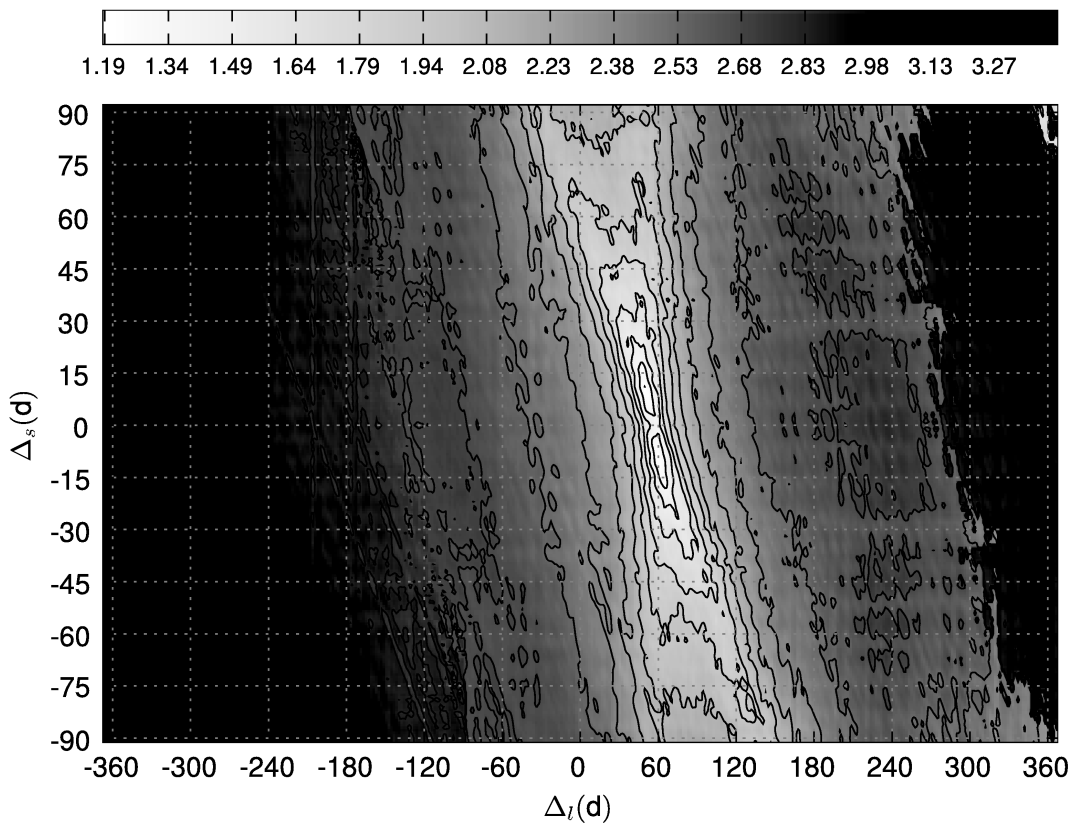


Fig. 2. The two-dimensional grid of merit function values for a computer generated random walk and a blend computed from it. The general minimum must indicate the true pair of long and short delay values. The plot demonstrates degeneracy in full-scale computations well – there is obvious symmetry between the areas for positive and negative values of short delays. Values on the colour key represent the $\log(D^2)$ and spacing of the contours. The same type of colour key is used in all two-dimensional plots.

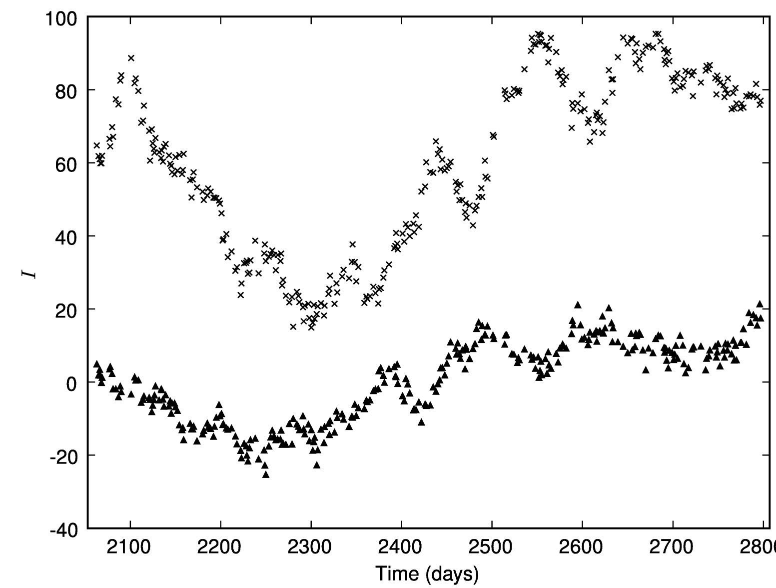


Fig. 3. A basic computer-generated random walk C (lower curve) and a computer generated blend curve A ($\Delta_l = 50.2$, $\Delta_s = 10.6$, the amplification parameter $\alpha = 1.3$; we added 5% noise to both curves and shifted the blend up by 60 units).

Random sampling, daylight caps.
Recovered parameters:
$\Delta_l = 50$ days, $\Delta_s = 11$ days, and $\alpha = 1.1$.

Assuming pure curves, we get 59 days, which is incorrect answer.

Visual representation of the results (one blend, Schild's sampling and errors)

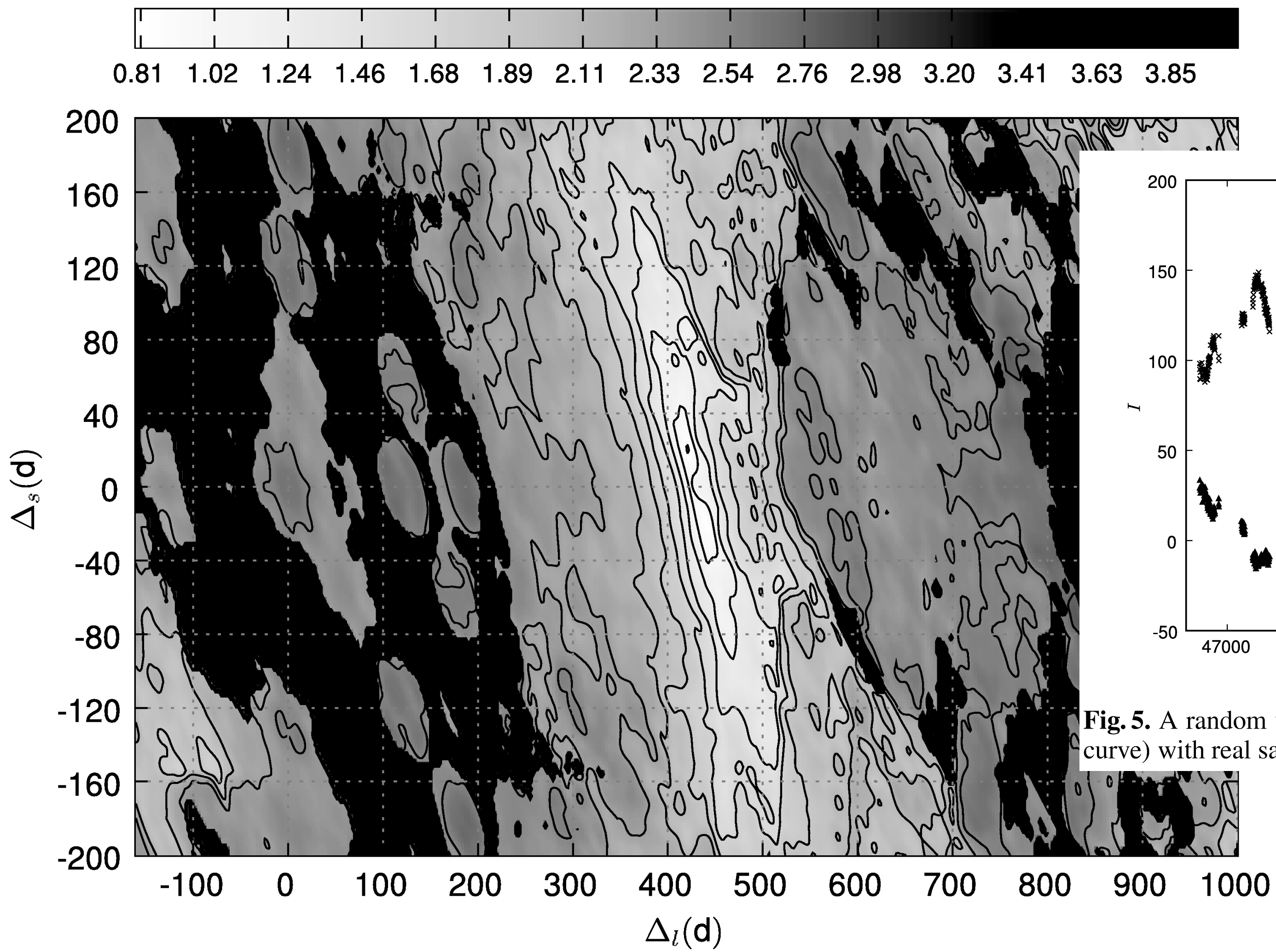


Fig. 6. Merit function values for simulated data and real sampling.

420.15, 20.21, 0.8 were put in

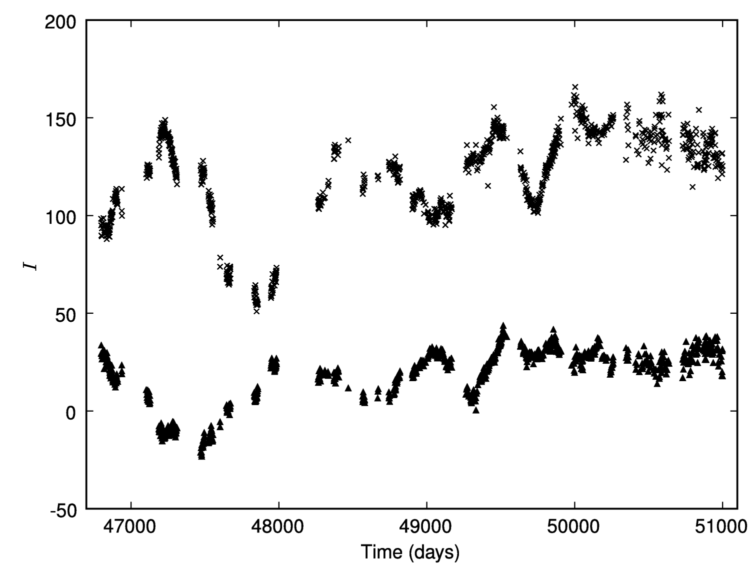


Fig. 5. A random walk *C* (lower curve) and a blend *A* (upward-shifted curve) with real sampling.

420.2, 20.0, 0.9 came out

Visual representation of the results (two blends)

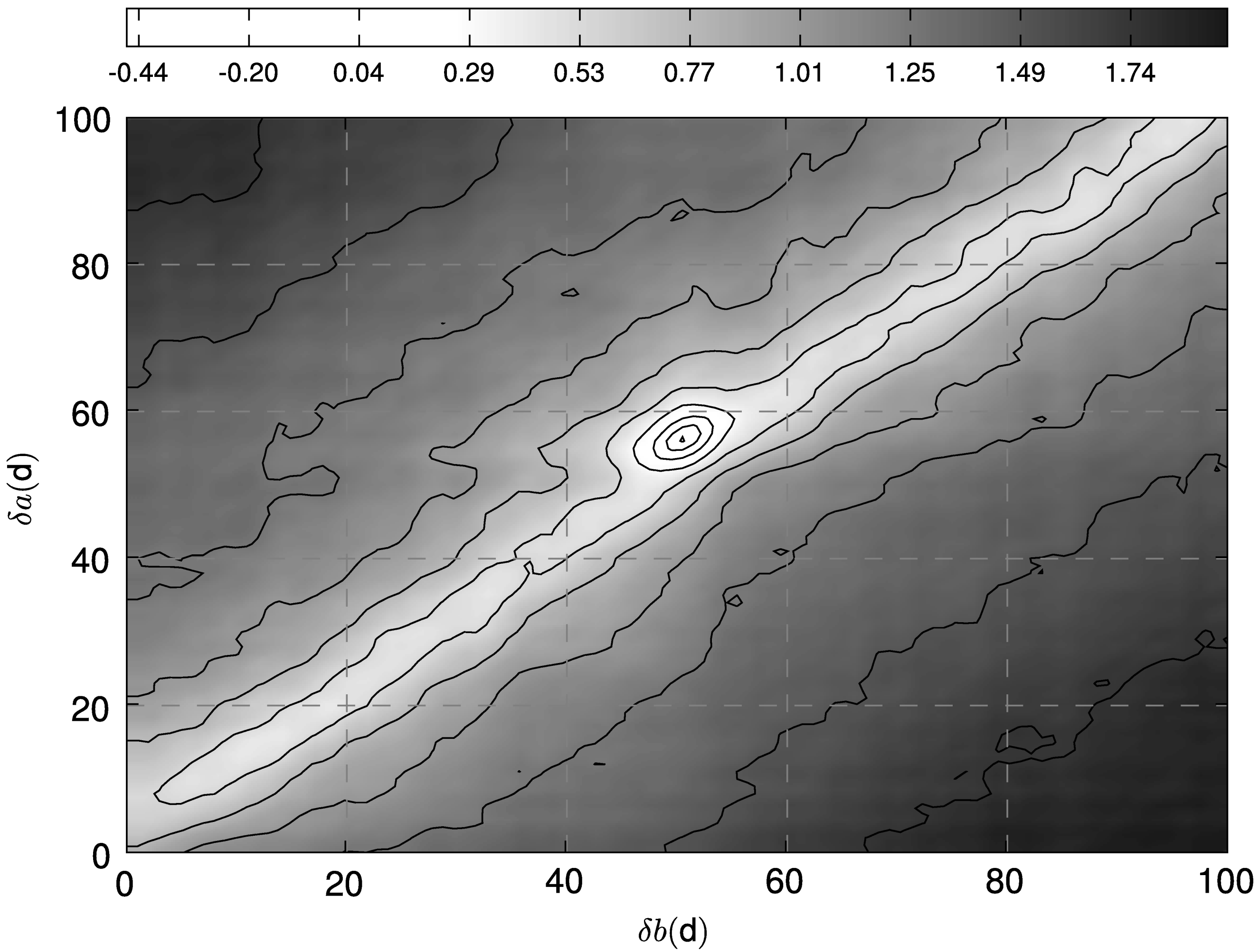
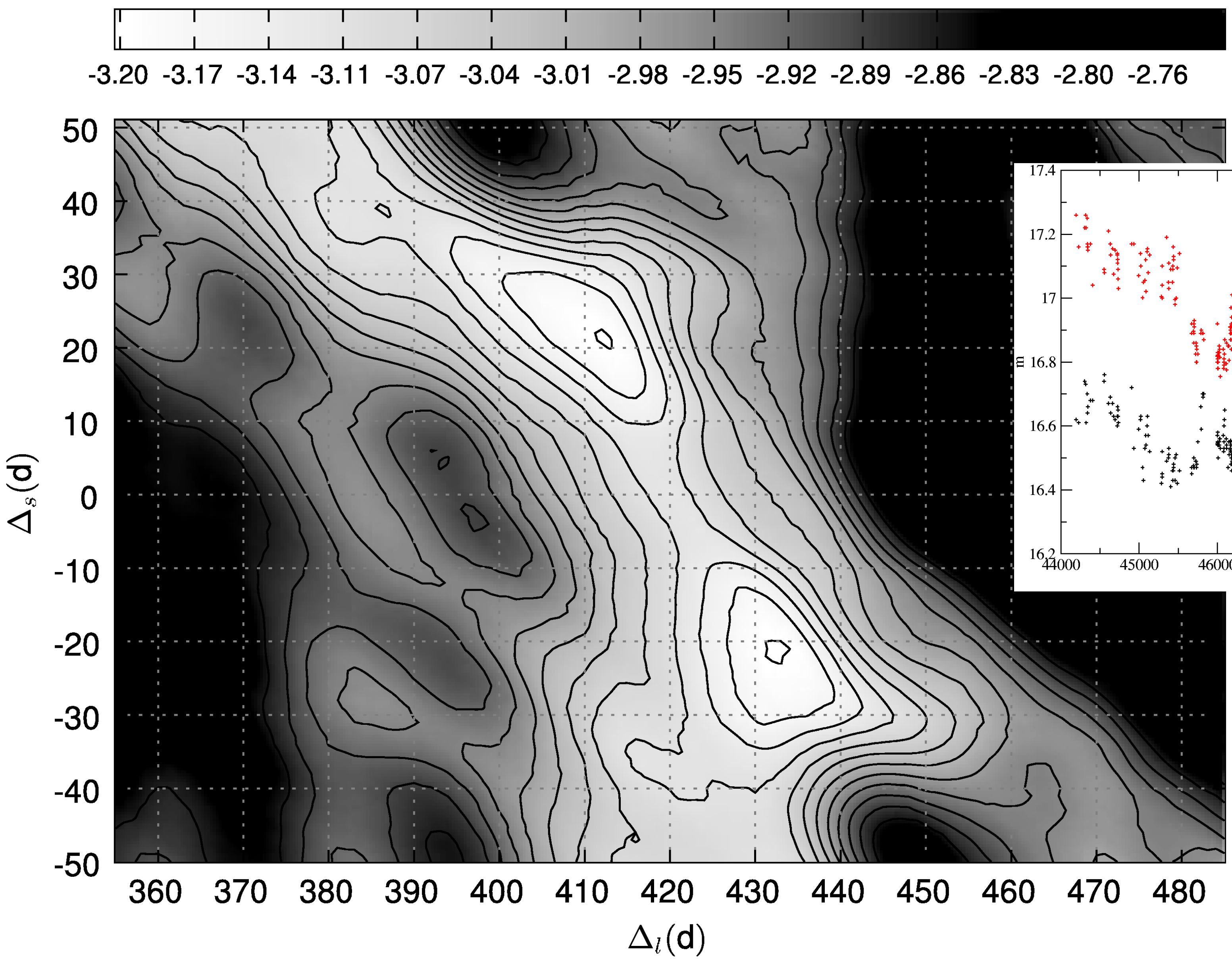
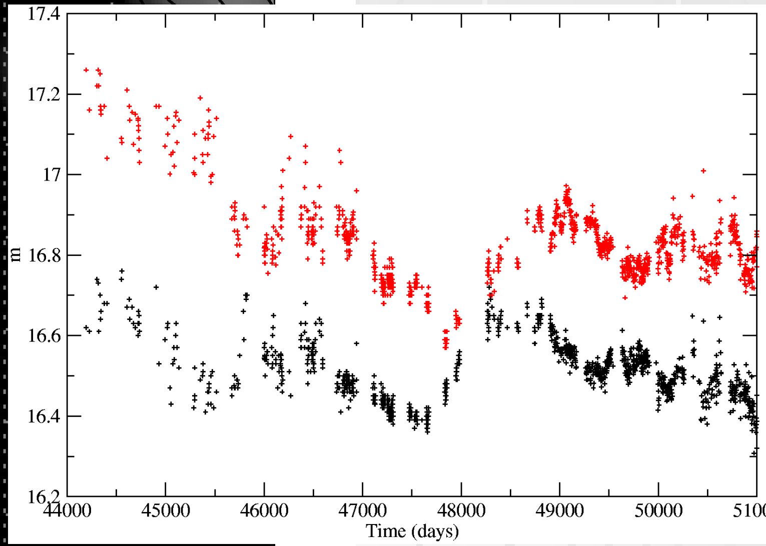


Fig. 4. D^2 values for very close short delays. $\Delta c = 420.2$, $\Delta a = 56.5$, $\Delta b = 50.1$ days and $a = 0.8$.

- Even more degenerate
- Does not work if short delays are equal
- Higher S/N is needed

Results from Schild's data





Assuming that B is blend, we got 412 and 22 days for time delays.

What we were talking about.

- It is possible to find time delays from blended data using artificial blends
  - adding and subtracting blended data
  - statistic for measuring the difference of two time series that is insensitive to gaps

References

- Hirv A., Eenmäe T., Liimets T., Liivamägi L. J., Pelt J. 2007, A&A, 464, 471
- Hirv A., Eenmäe T., Liivamägi L. J., Pelt J. 2007, Baltic Astronomy, 16, 241