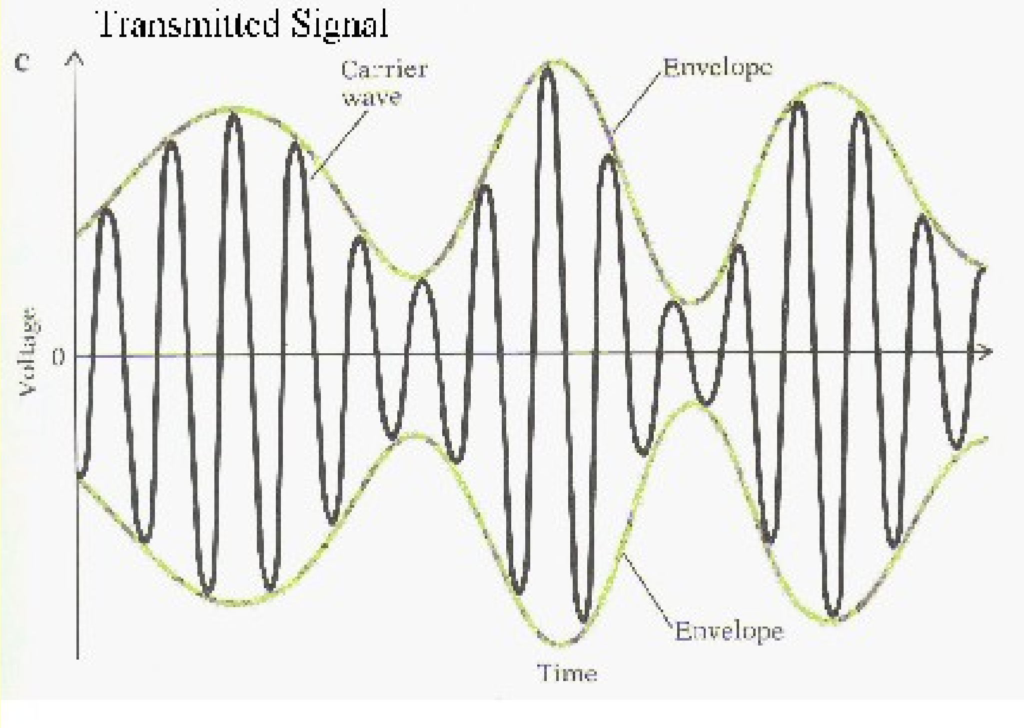
The Finnish Graduate School in Astronomy and Space Physics Summer School 2007

Time Series Analysis

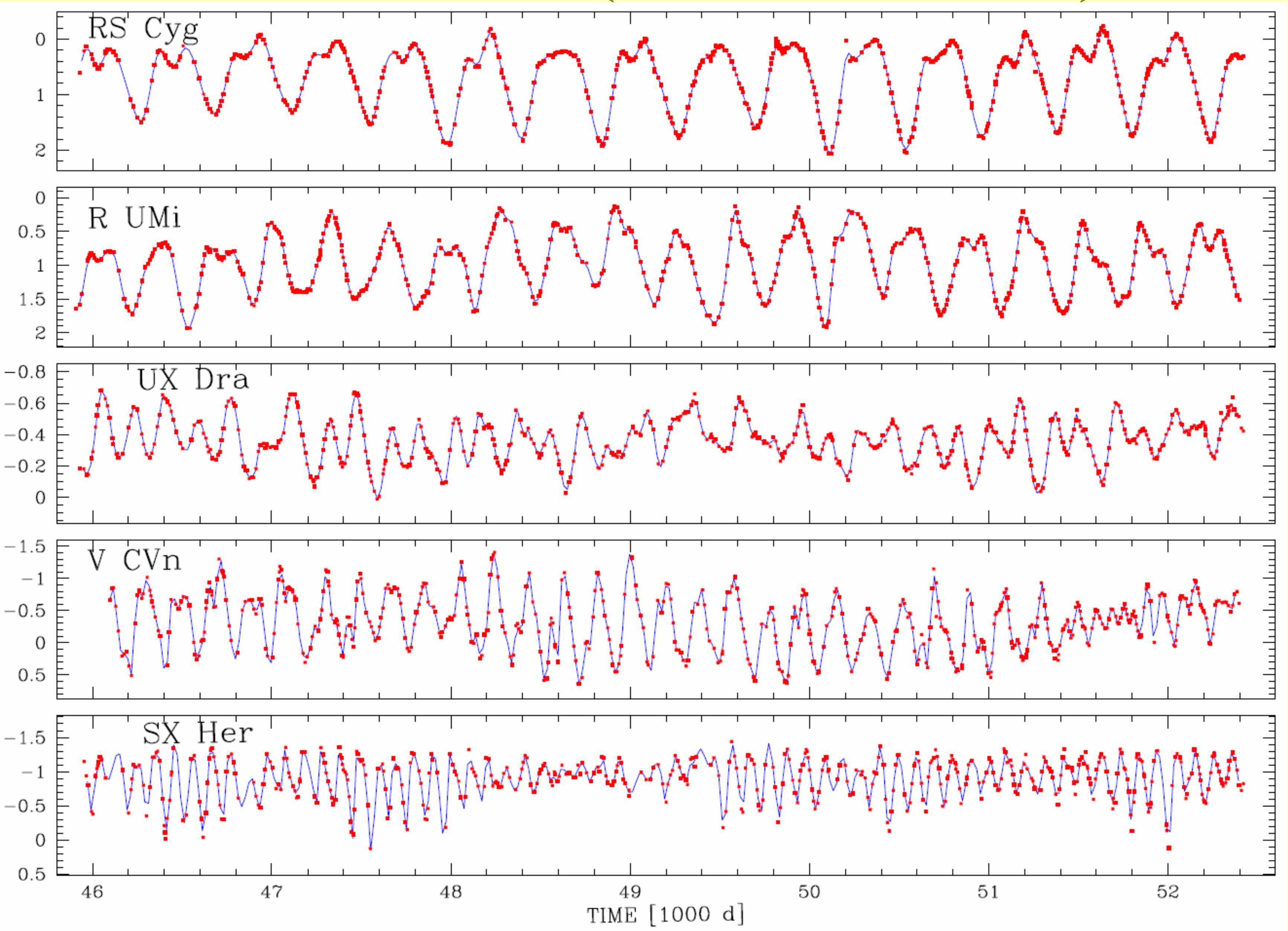
Part IV. Solutions

Time series models with time dependent parameters

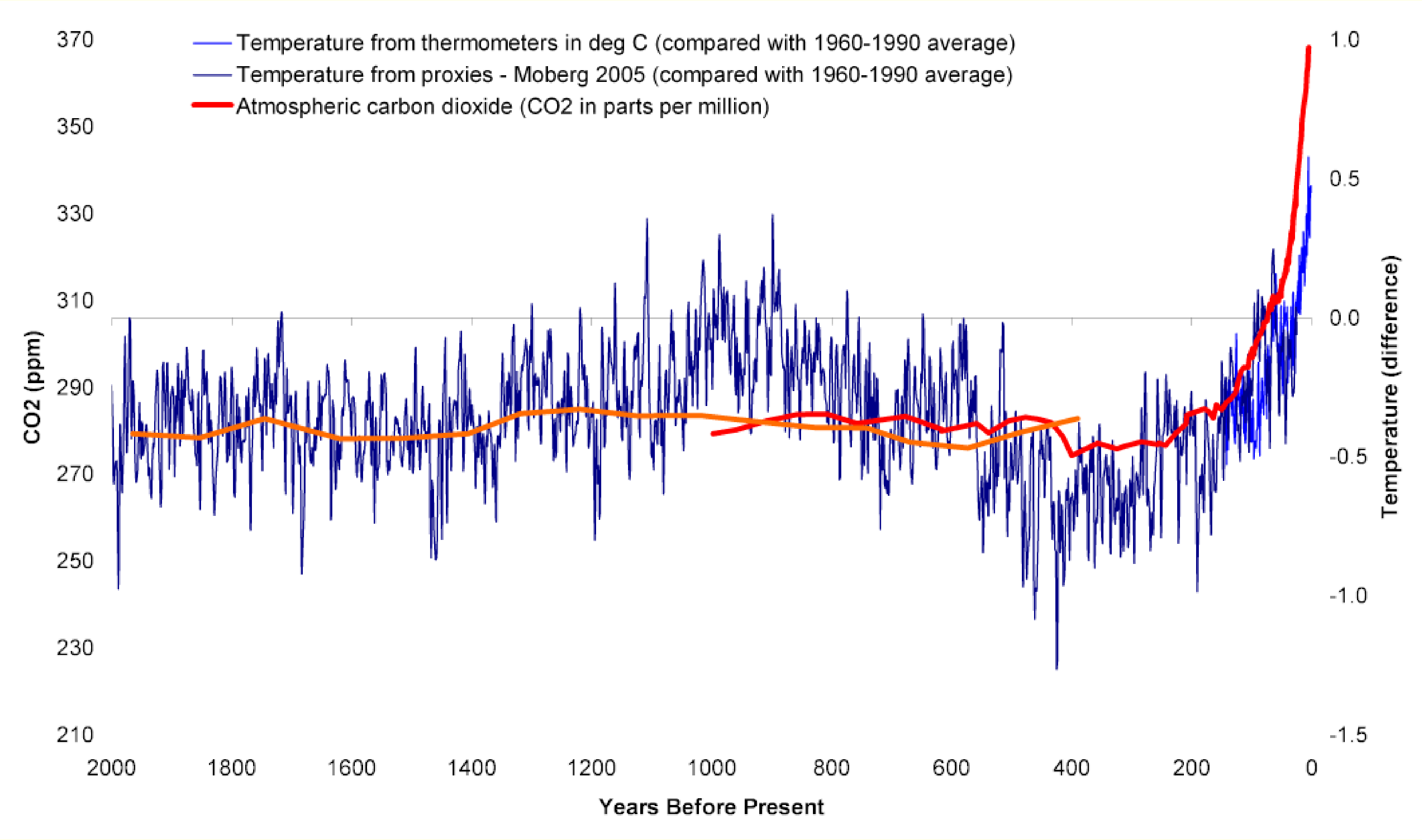


Modulations in long time series

Motivation (variable stars)



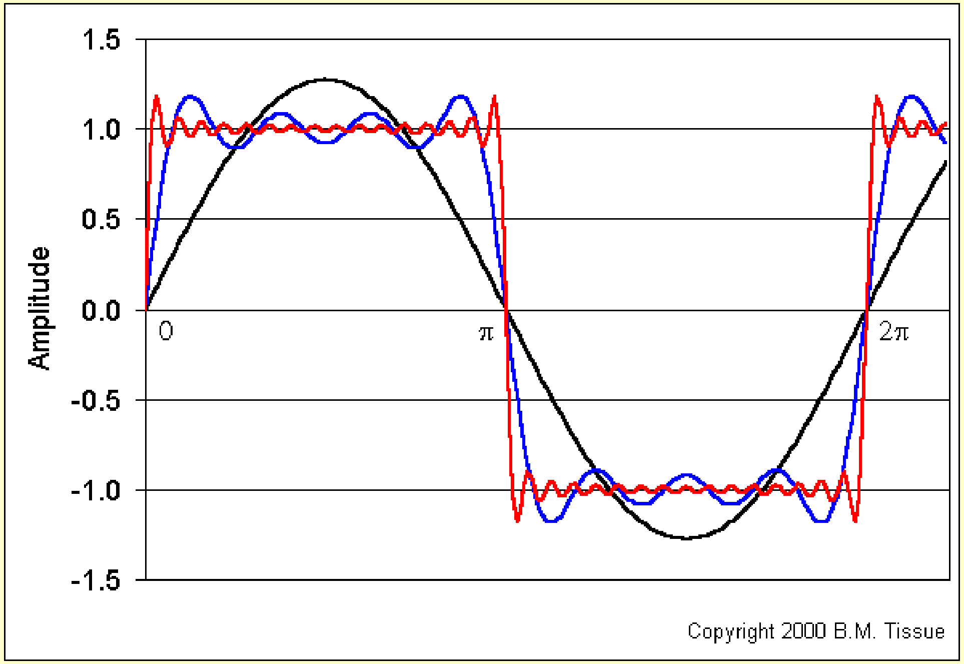
Motivation (climatic variations)



Jean Baptiste Joseph Fourier

(March 21, 1768 - May 16, 1830)

French mathematician and physicist who is best known for initiating the investigation of Fourier series and their application to problems of heat flow. The Fourier transform is also named in his honor.



The Fourier Transform and its Inverse

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\exp(-i\omega t)\,dt$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\exp(i\omega t)\,d\omega$$

Why Fourier Transform is not enough?

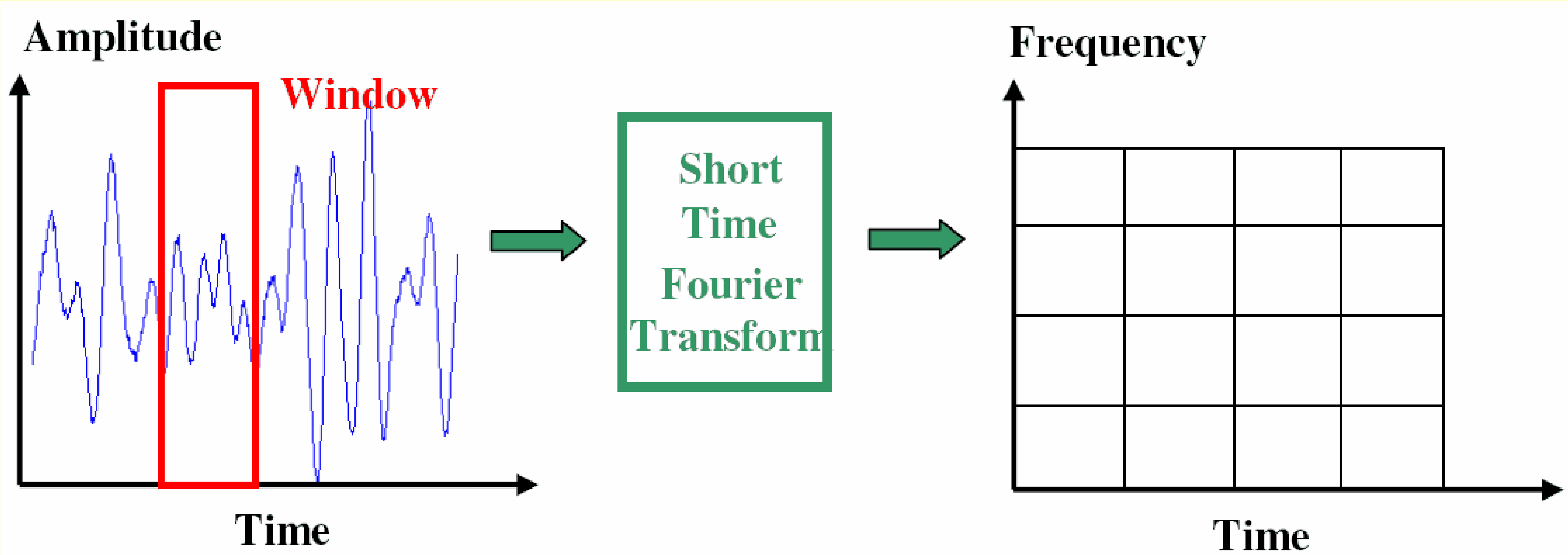
- Historically, researchers have used Fourier analysis to analyze signals. As a result, the term “spectrum” has become almost synonymous with the Fourier transform of the data.

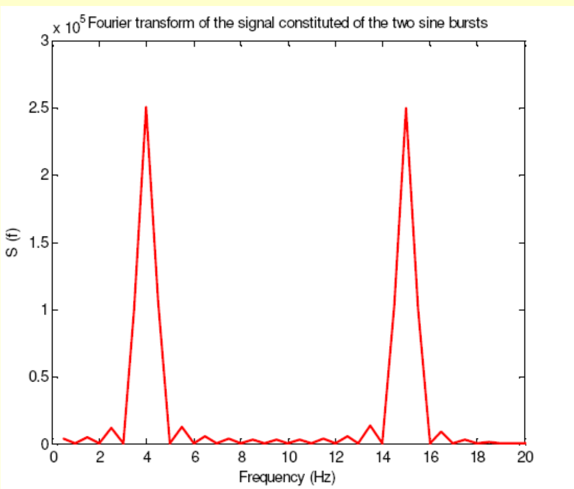
- The spectral domain approach is motivated by the observation that most regular, and hence predictable, behavior of a time series is to be periodic.

- While Fourier transform is valid under extremely general condition, there are some crucial restrictions of the method: (a) the system must be linear (b) the data must be strictly periodic or stationary.

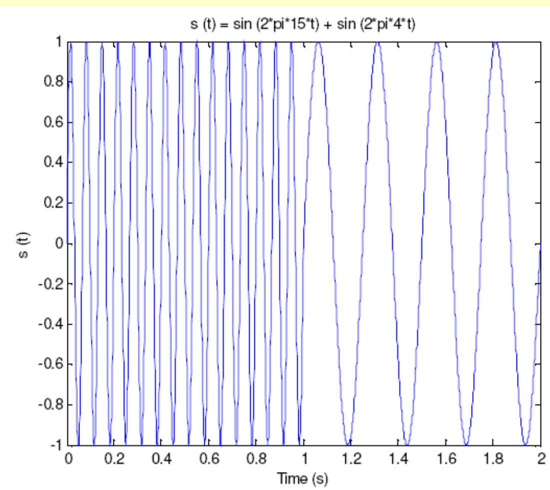
- But this is far the case when dealing with real observations.

Short time Fourier Transform

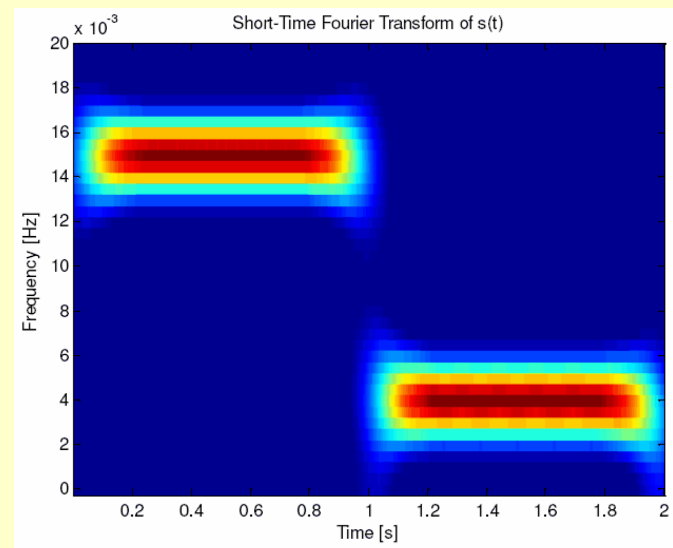




(a) Frequency domain spectrum

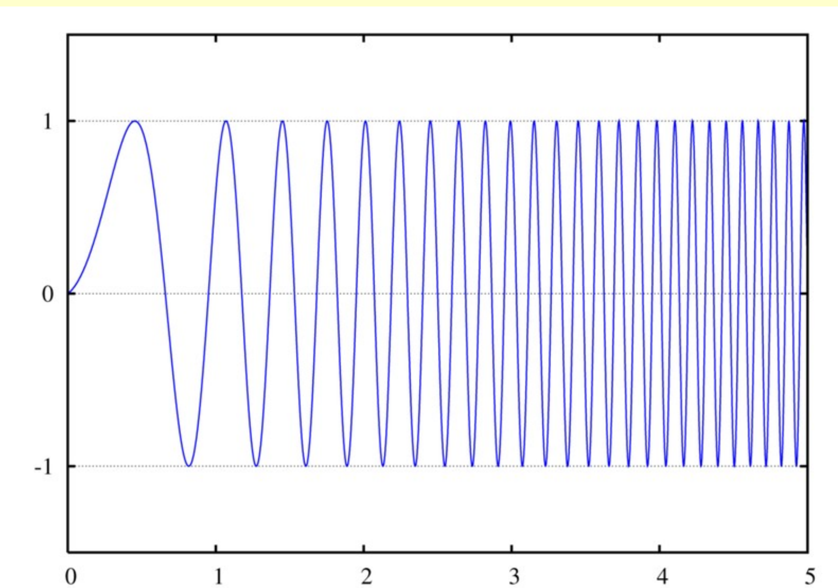


(b) Time domain signal



Gabor Transform

$$X_g(\mu, \omega) = \int_{-\infty}^{\infty} x(t) e^{-\frac{(t-\mu)^2}{2\sigma^2}} e^{-j\omega t} dt$$



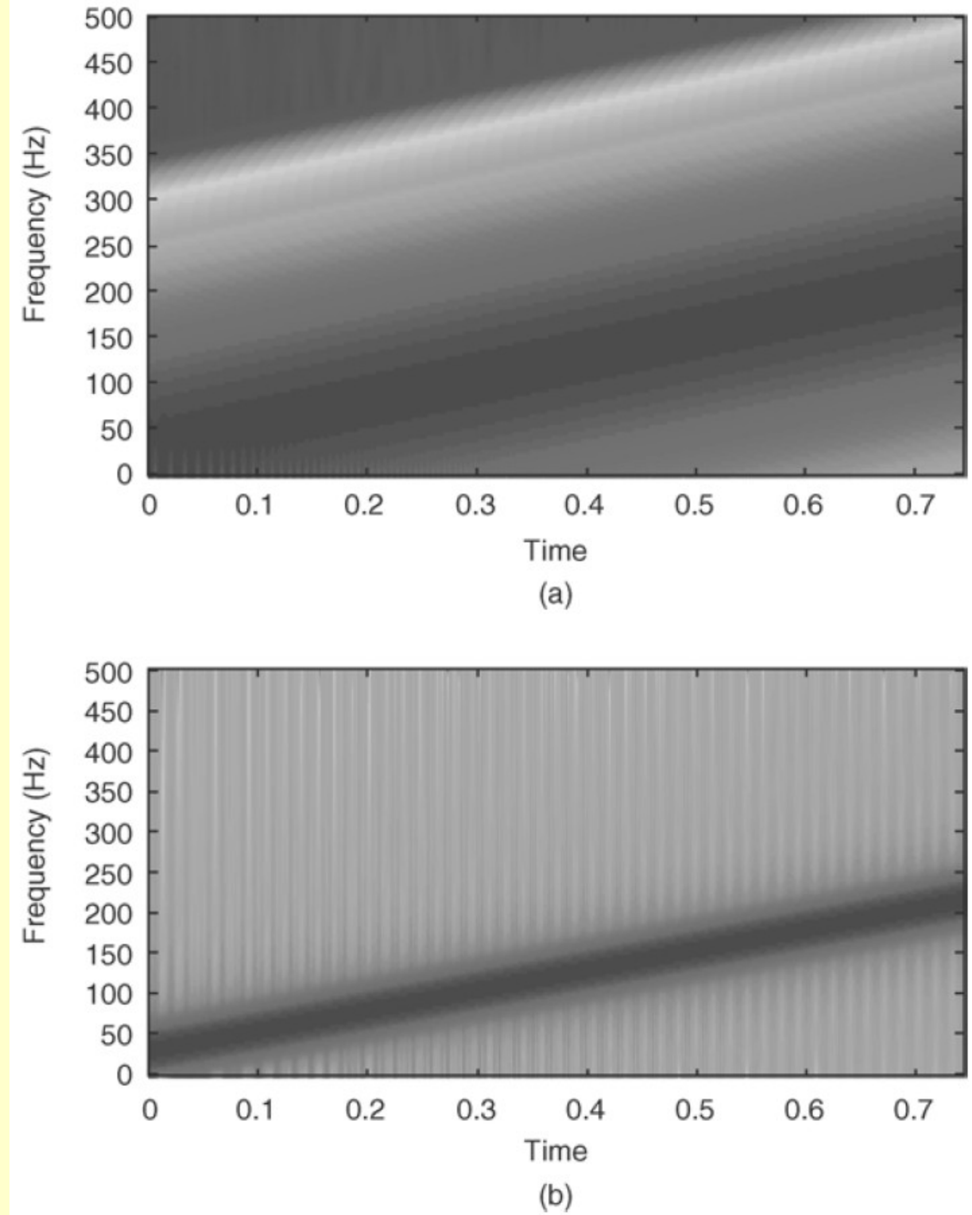
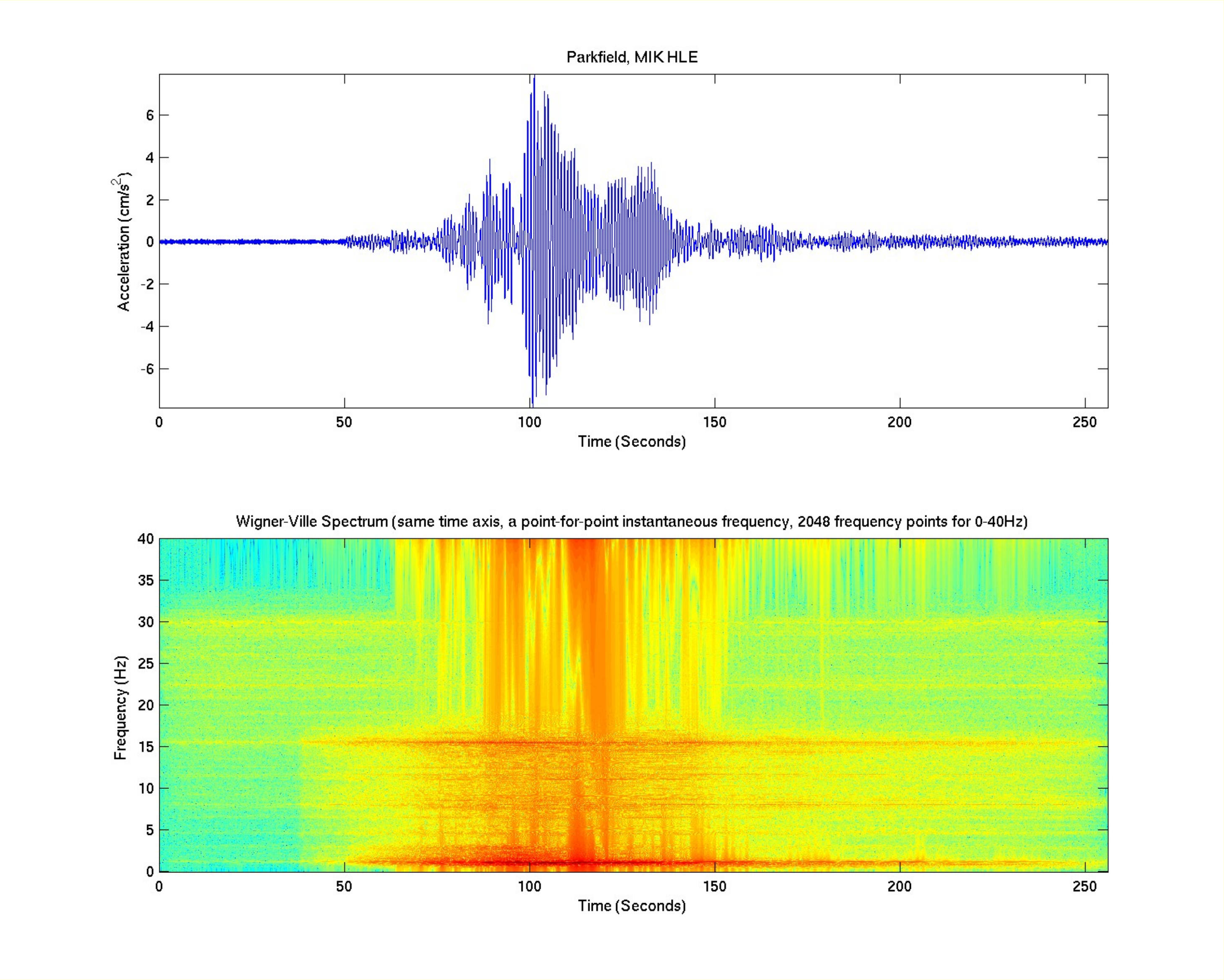


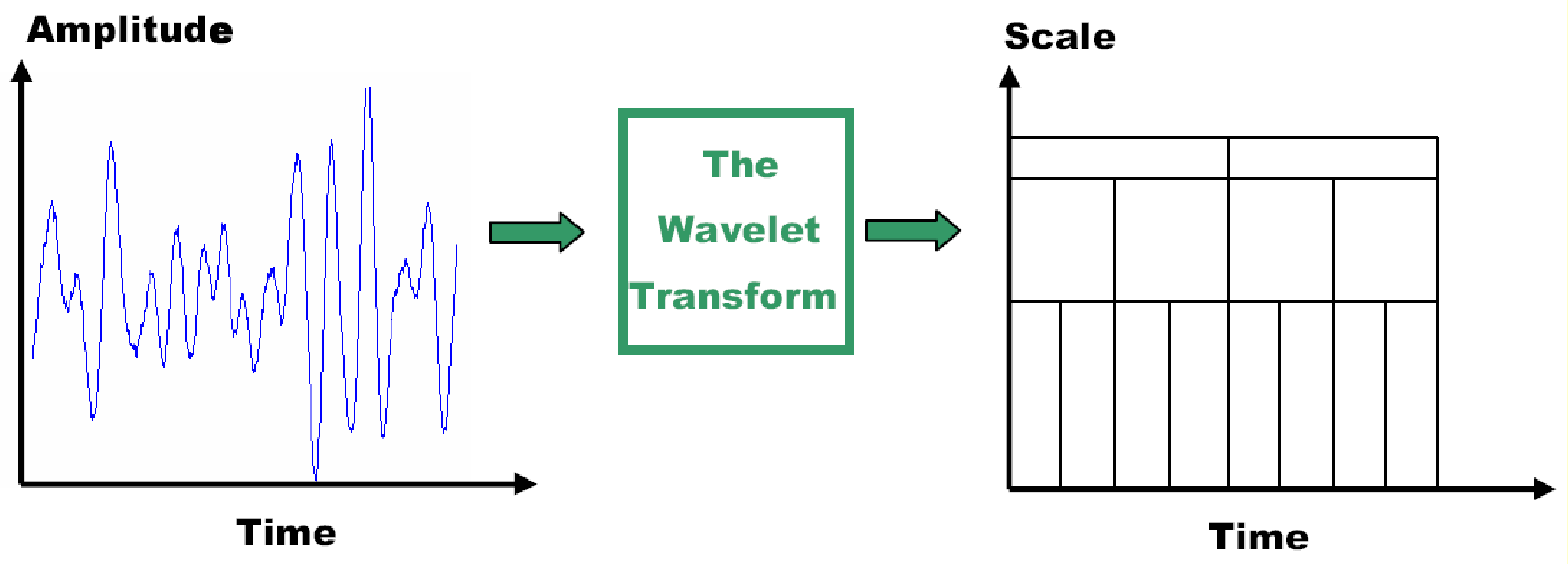
Fig. 10.6. Window width effects in the Gabor transform of a linear chirp. Windowing with a Gaussian of $\sigma = 4$ is shown in panel (a). Panel (b) shows the case of $\sigma = 64$.

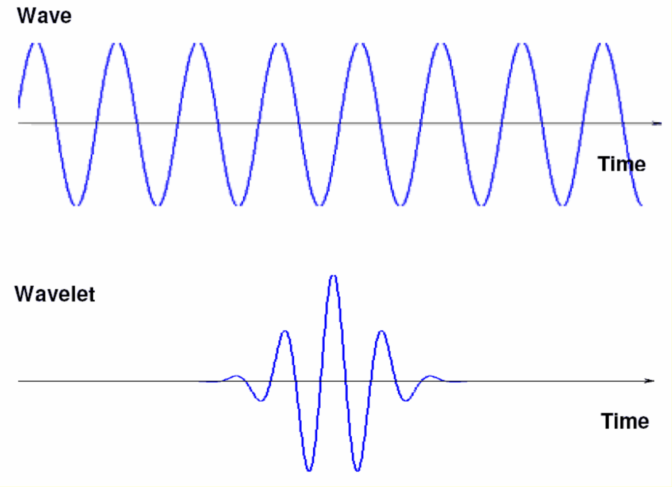
Wigner-Ville Transform

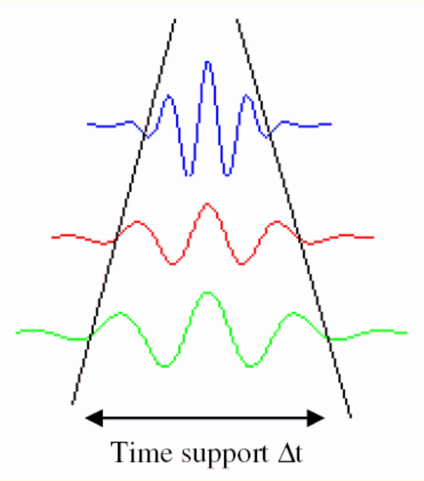
$$W_x(t,\omega) = \int_{-\infty}^{\infty} x(t+\frac{\tau}{2})x^*(t-\frac{\tau}{2})e^{-i\omega\tau}d\tau$$



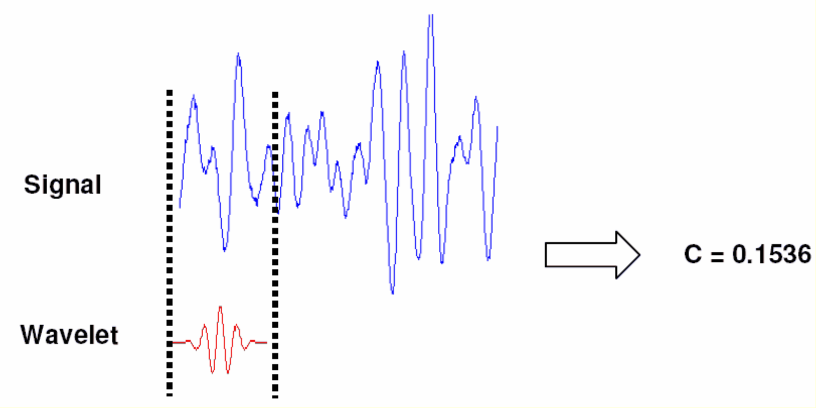
Wavelet Transforms

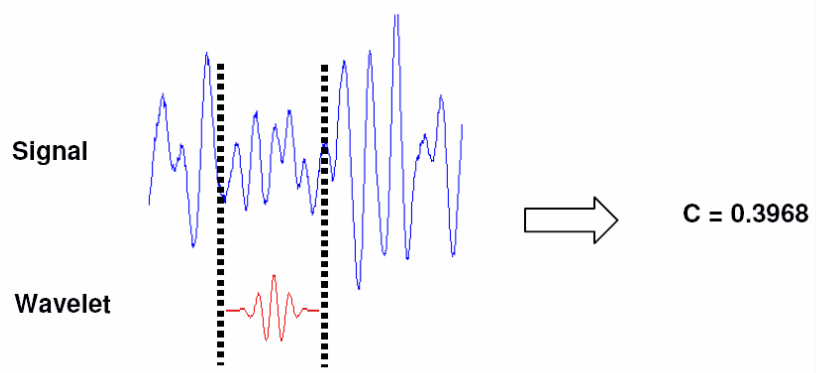


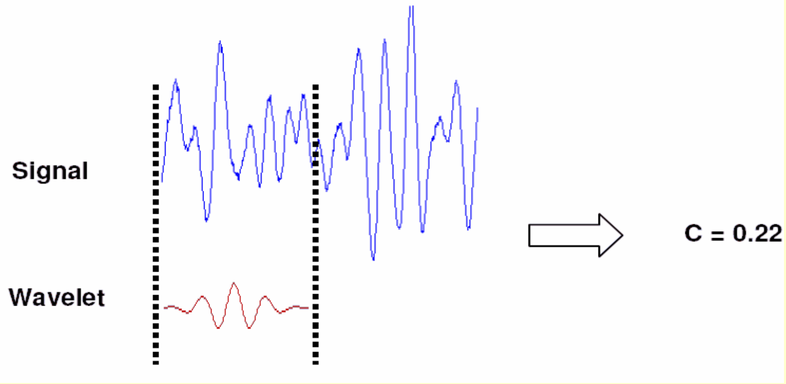


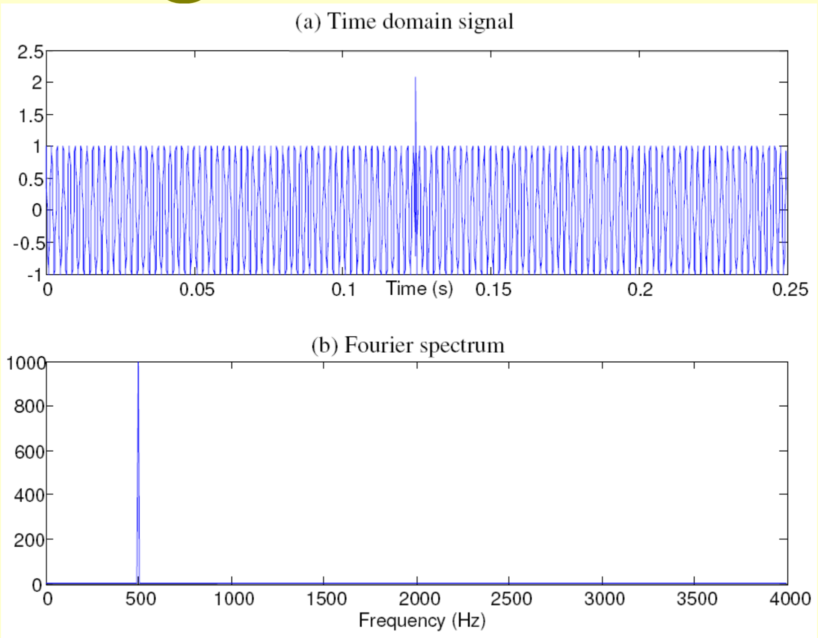


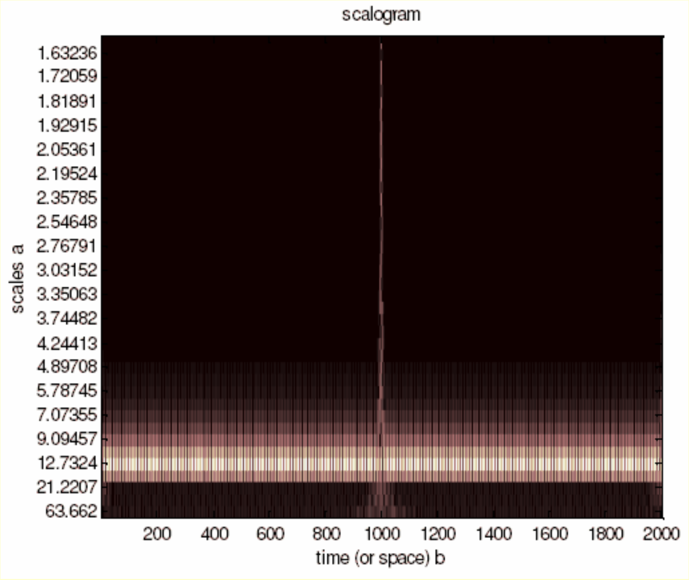
Wavelet scalograms



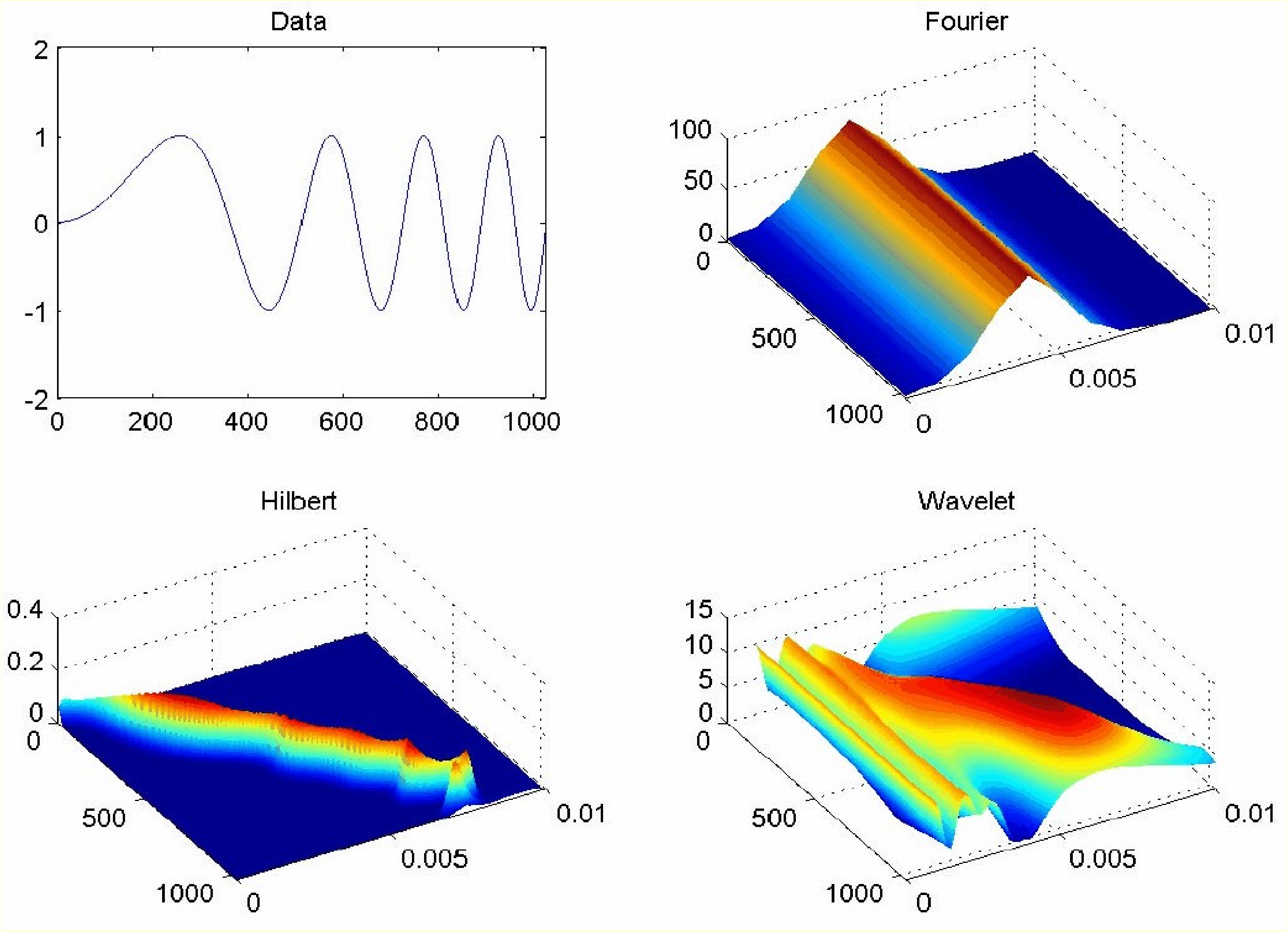








Comparison of the Hilbert-Huang Transform with others



Comparison

	Fourier	Wavelet	Hilbert
Basis	*a priori*	*a priori*	adaptive
Frequency	convolution: global uncertainty	convolution: regional uncertainty	differentiation: local, certainty
Presentation	energy-frequency	energy-time-frequency	energy-time-frequency
Nonlinear	no	no	yes
Nonstationary	no	yes	yes
Feature Extraction	no	discrete: no; continuous: yes	yes
Theoretical base	theory complete	theory complete	empirical

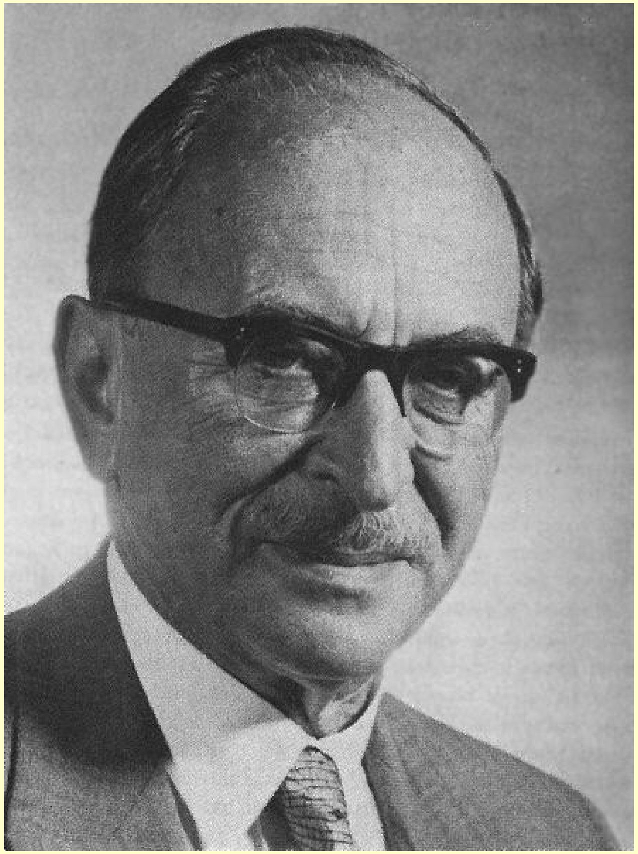
David Hilbert

January 23, 1862, Königsberg

February 14, 1943, Göttingen

German mathematician, recognized as one of the most influential and universal mathematicians of the 19th and early 20th centuries. He invented or developed a broad range of fundamental ideas, in invariant theory, the axiomatization of geometry, and with the notion of Hilbert space, one of the foundations of functional analysis.

Dennis Gabor



Ungaris Gábor Dénes
June 5, 1900, Budapest
February 9, 1979, London
Hungarian physicist and inventor, most notable for inventing holography, for which he later received the Nobel Prize in Physics.

Norden Huang

Born: Wuhan, Hubei, China, December 13, 1937

(Day of the Rape in Nanjing)

Before retirement:

Director of NASA's Goddard Institute of Data Analysis

Chief Scientist for Oceanography

NASA Goddard Space Flight Center

Greenbelt, Maryland 20771

Senior Fellow

NASA Goddard Space Flight Center

Greenbelt, Maryland 29771

Guest Investigator (October 1999 -)

Harvard University, Medical School

Electrocardiography and Arrhythmia Monitoring Laboratories

Margaret and H. A. Rey Laboratory for Nonlinear Dynamics in Medicine

Hilbert Transform and analytic signal

- Hilbert Transform is a convolution integral of X(t) and ($1/\pi t$)
- Hilbert transform consists of passing X(t) through a system which leaves the magnitude unchanged, but changes the phase of all frequency components by $\pi/2$.
- Z(t) is called an analytic signal, in which the imagery part is the Hilbert transform of X(t).
- A(t) is called the envelope of X(t) and θ(t) is called the instantaneous phase of X(t).
- The "instantaneous frequency" f_0 is given by the time derivative of θ(t).

$$Y(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{X(\tau)}{t-\tau} d\tau$$

$$Z(t) = X(t) + jY(t)$$
$$= A(t)\exp(j\theta(t))$$

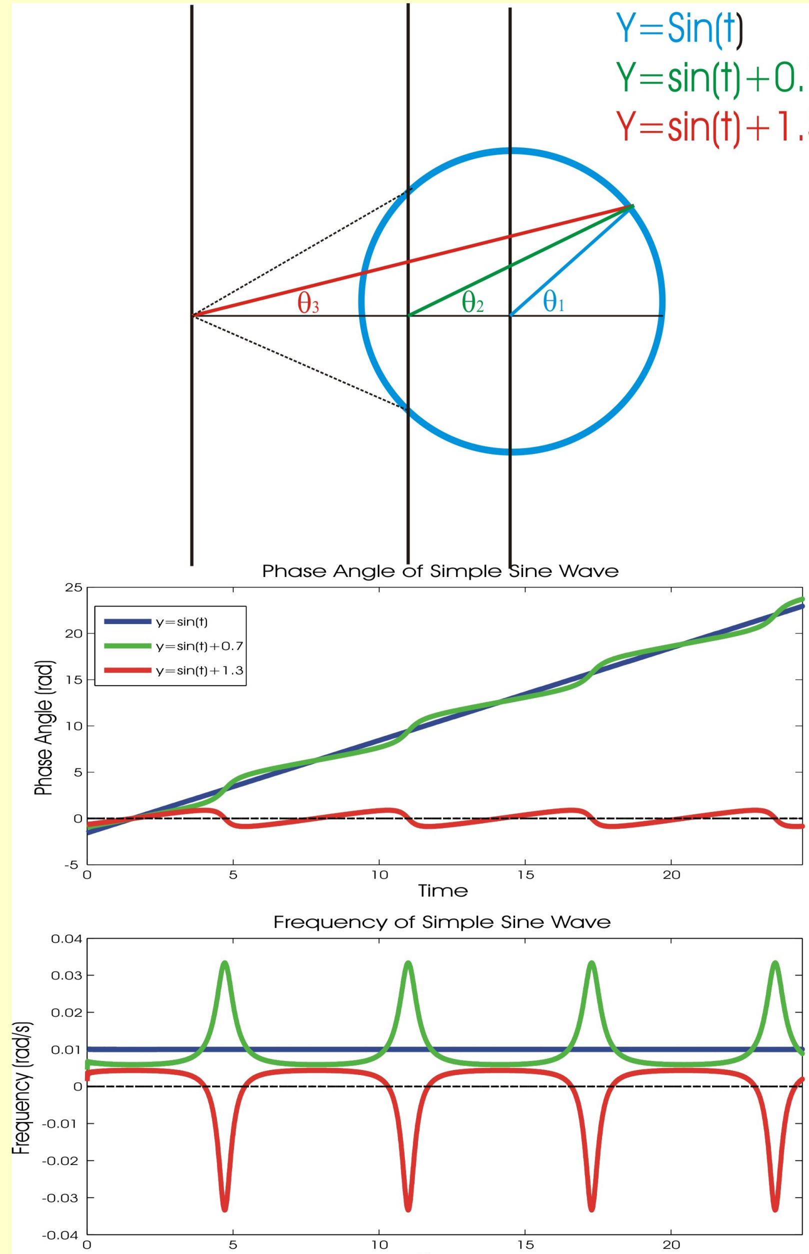
$$A(t) = [X^2(t) + Y^2(t)]^{1/2}$$

$$\theta(t) = \tan^{-1}\left(\frac{Y(t)}{X(t)}\right)$$

$$\omega = 2\pi f_0 = \frac{d\theta(t)}{dt}$$

Meaningful instantaneous frequency

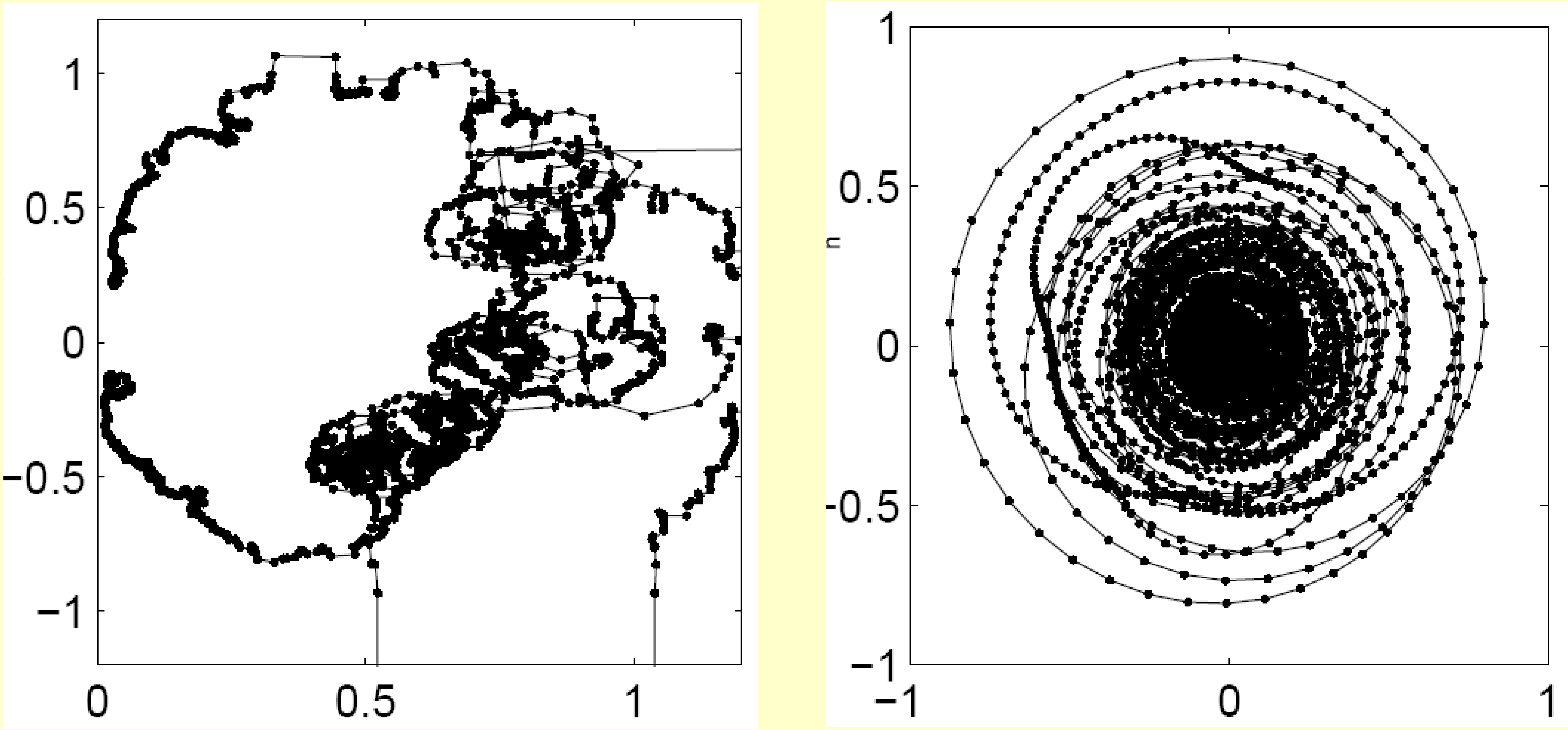
- For a signal to have a meaningful instantaneous frequency, the frequency must be postive (Gabor 1946; Boashash 1992).
- The phase function of sin(t) is a straight line. The phase plot of x-y plane is a simple circle of unit radius.
- If we change the signal mean by adding a small amount , i.e. x(t)= +sin(t) the phase plot is still a simple circle independent of the value of . However, the center of the circle is displaced by the amount .
- If <1 the Fourier spectrum has a DC term but the mean zero-crossing frequency is still the same.
- If >1, both phase function and instantaneous frequency will assume negative value.



From Hilbert to Huang

- Hilbert transform was originally developed to solve integral equations. The Hilbert transform yields another time series that has been phase shifted by 90° via its integral definition. By itself, this hold little interest for us.
- However, when Gabor (1946) developed his concept of analytic signal $z(t)= X(t)+jY(t)$. A particularly interesting case occurs when the signal is band-limited. Then we can rewrite $z(t)=A(t)\exp(j(\theta(t))$, a local time-varying wave with amplitude of $A(t)$ and phase $\theta(t)$.
- Unfortunately, most signals are not band limited. Huang's contribution is the development of a method which he calls a sifting process that decomposes a wide class of signals into a set of band-limited functions (Intrinsic Mode Functions, IMFs)
- It is now a possibility for us to extract instantaneous information from the signal.

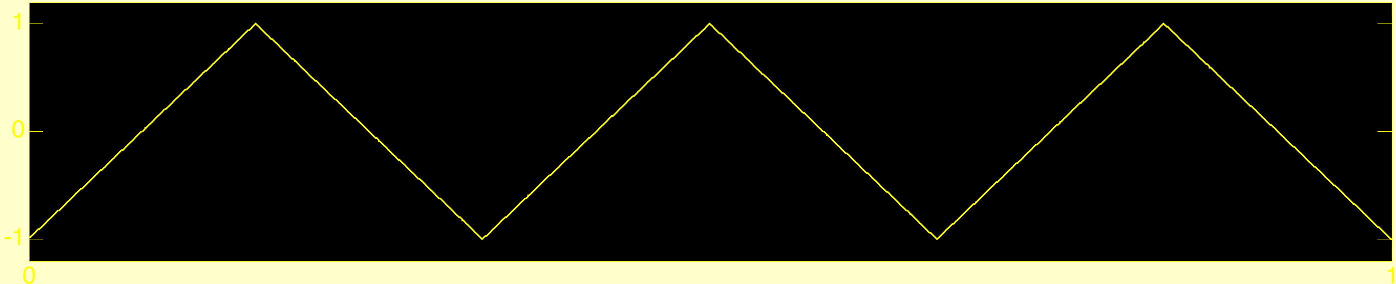
Hilbert Transform for signal and IMF (Intrinsic Mode Function)



Empirical Mode Decomposition

Algorithmic definition

A LF sawtooth



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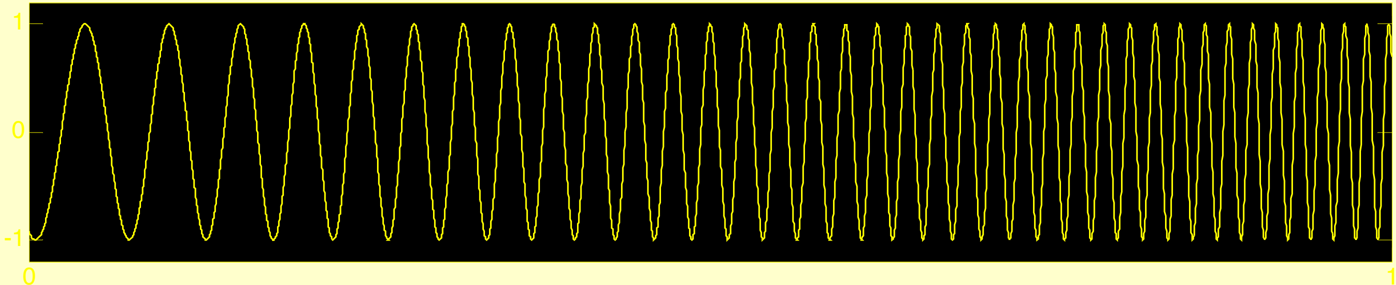
A linear FM



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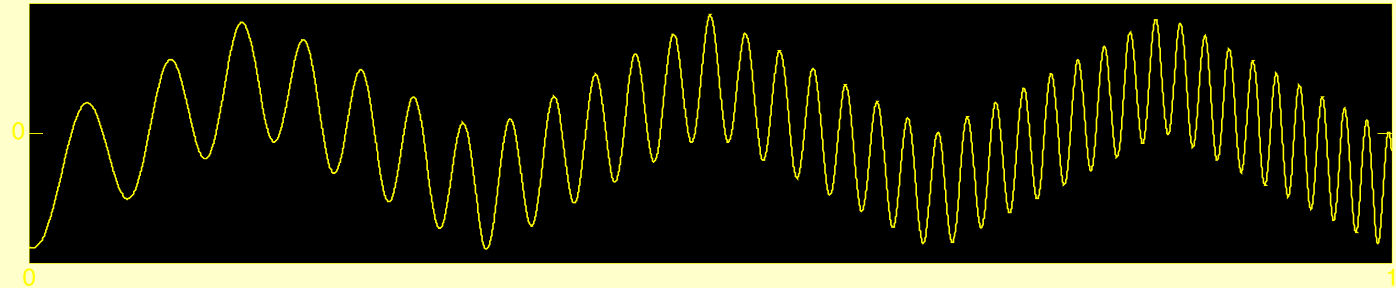

Empirical Mode Decomposition

Algorithmic definition

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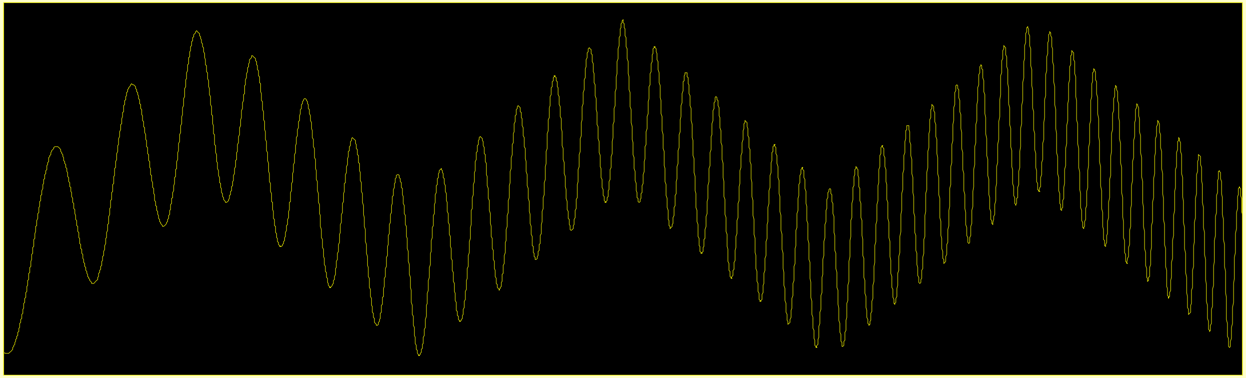

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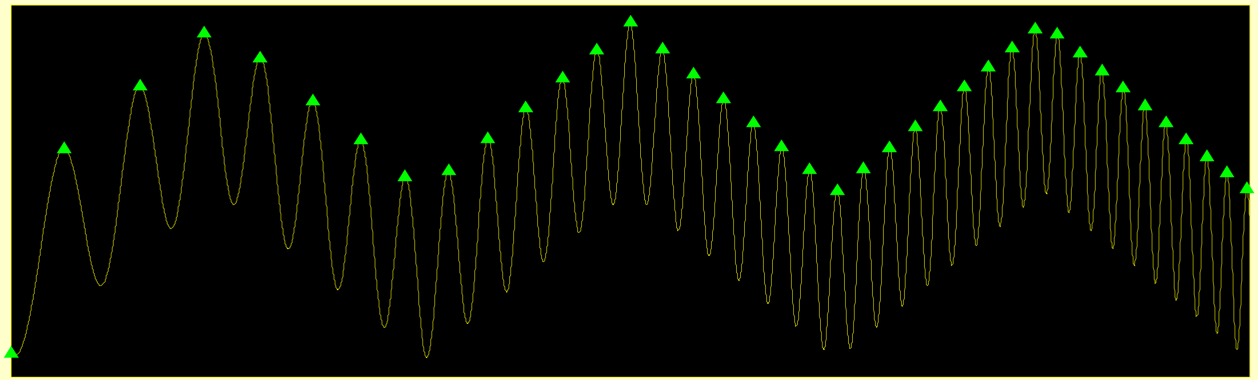

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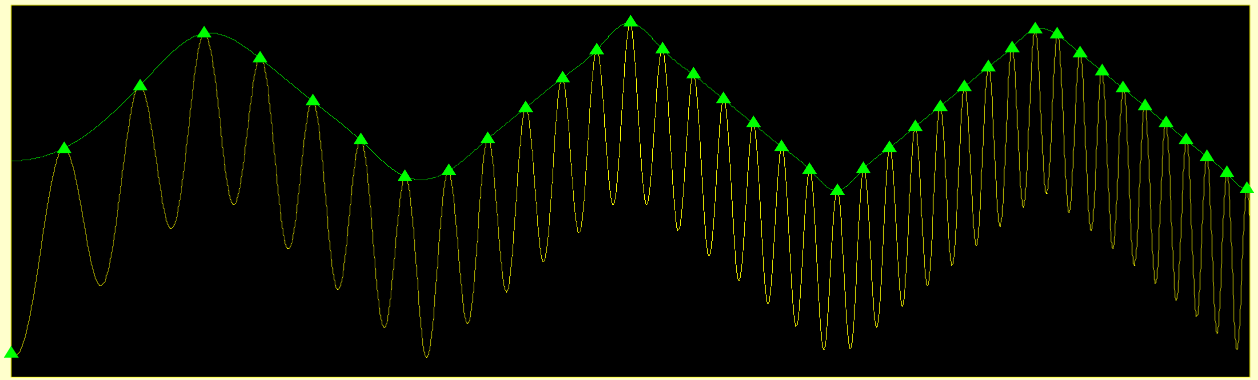

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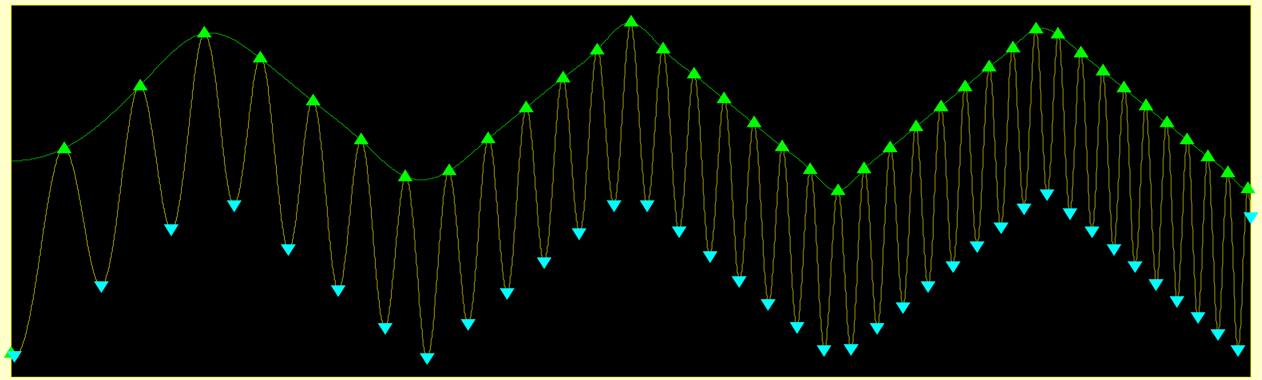

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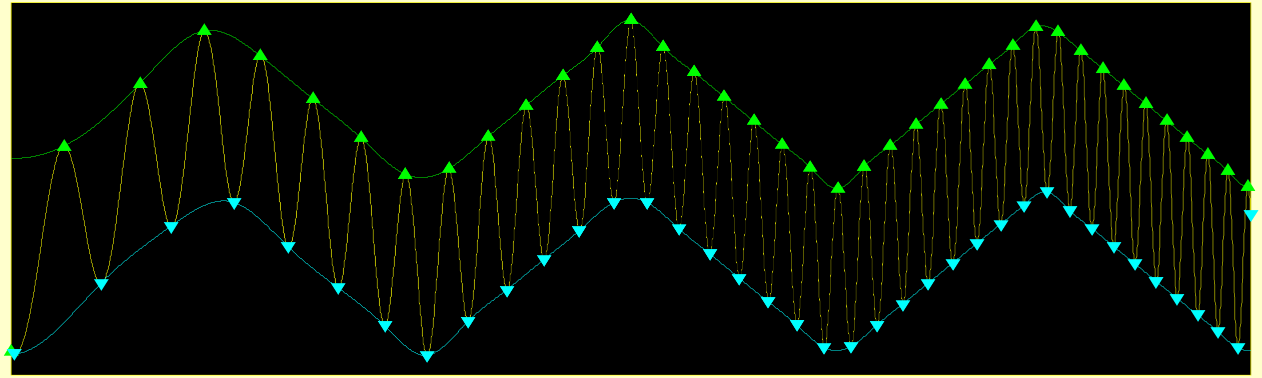

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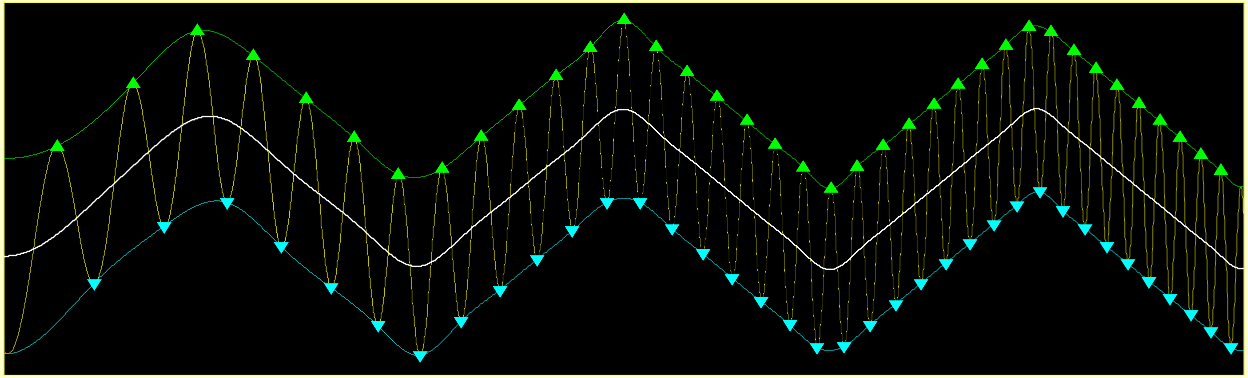

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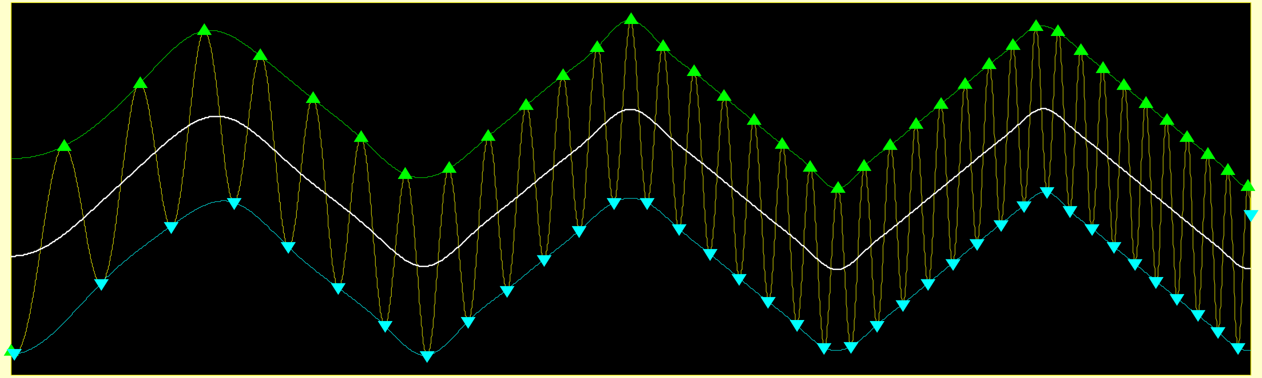

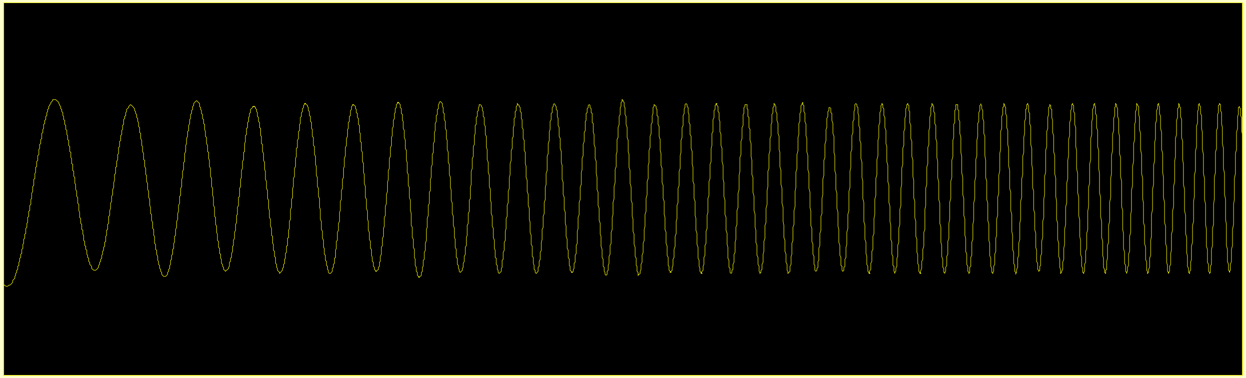

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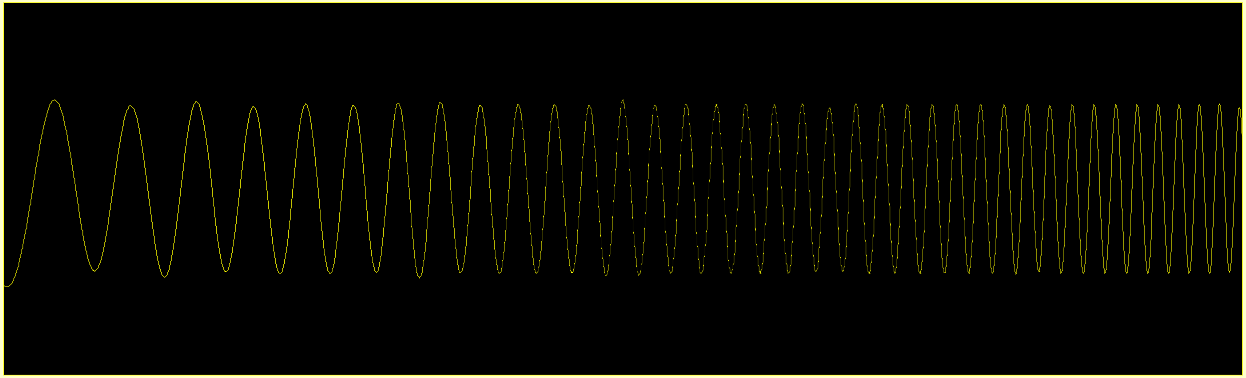

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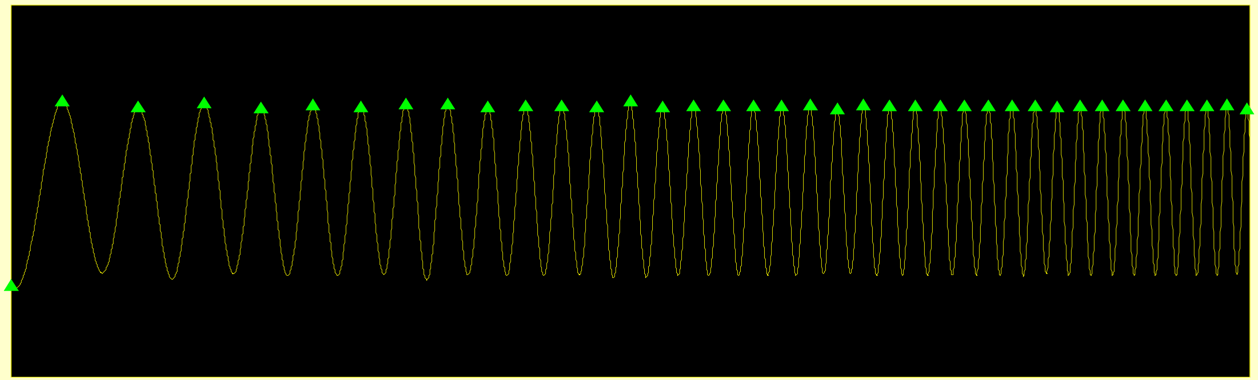

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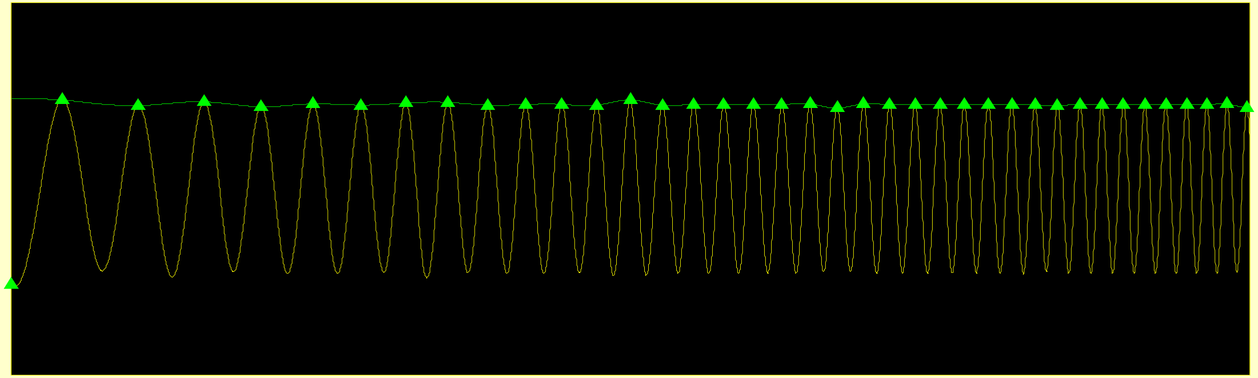

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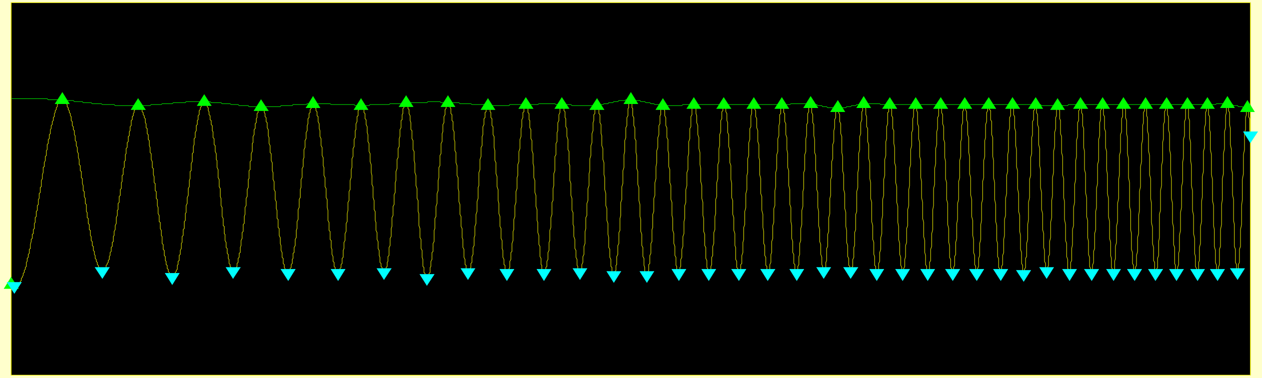

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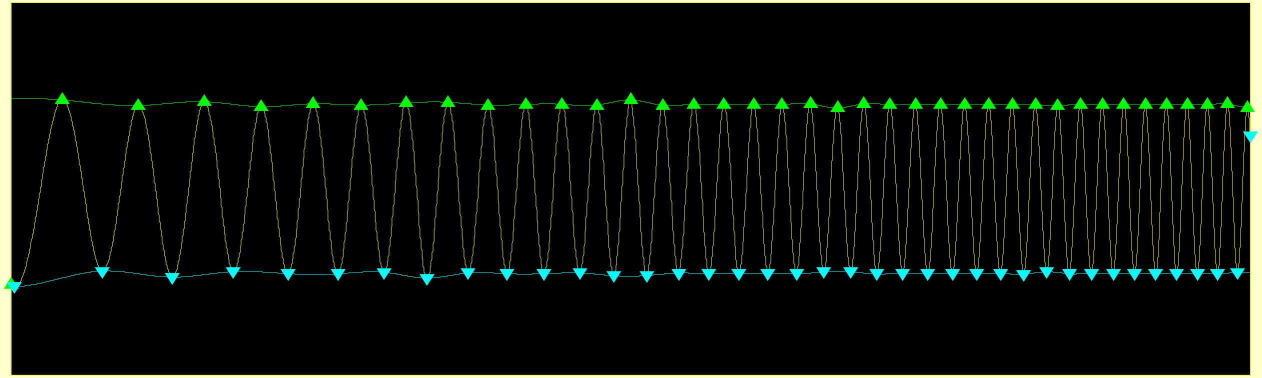

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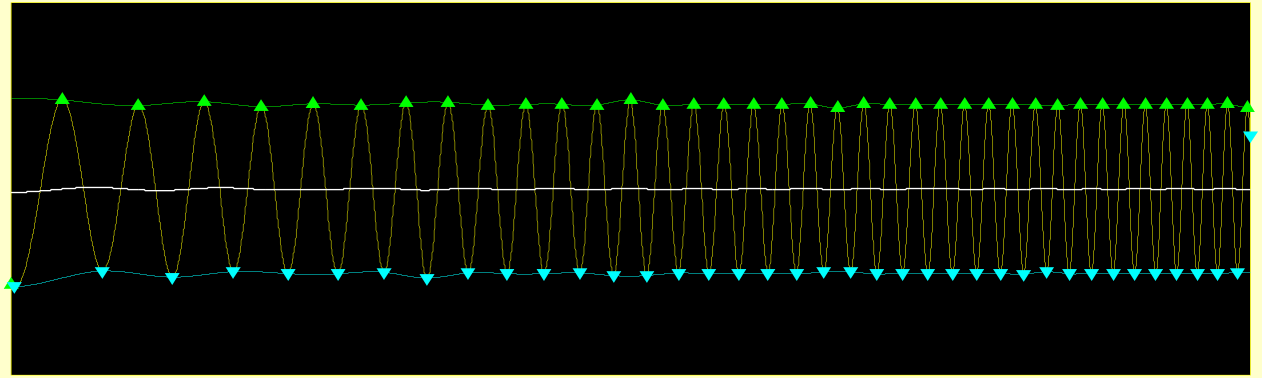
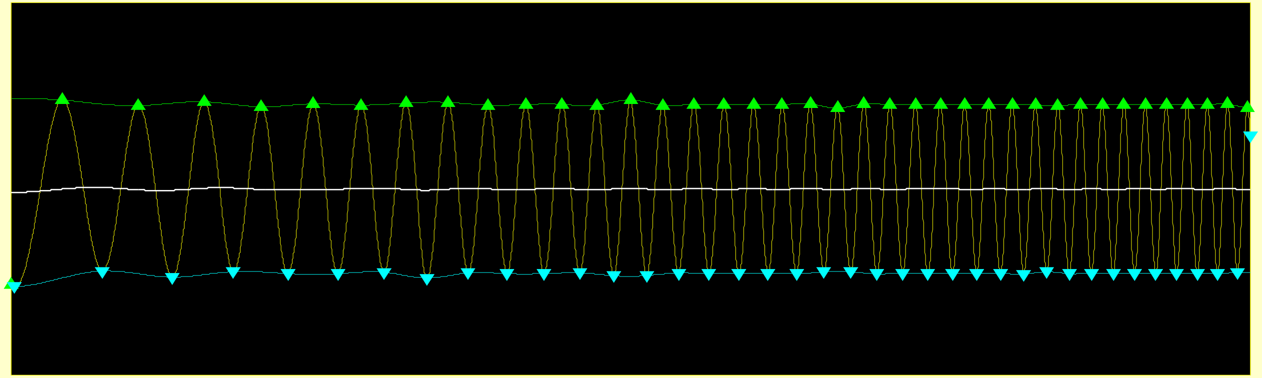
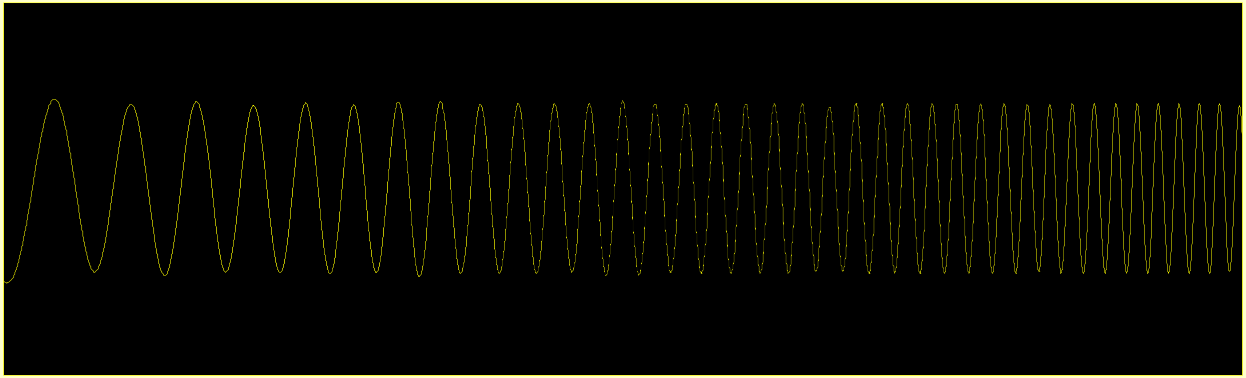


Empirical Mode Decomposition

Algorithmic definition

SIFTING PROCESS





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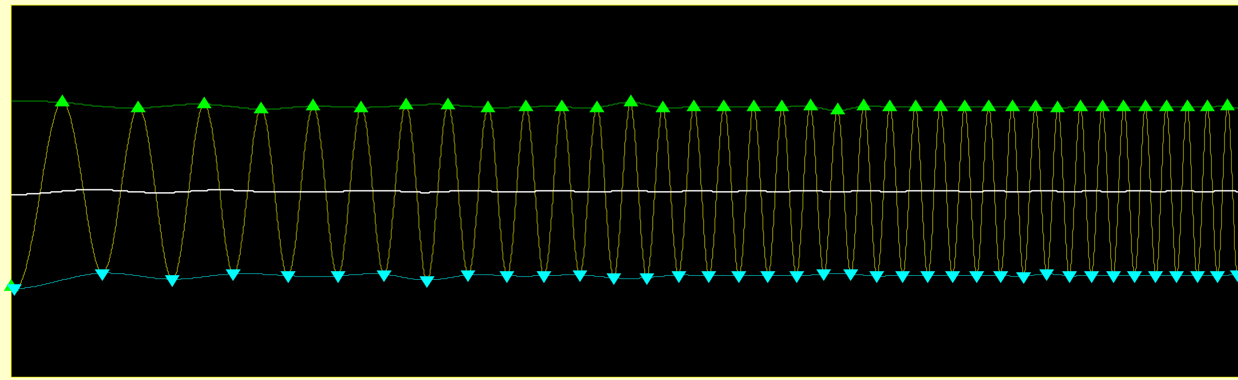
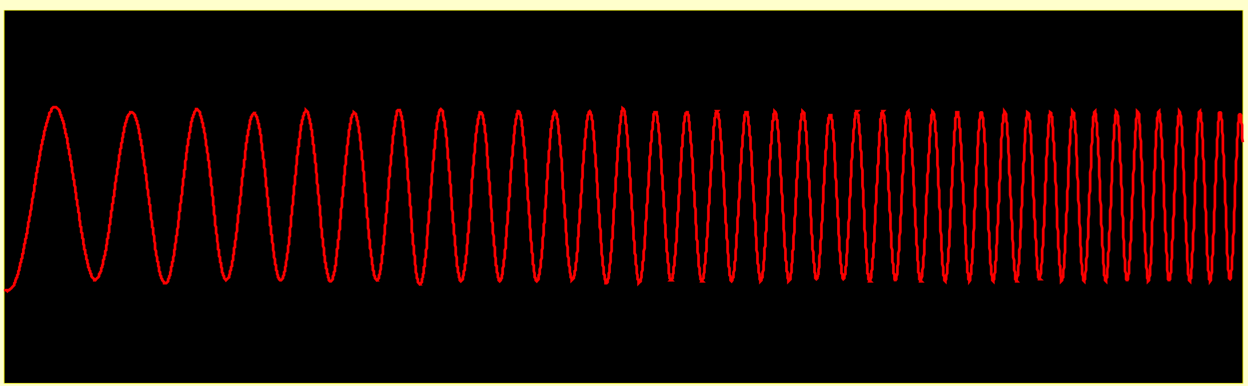


First Intrinsic Mode Function



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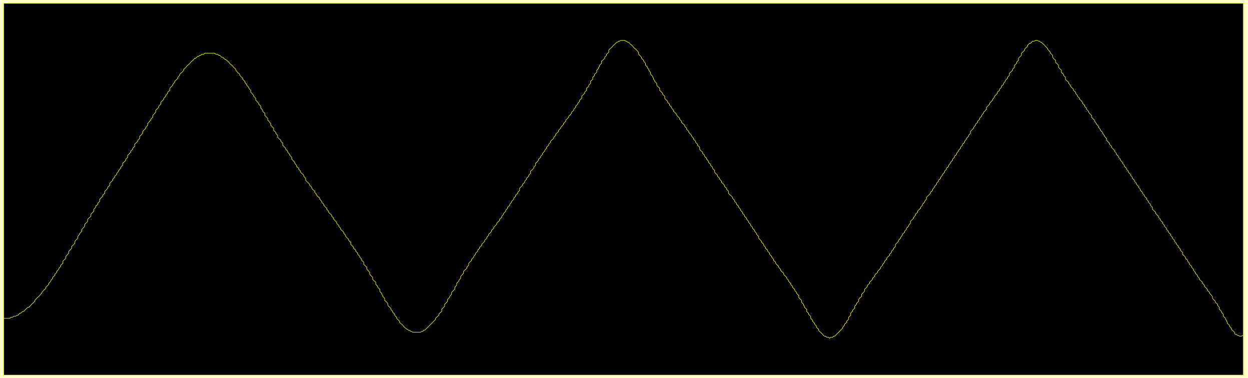

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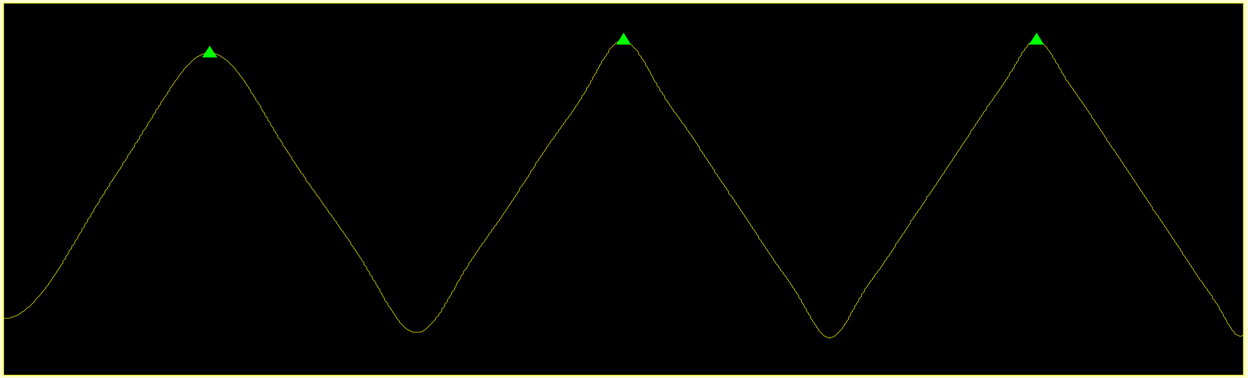

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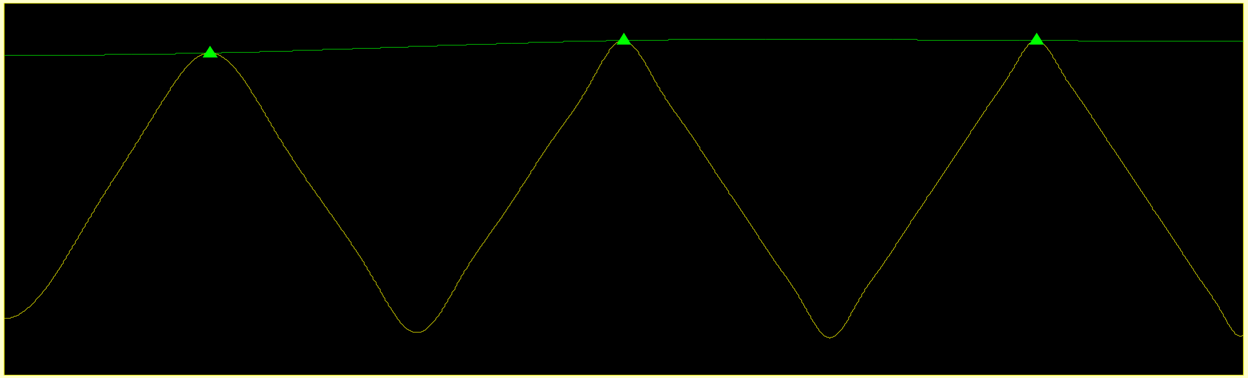

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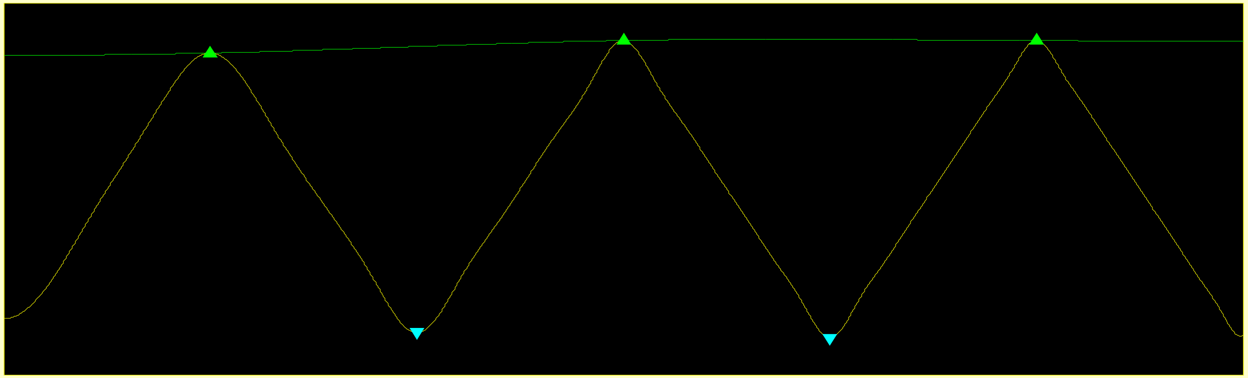

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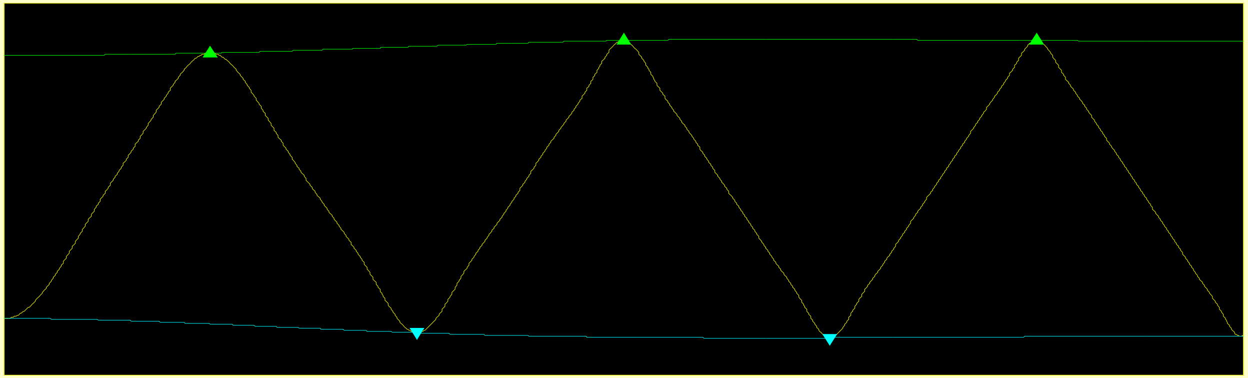

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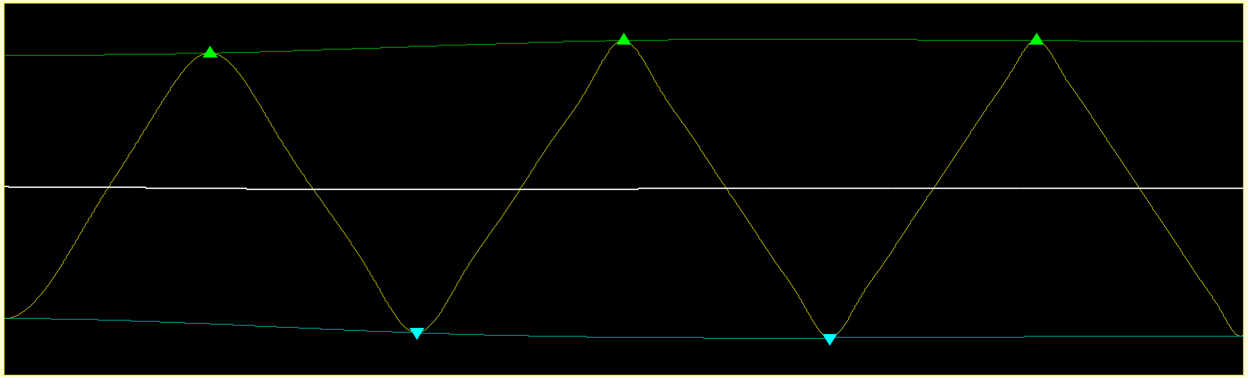

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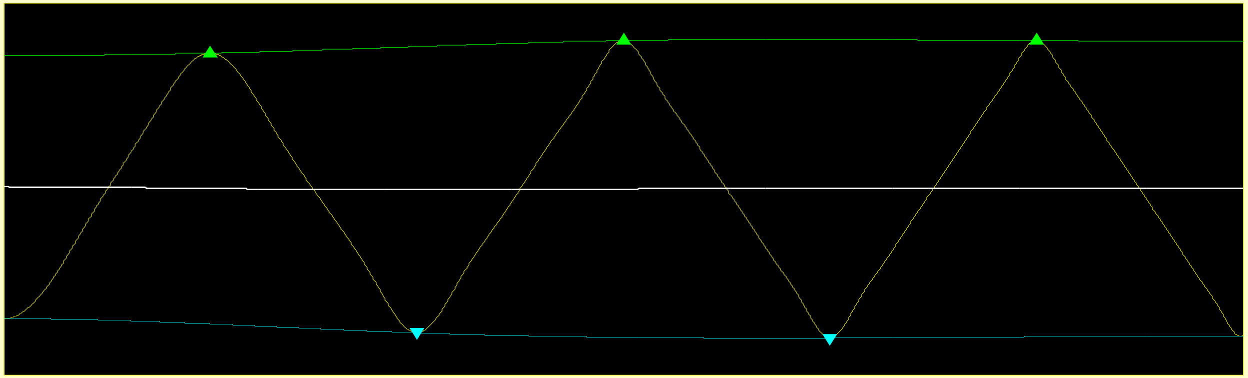
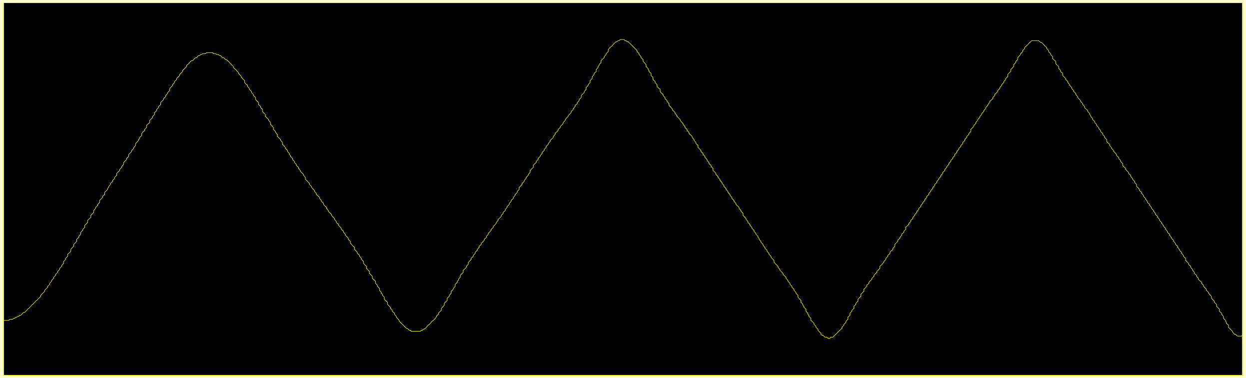


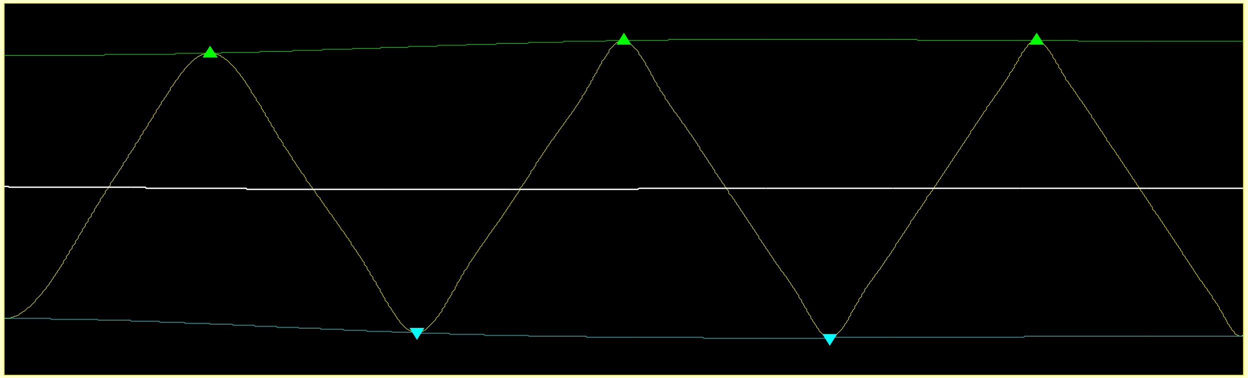


Empirical Mode Decomposition

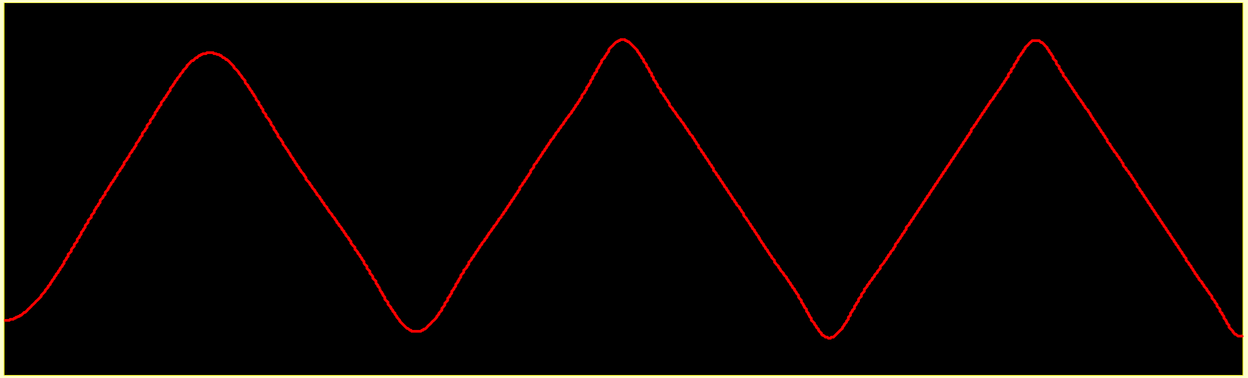
Algorithmic definition

S
I
F
T
I

O
C
E
S
S



Second Intrinsic Mode Function

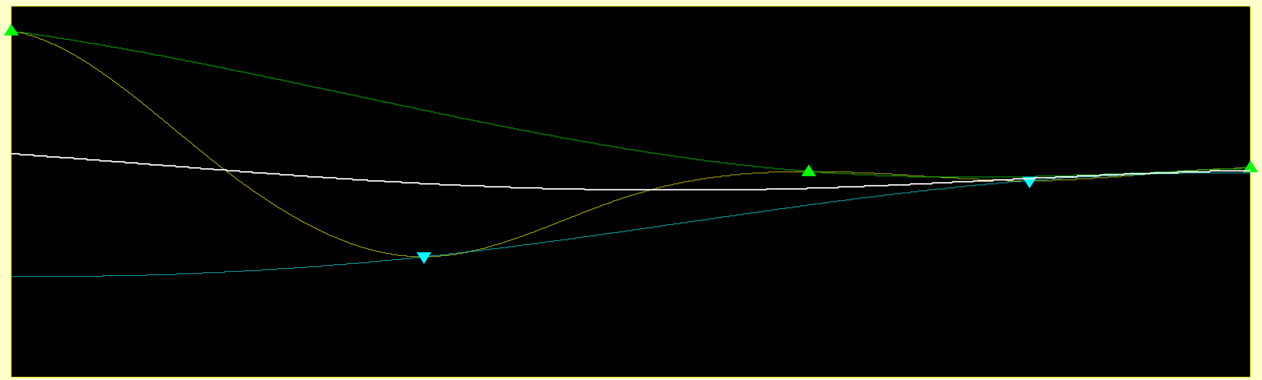


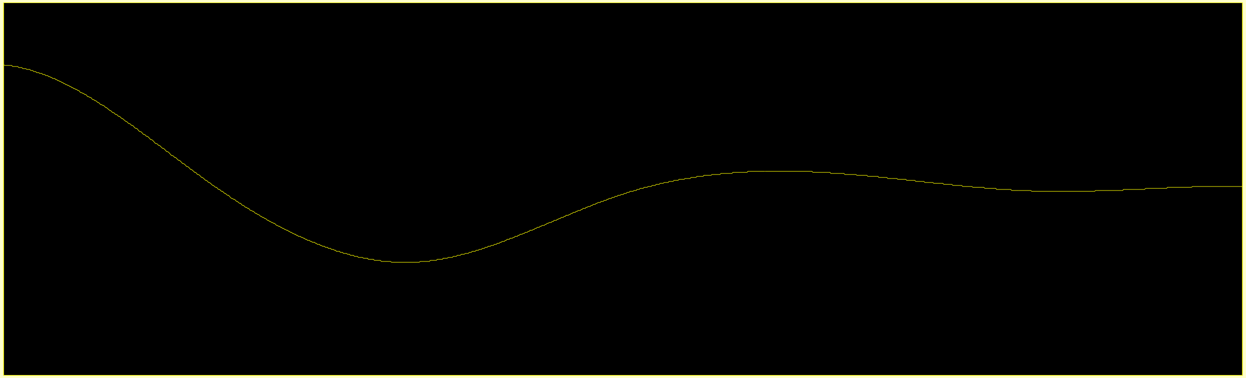
Empirical Mode Decomposition

Algorithmic definition

S
I
F
T
I
N
G

P
R
O
C
E
S
S

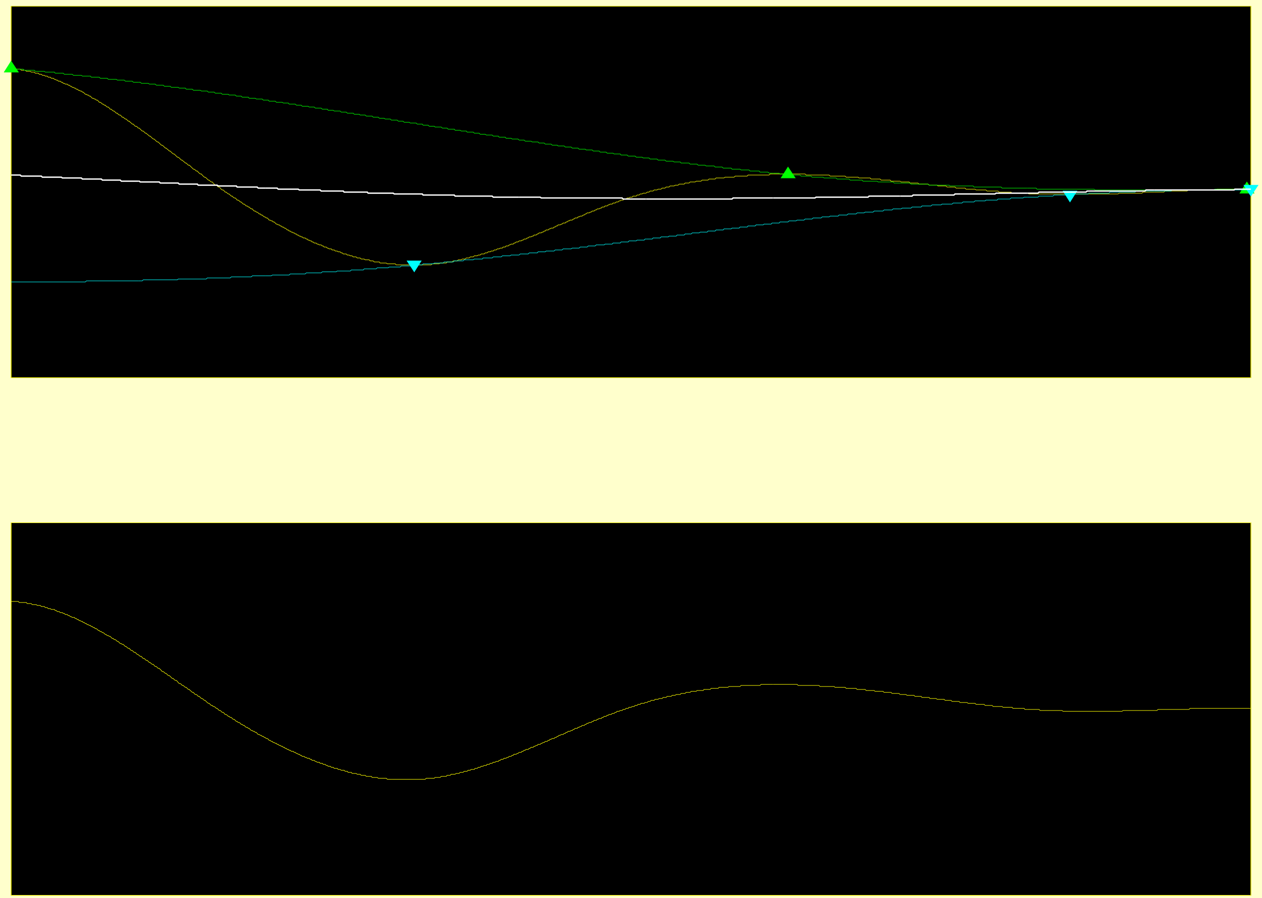




Empirical Mode Decomposition

Algorithmic definition

SIFTING PROCESS

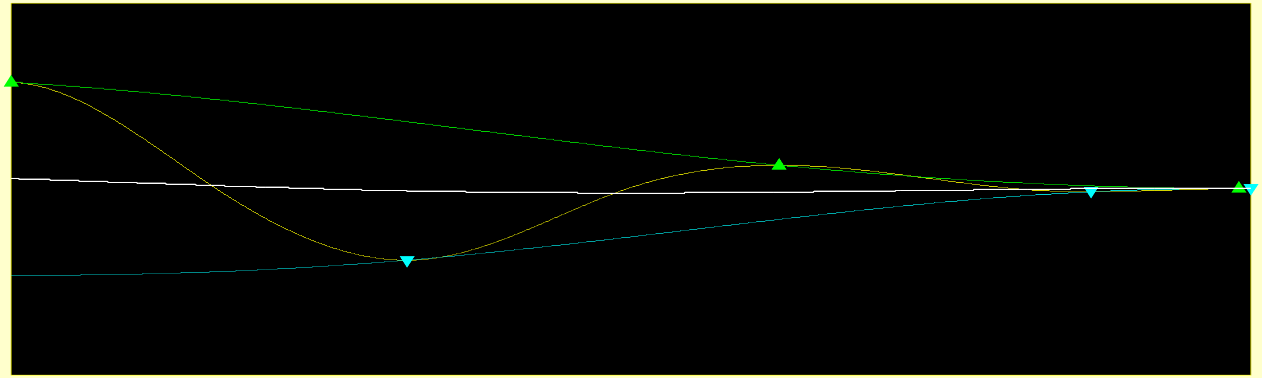


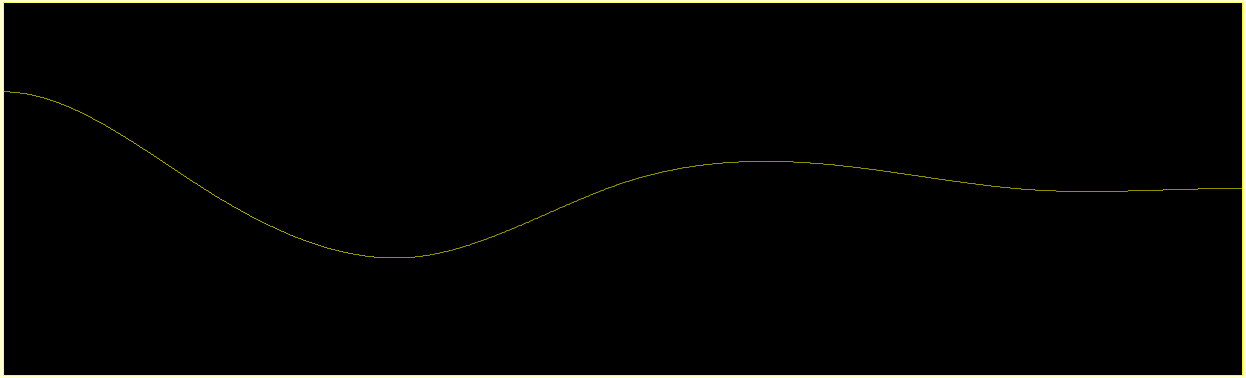
Empirical Mode Decomposition

Algorithmic definition

S I F T I N G

P R O C E S S



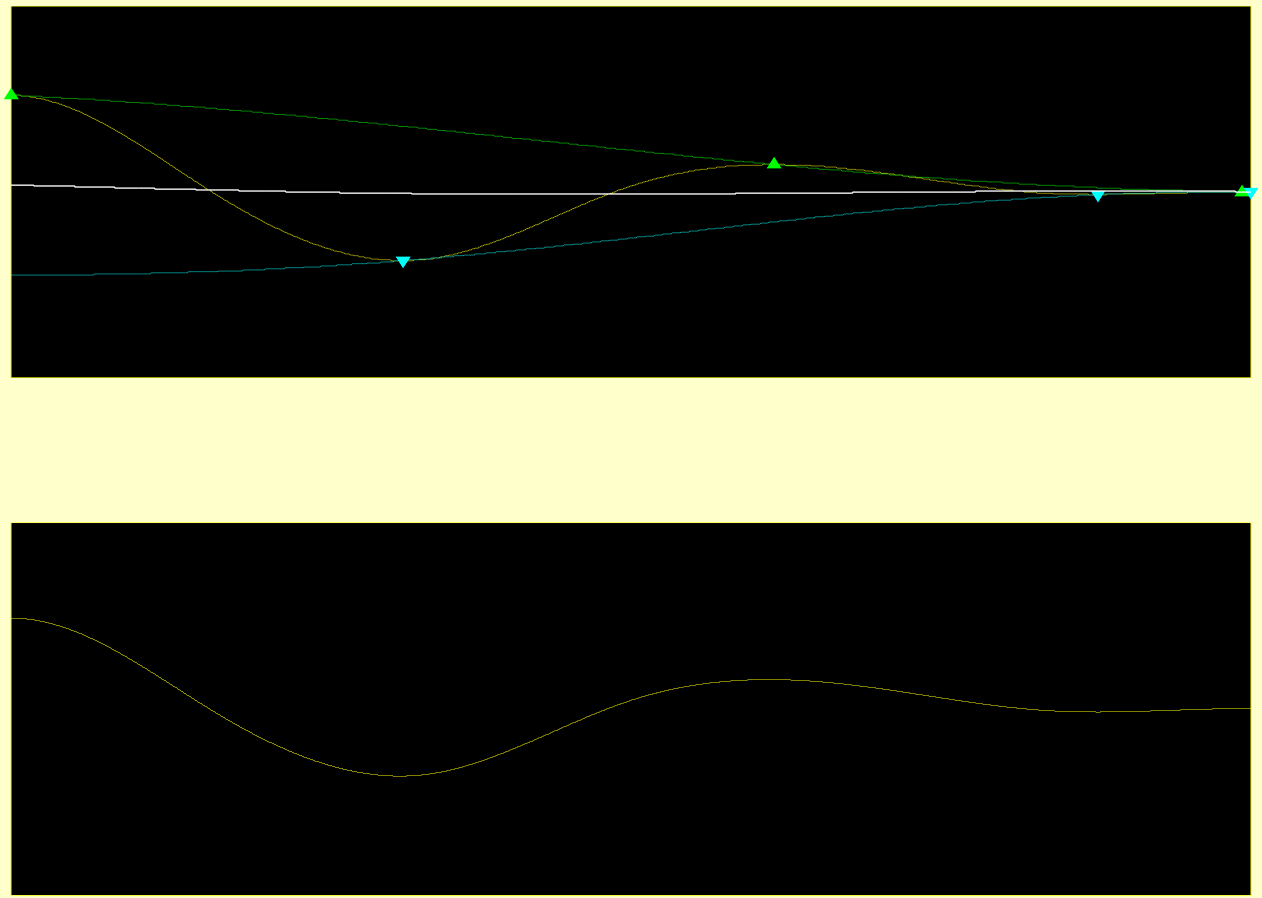


Empirical Mode Decomposition

Algorithmic definition

S
I
F
T
I
N
G

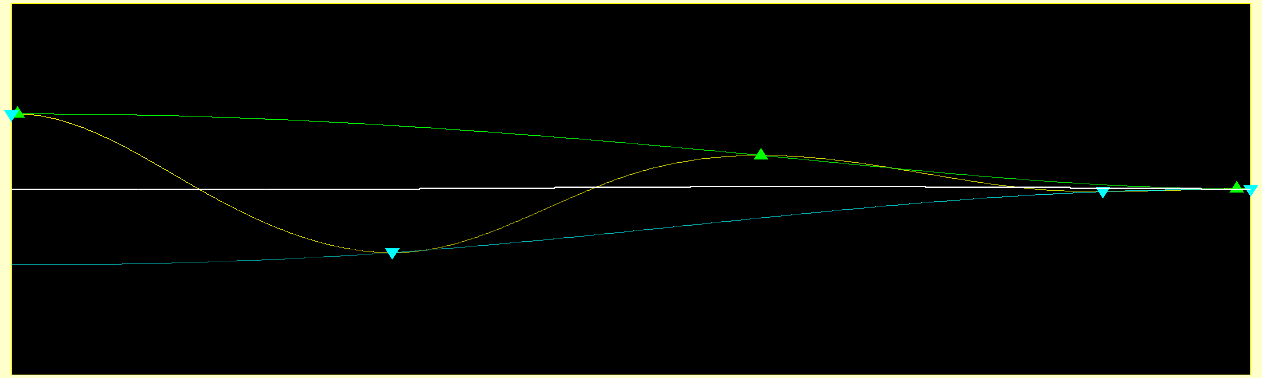
P
R
O
C
E
S
S

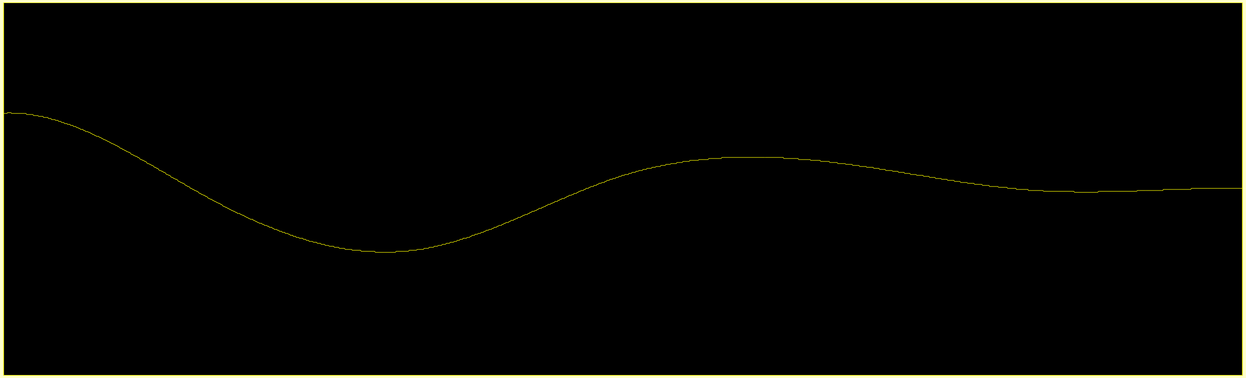


Empirical Mode Decomposition

Algorithmic definition

SIFTING PROCESS



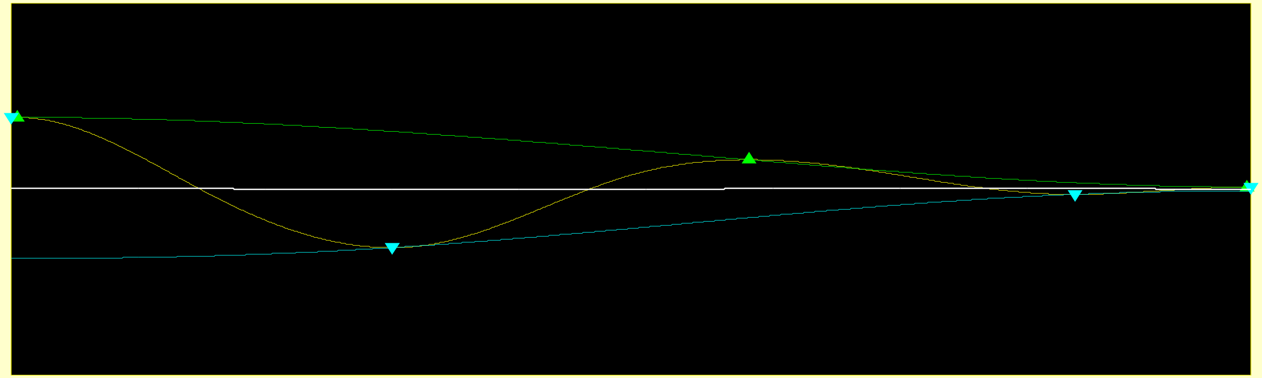


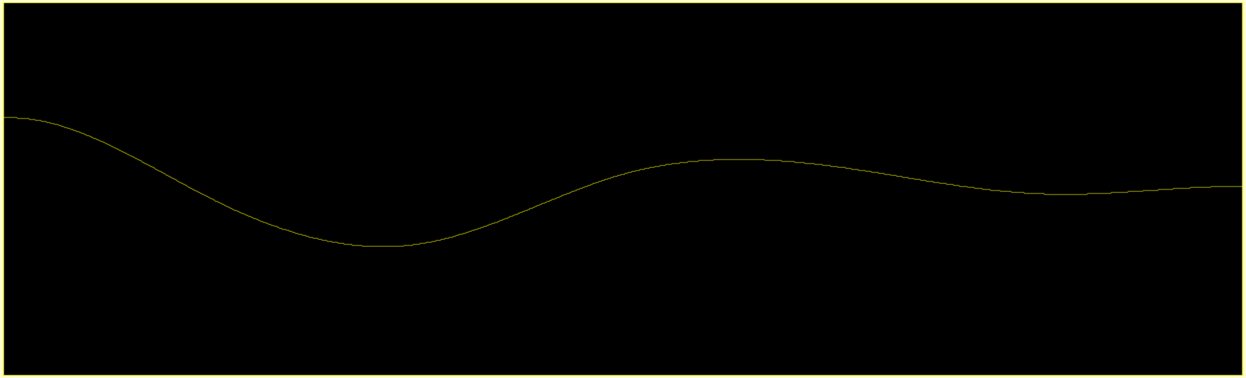
Empirical Mode Decomposition

Algorithmic definition

S I F T I N G

P R O C E S S

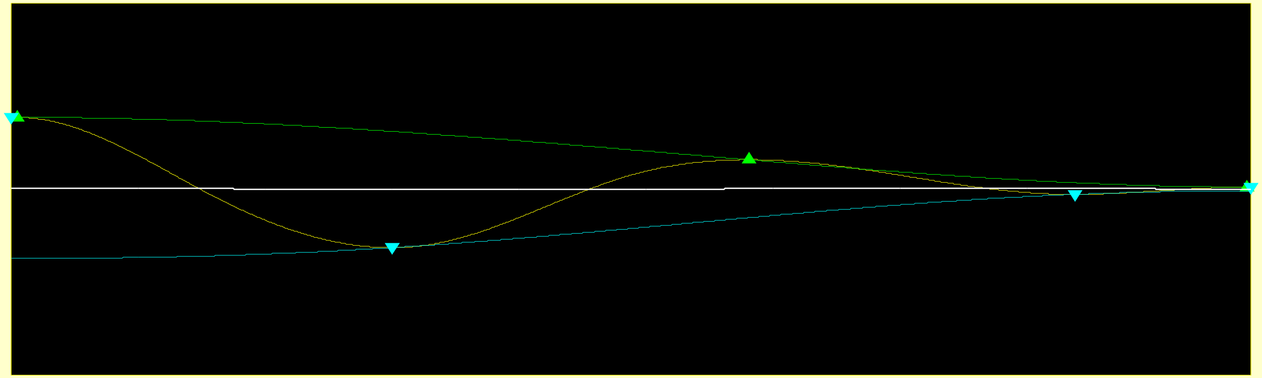




Empirical Mode Decomposition

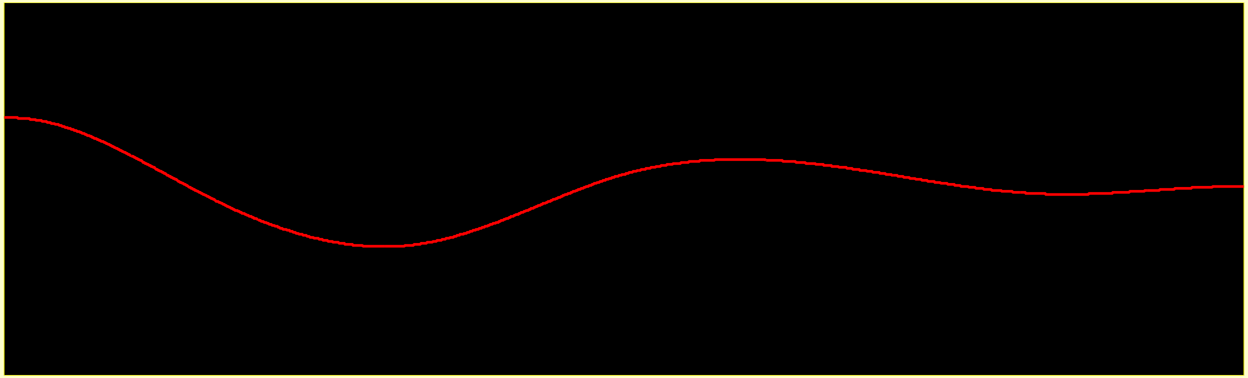
Algorithmic definition

S
I
F
T
I



O
C
E
S
S

Third Intrinsic Mode Function

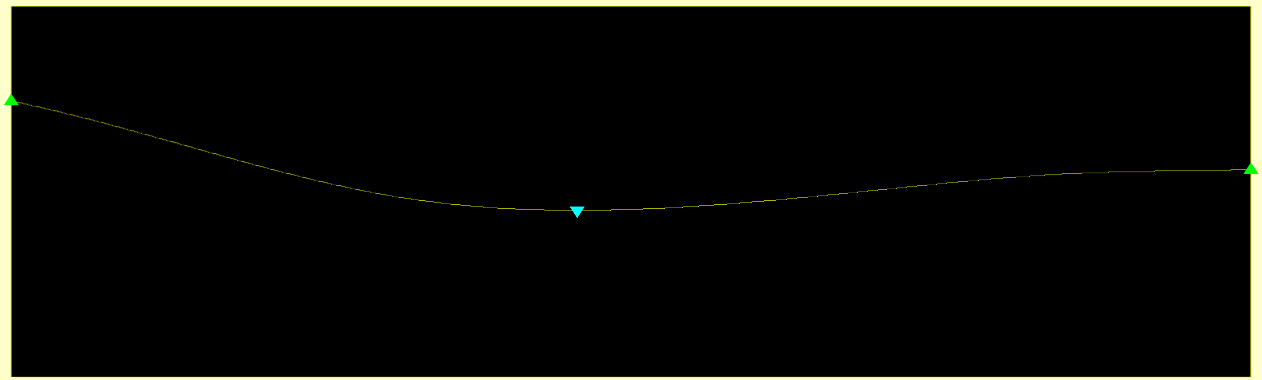


Empirical Mode Decomposition

Algorithmic definition

S
I
F
T
I
N
G

P
R
O
C
E
S
S

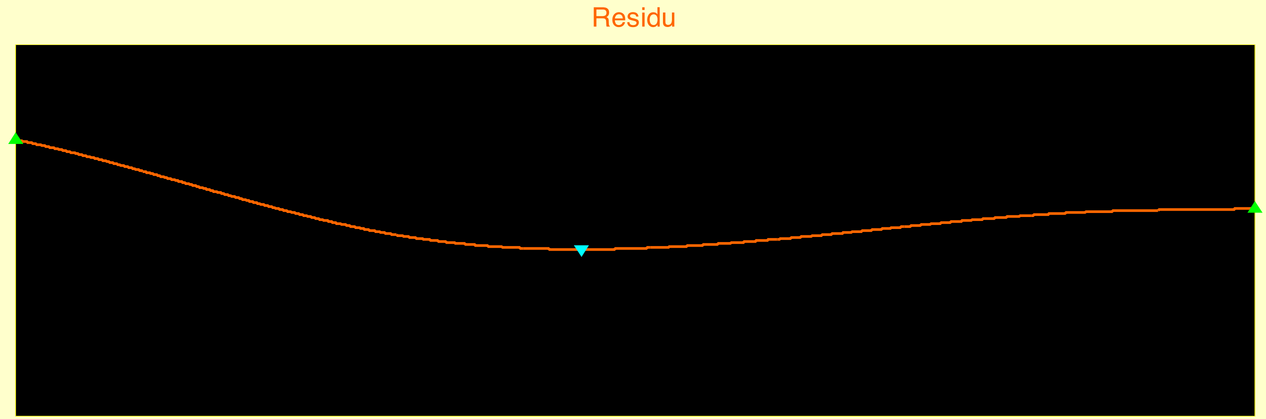


Empirical Mode Decomposition

Algorithmic definition

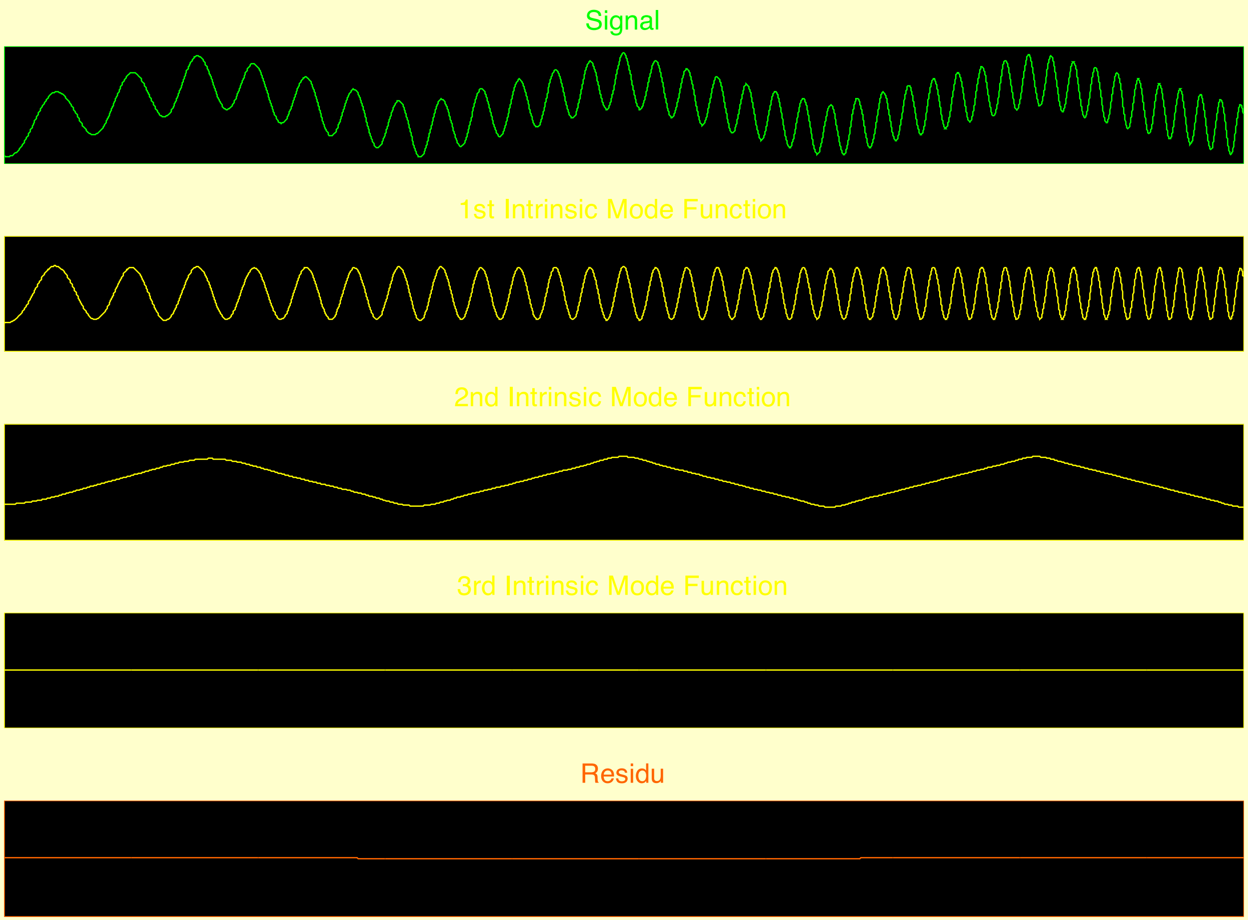
S
I
F
T
I

O
C
E
S
S

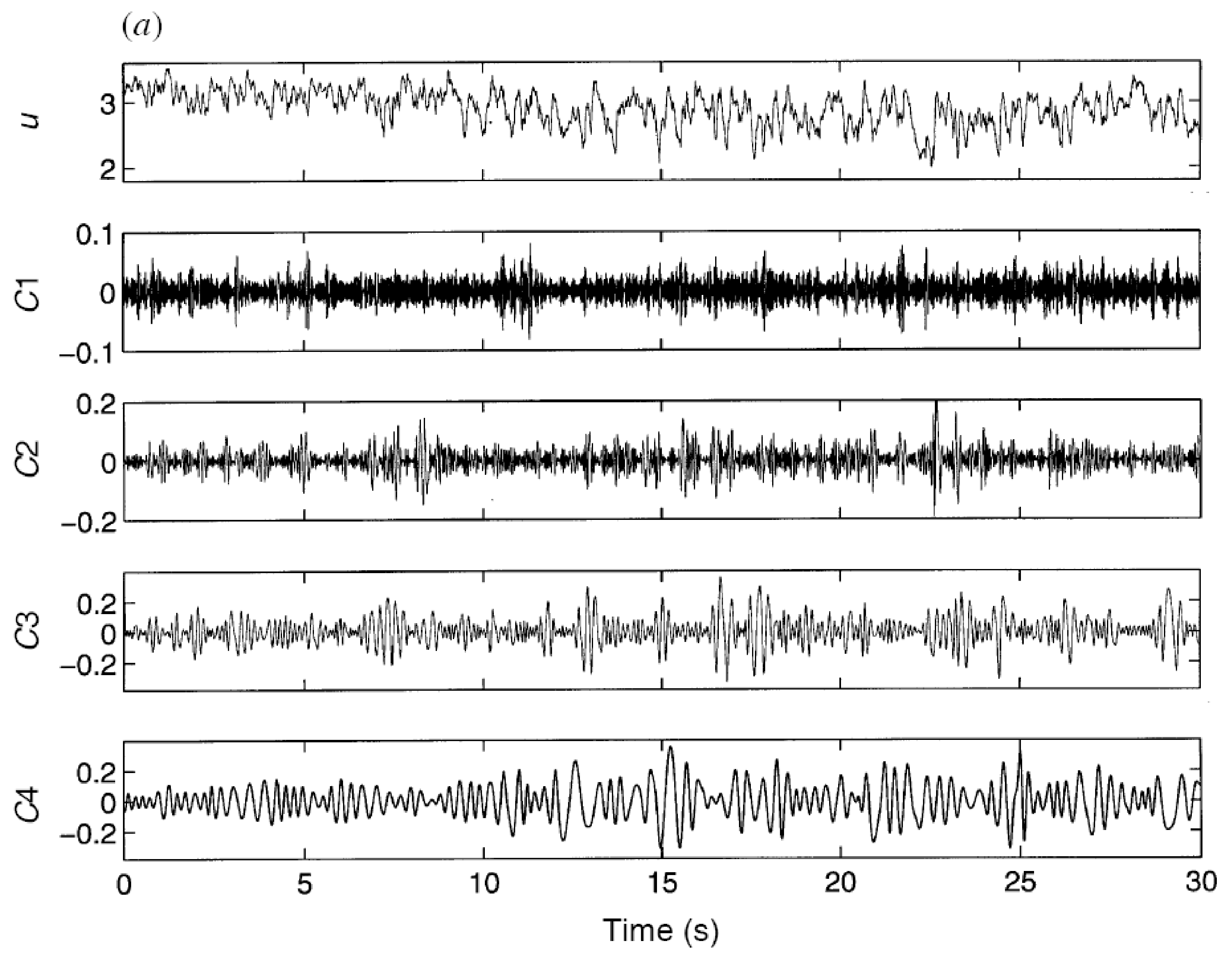


Empirical Mode Decomposition

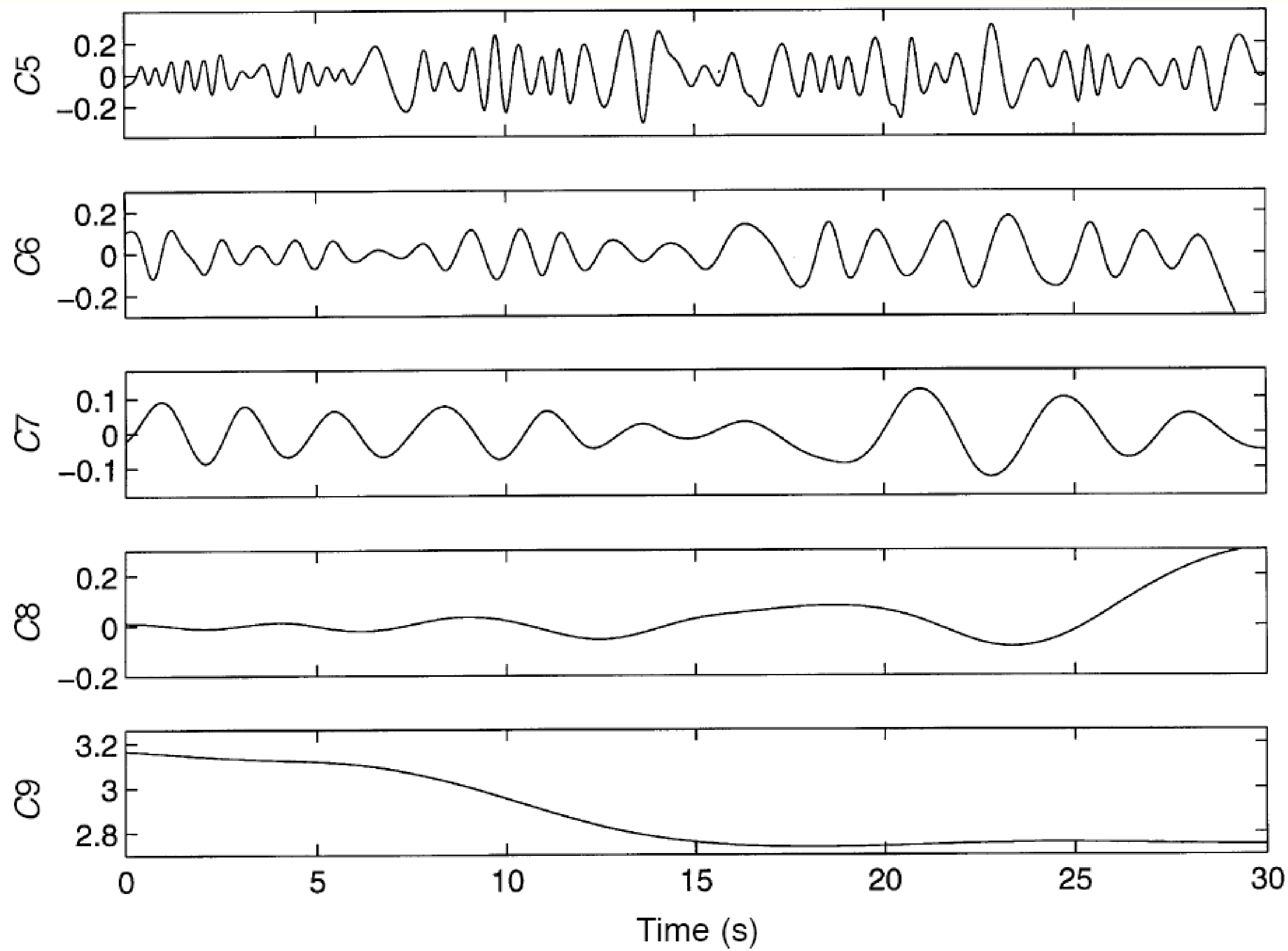
Algorithmic definition



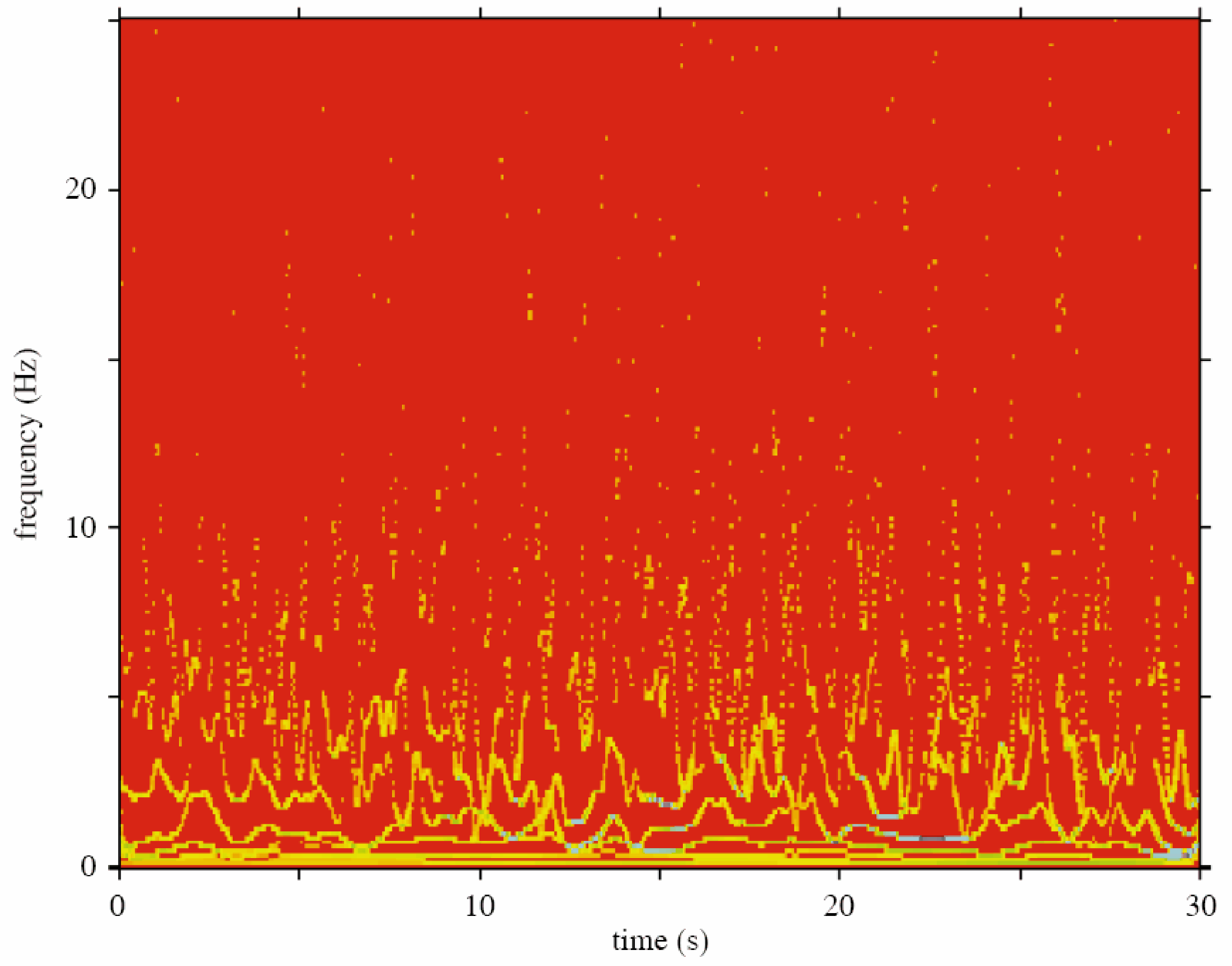
Original data and 4 modes



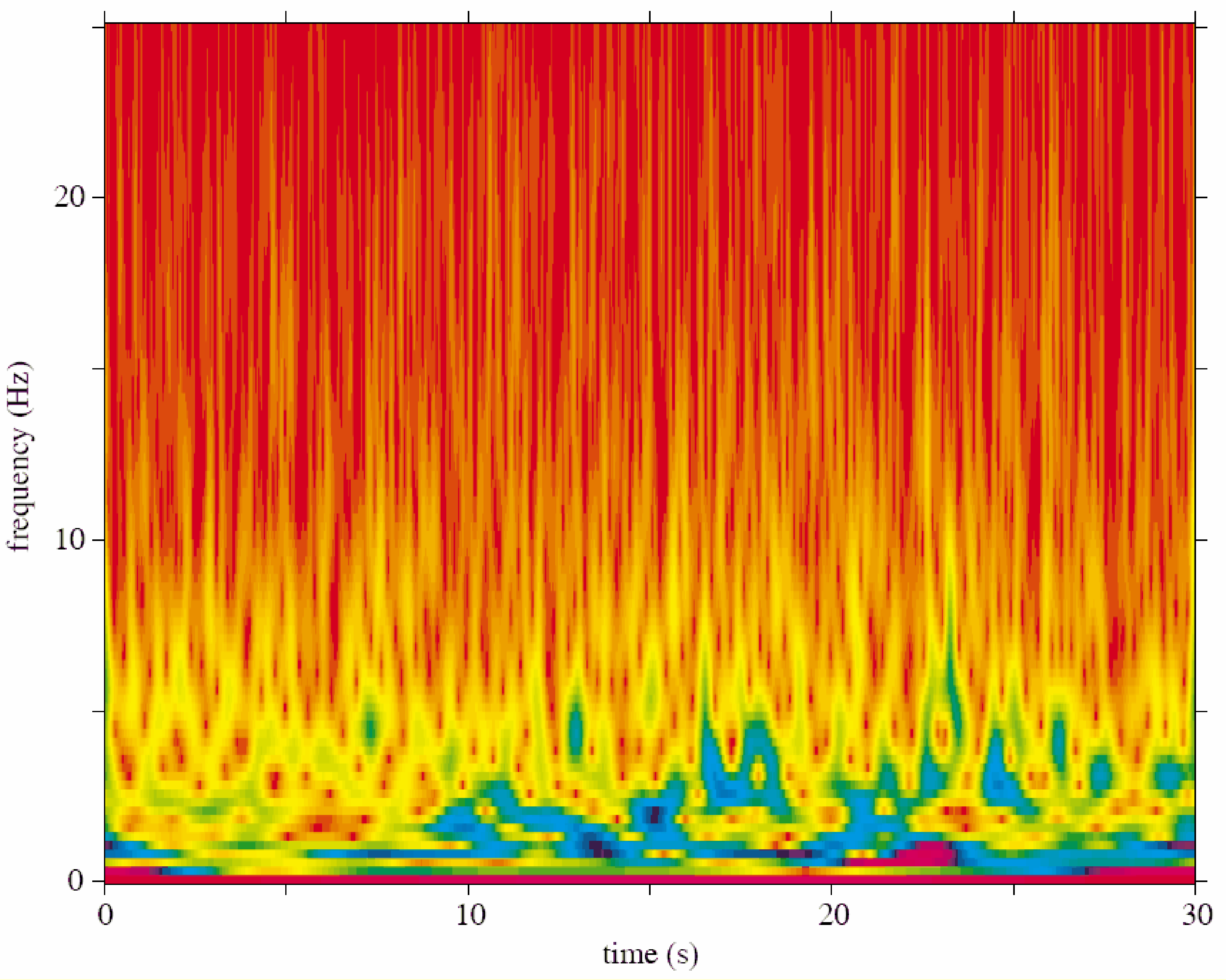
More modes


C5
0.2
0
−0.2
C6
0.2
0
−0.2
C7
0.1
0
−0.1
C8
0.2
0
−0.2
C9
3.2
3
2.8
0
5
10
15
20
25
30
Time (s)

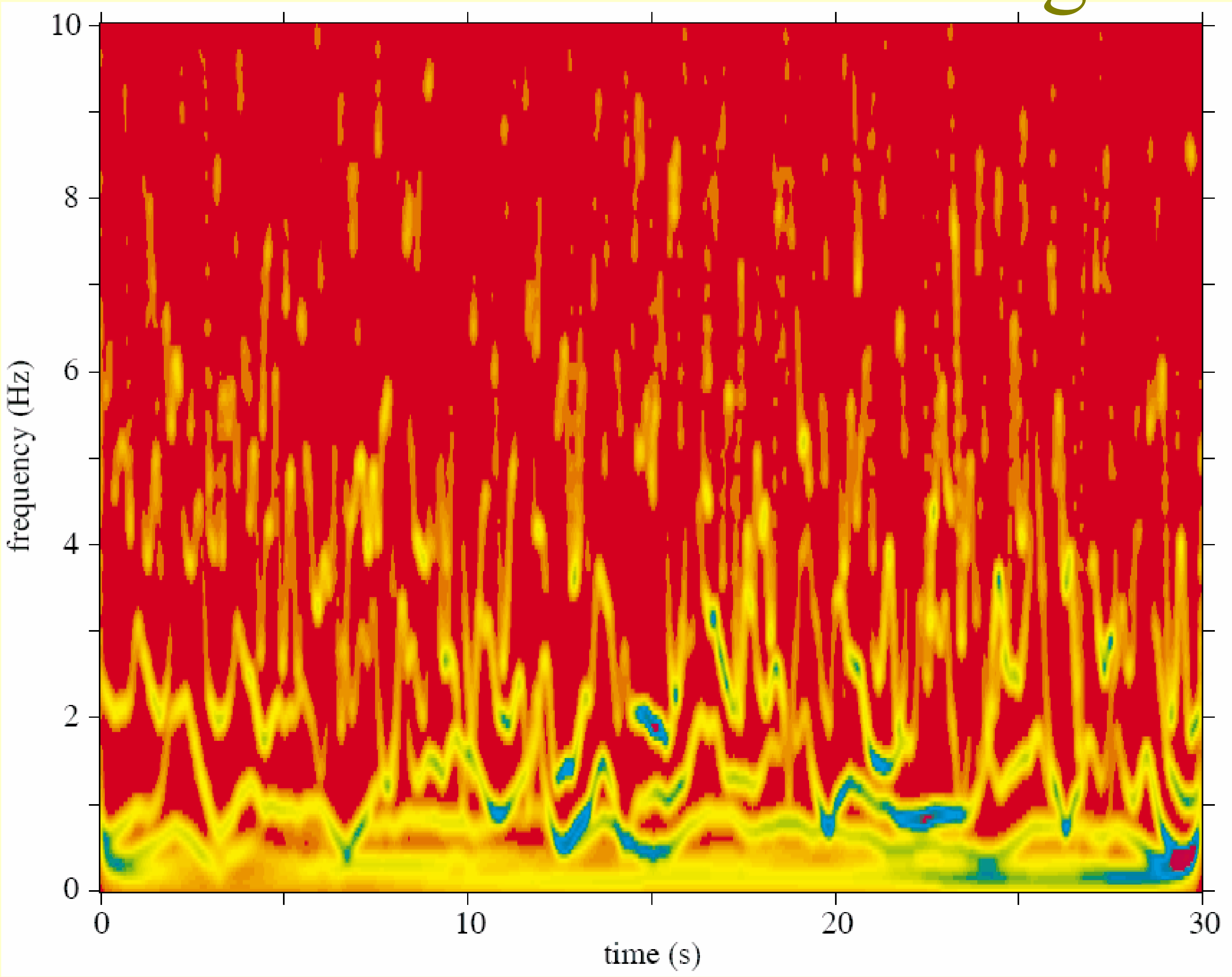
Hilbert-Huang Transform



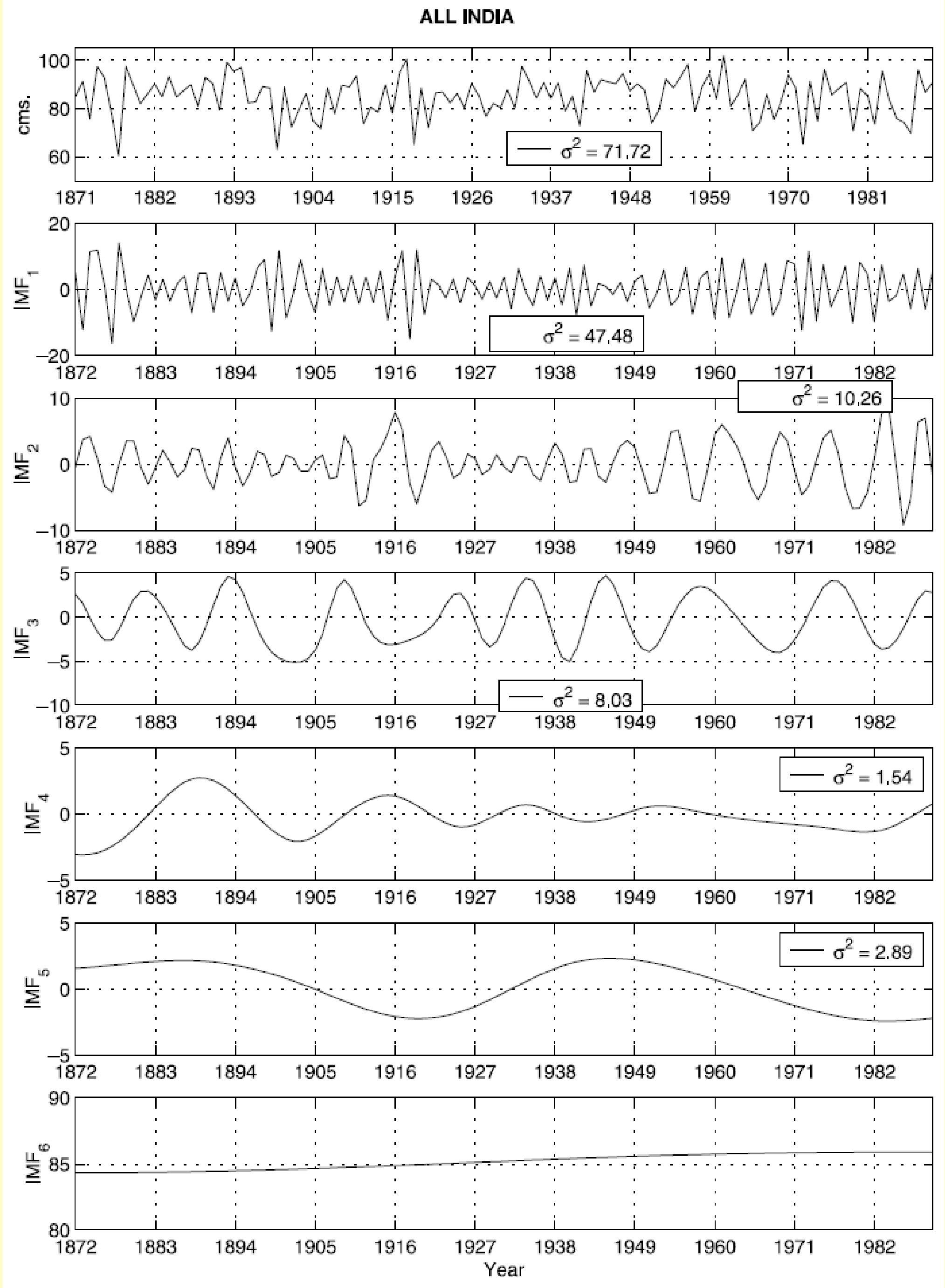
Wavelet Transform



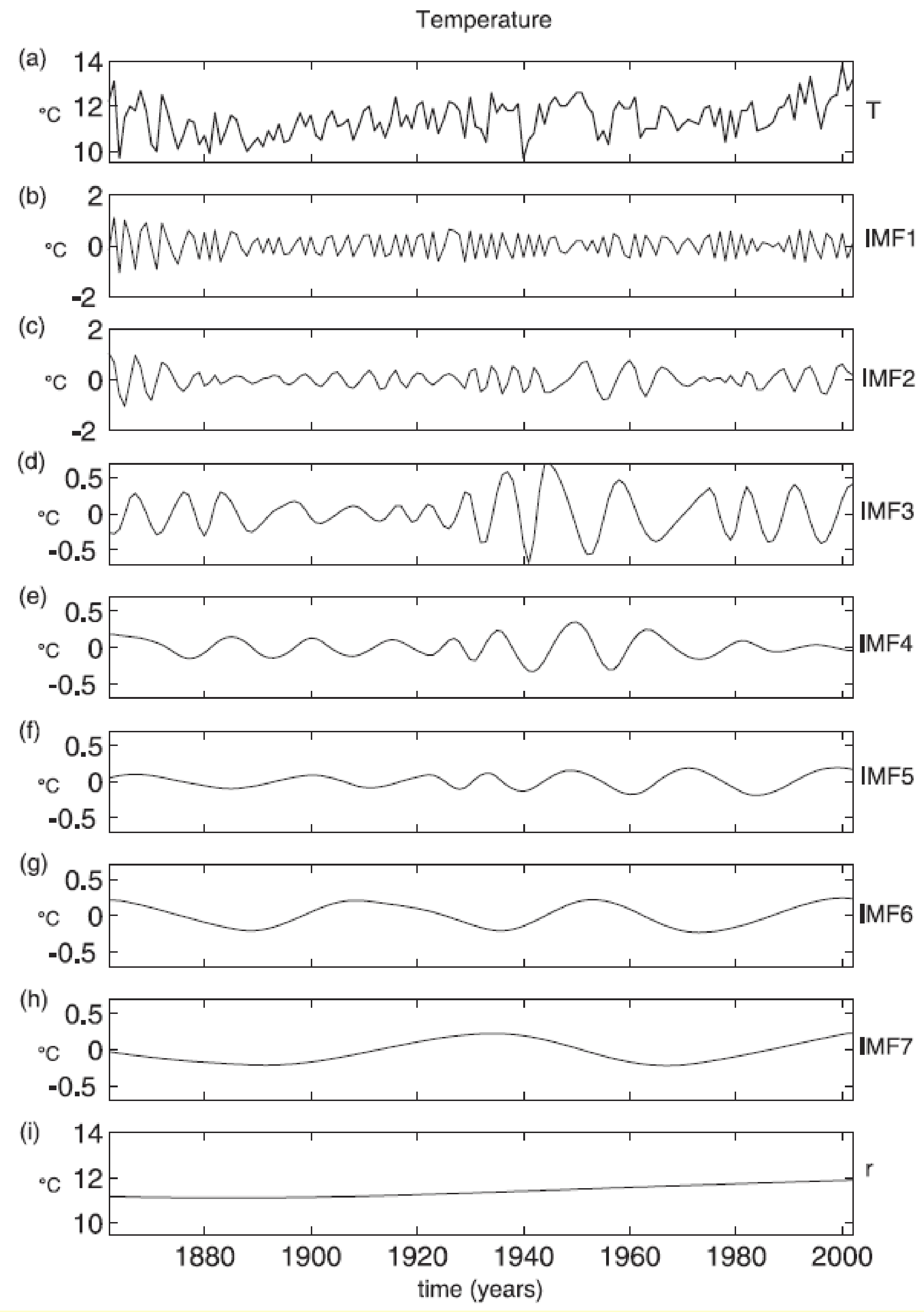
Smoothed Hilbert-Huang



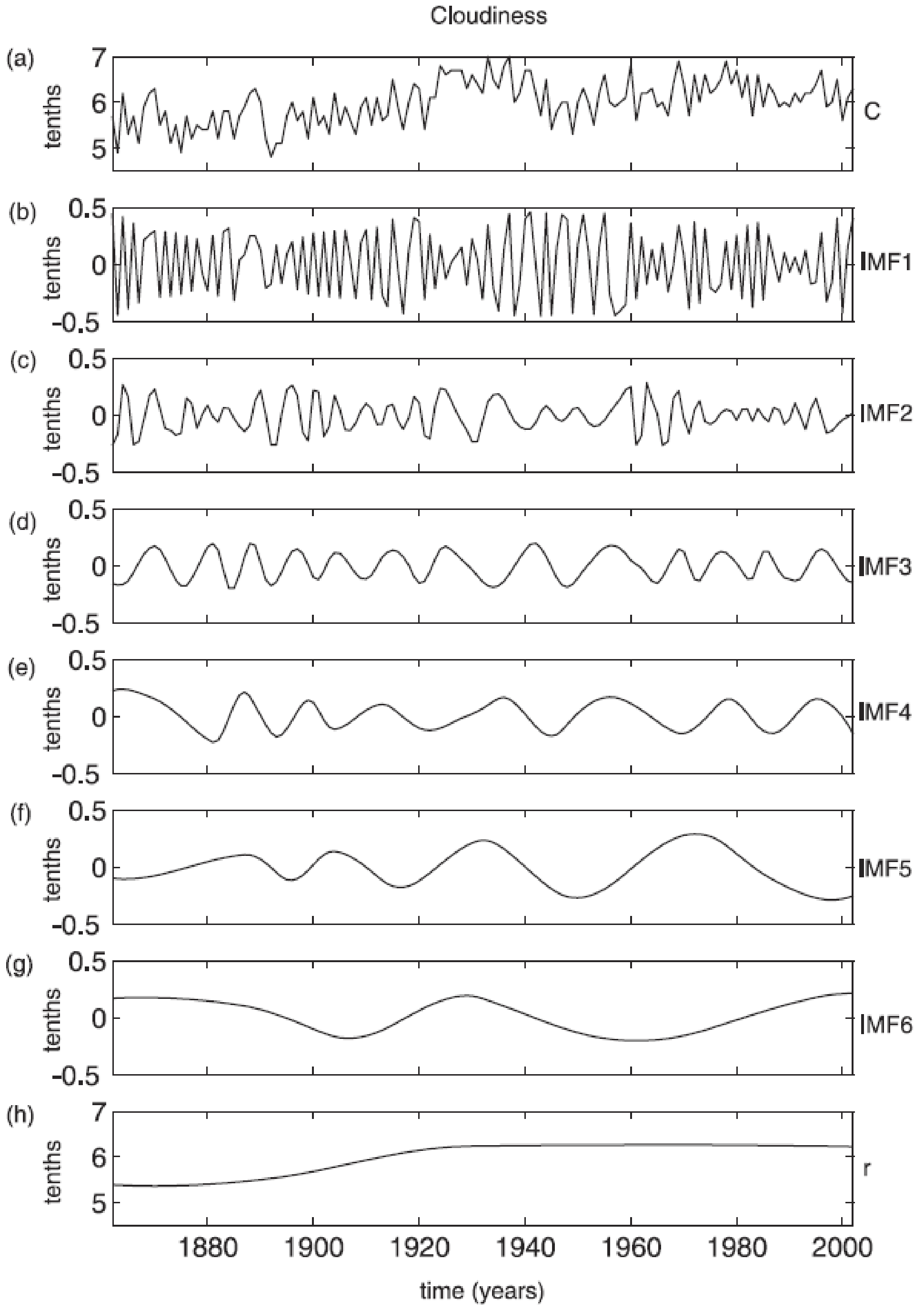
Temperature in India



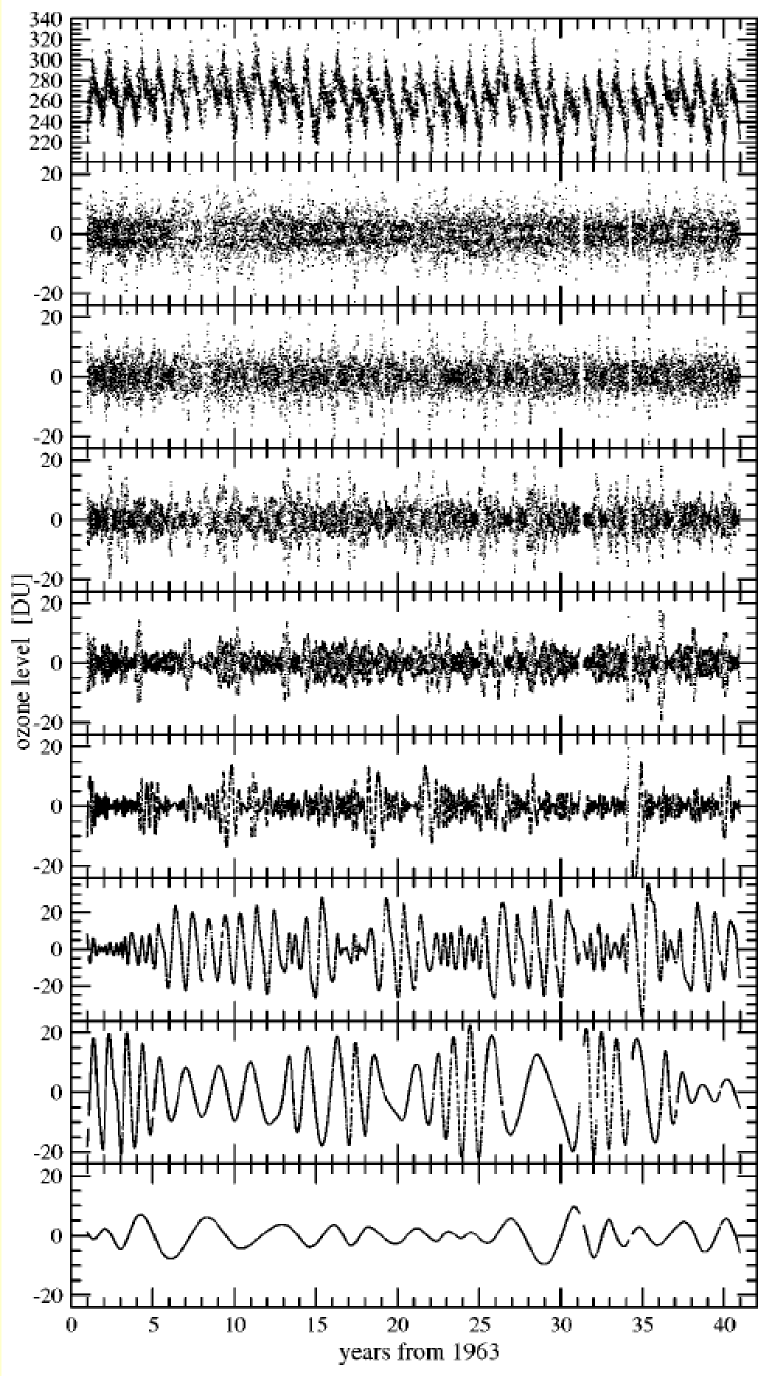
Temperature in Zagreb



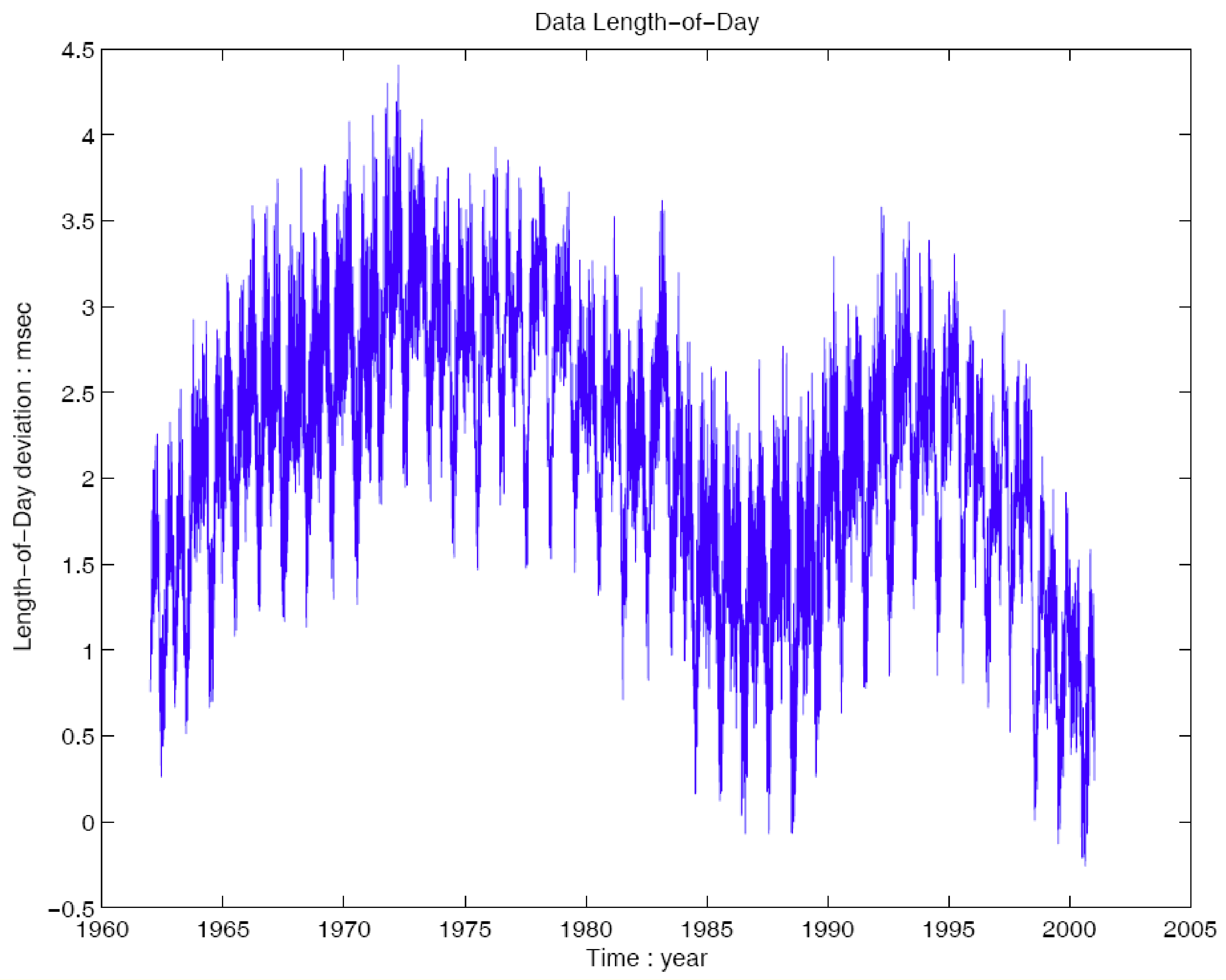
Cloudiness in Zagreb

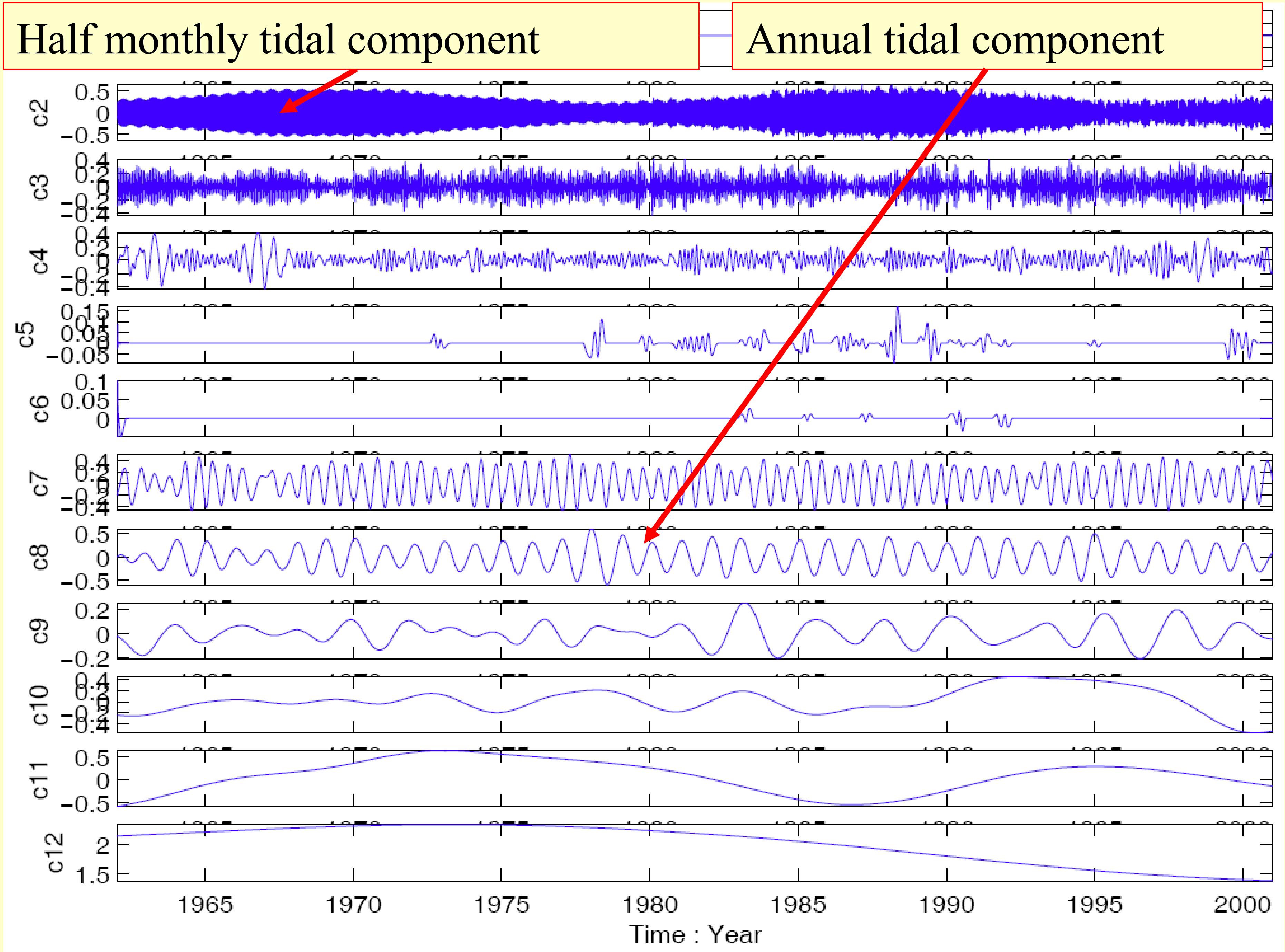


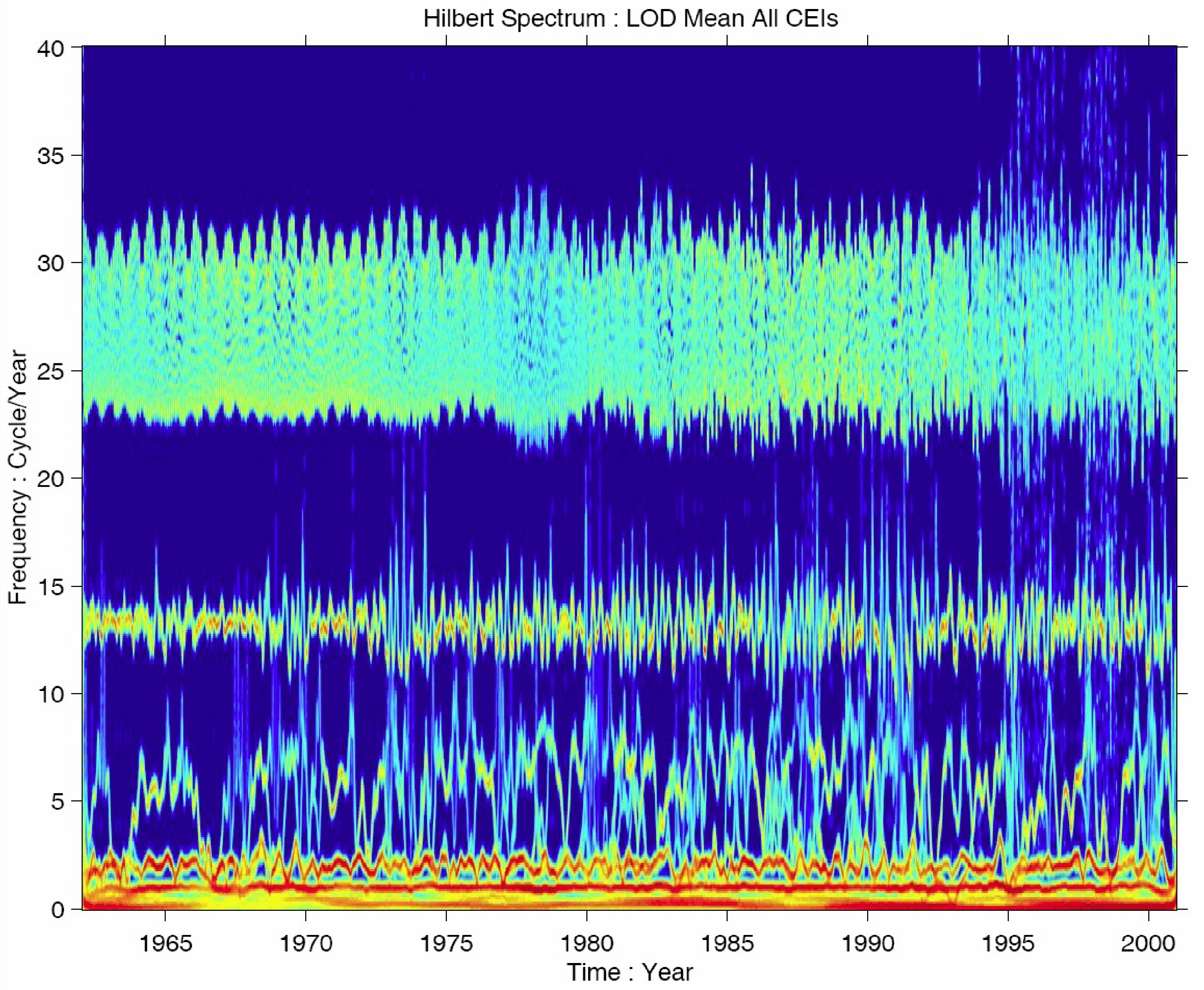
Ozone measured at Mauna Loa station



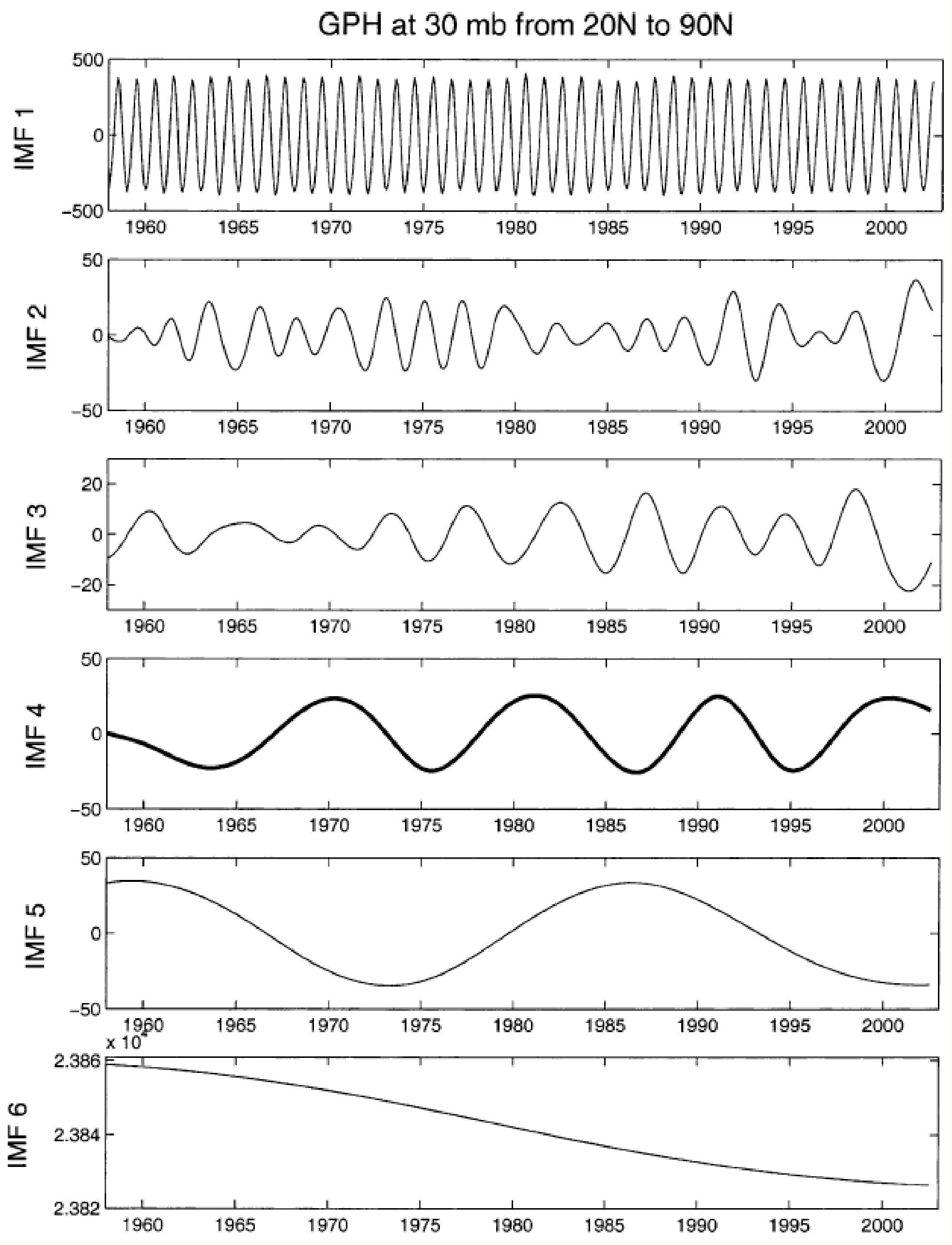
Length of day measurements




Half monthly tidal component
Annual tidal component
c2
c3
c4
c5
c6
c7
c8
c9
c10
c11
c12
1965
1970
1975
1980
1985
1990
1995
2000
Time : Year


Hilbert Spectrum : LOD Mean All CEIs
Frequency : Cycle/Year
0
5
10
15
20
25
30
35
40
1965
1970
1975
1980
1985
1990
1995
2000
Time : Year

Geopotential height



•The total geopotential height at 30 mb from 20N to 90N is decomposed into five modes and a trend.

•**The first mode** is the annual cycle.

•**The second mode** has an average frequency of 27 months and is anti-correlated with the equatorial QBO.

•**The third mode** has an average period of four years

•**Fourth mode** is highly correlated with the 11-year solar cycle.

•**The trend** indicates cooling in the stratosphere over the past four decades.

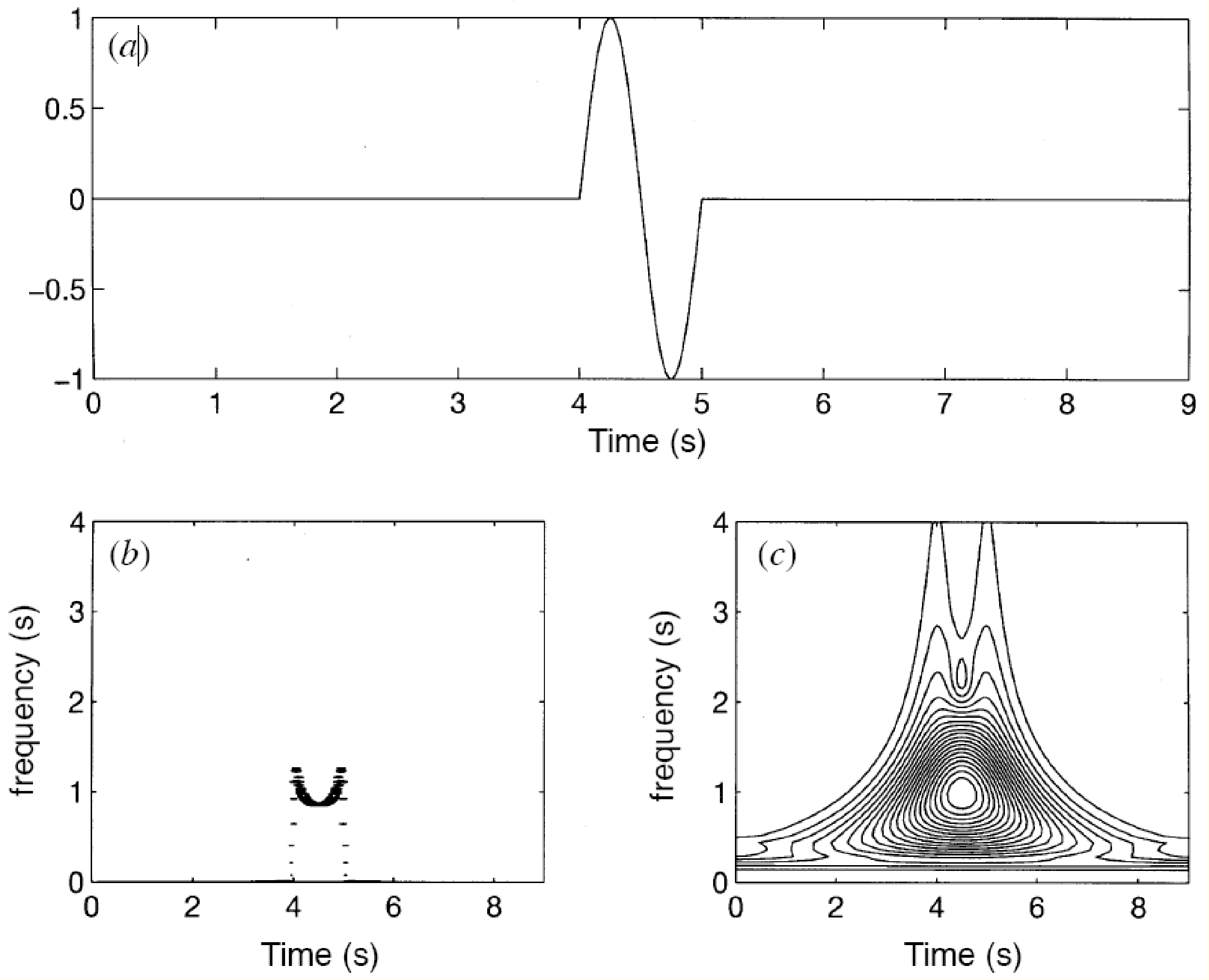
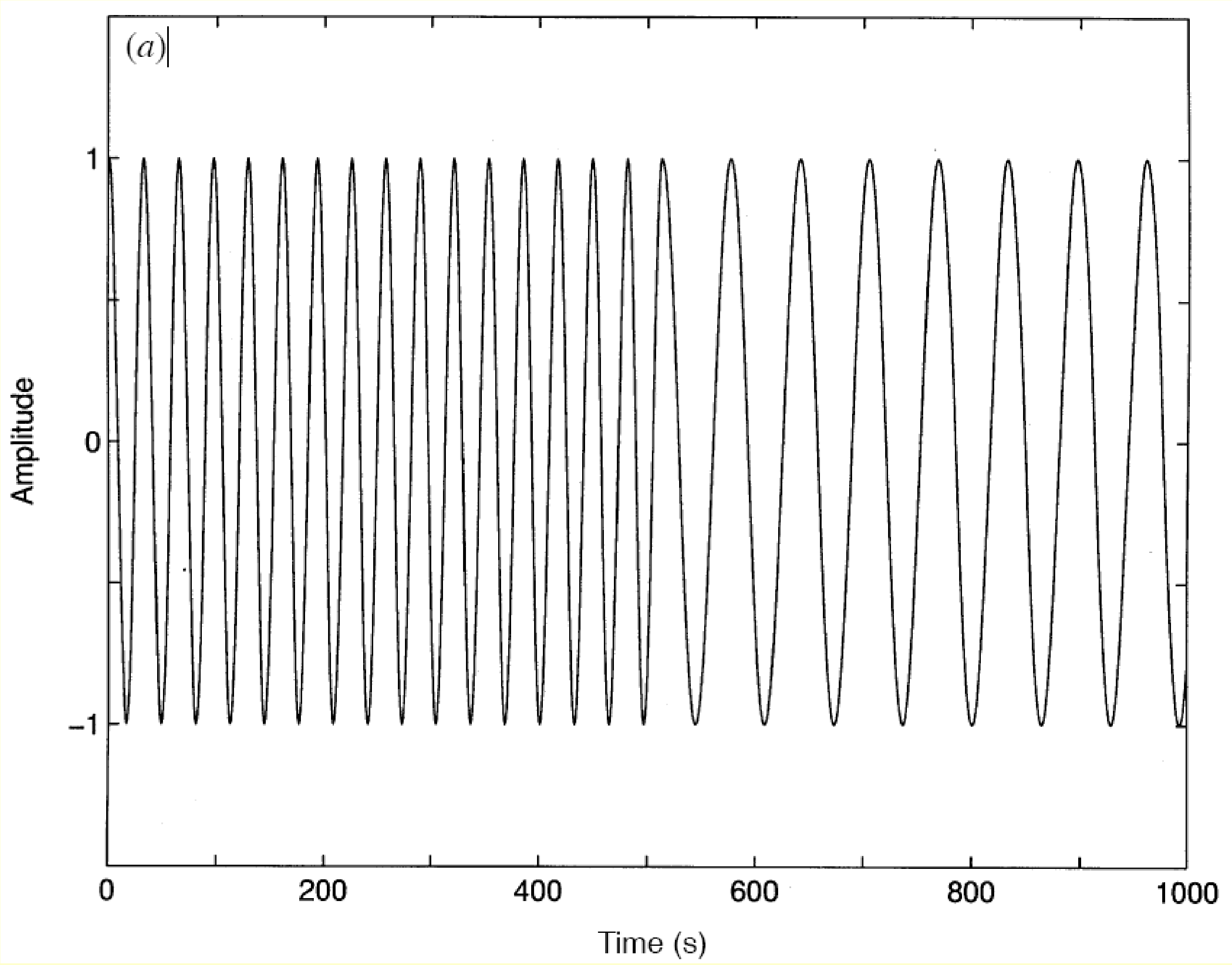
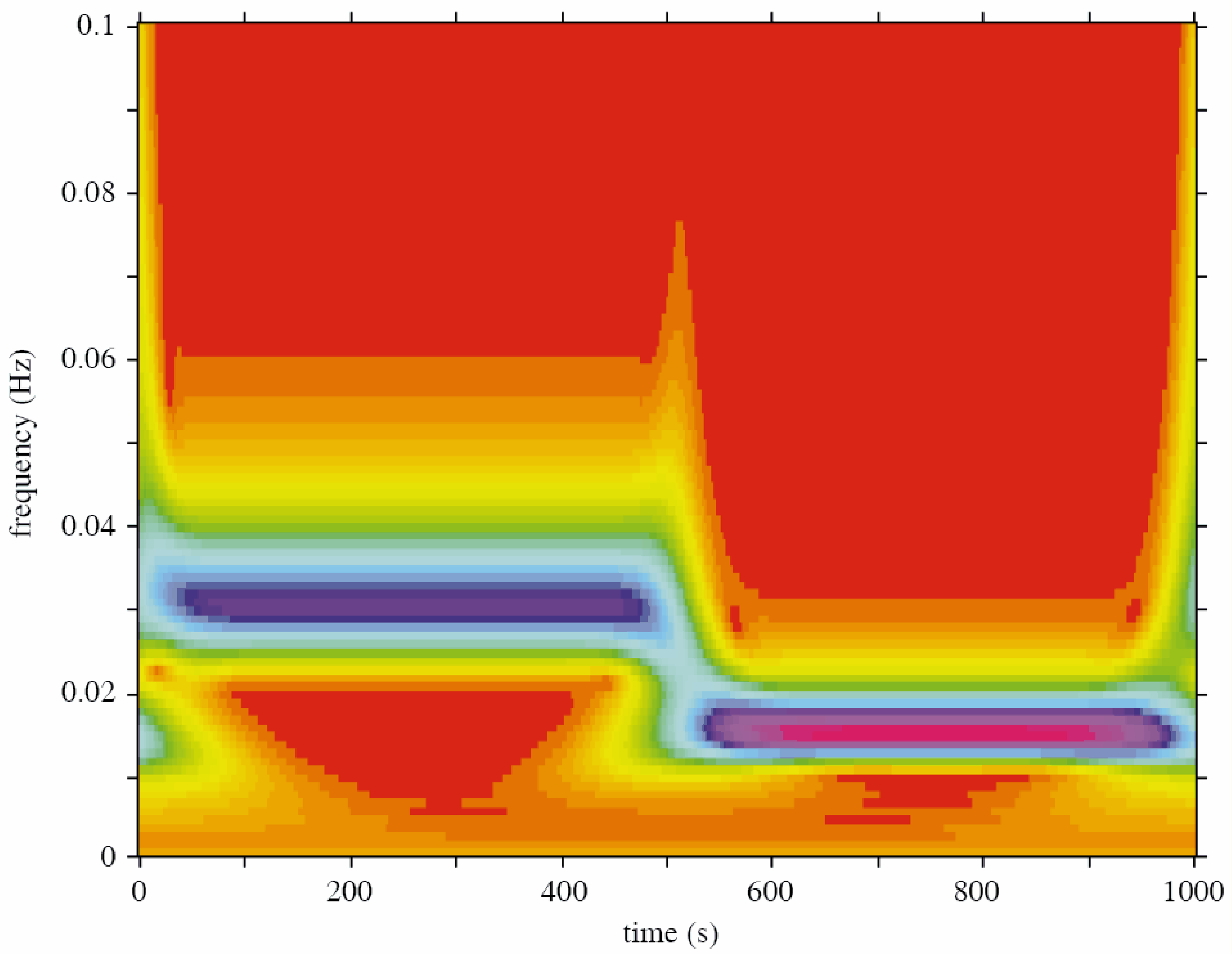


Figure 14. A calibration of time localization of the Hilbert spectrum analysis. (*a*) The calibration data, a single sine wave. (*b*) The Hilbert spectrum for the calibration signal: the energy is highly localized in time and frequency, though there are some end effects. (*c*) The Morlet wavelet spectrum for the calibration signal: the calibration signal is localized by the high-frequency components, yet the energy distribution in the frequency space spreads widely in comparison with the Hilbert spectrum.

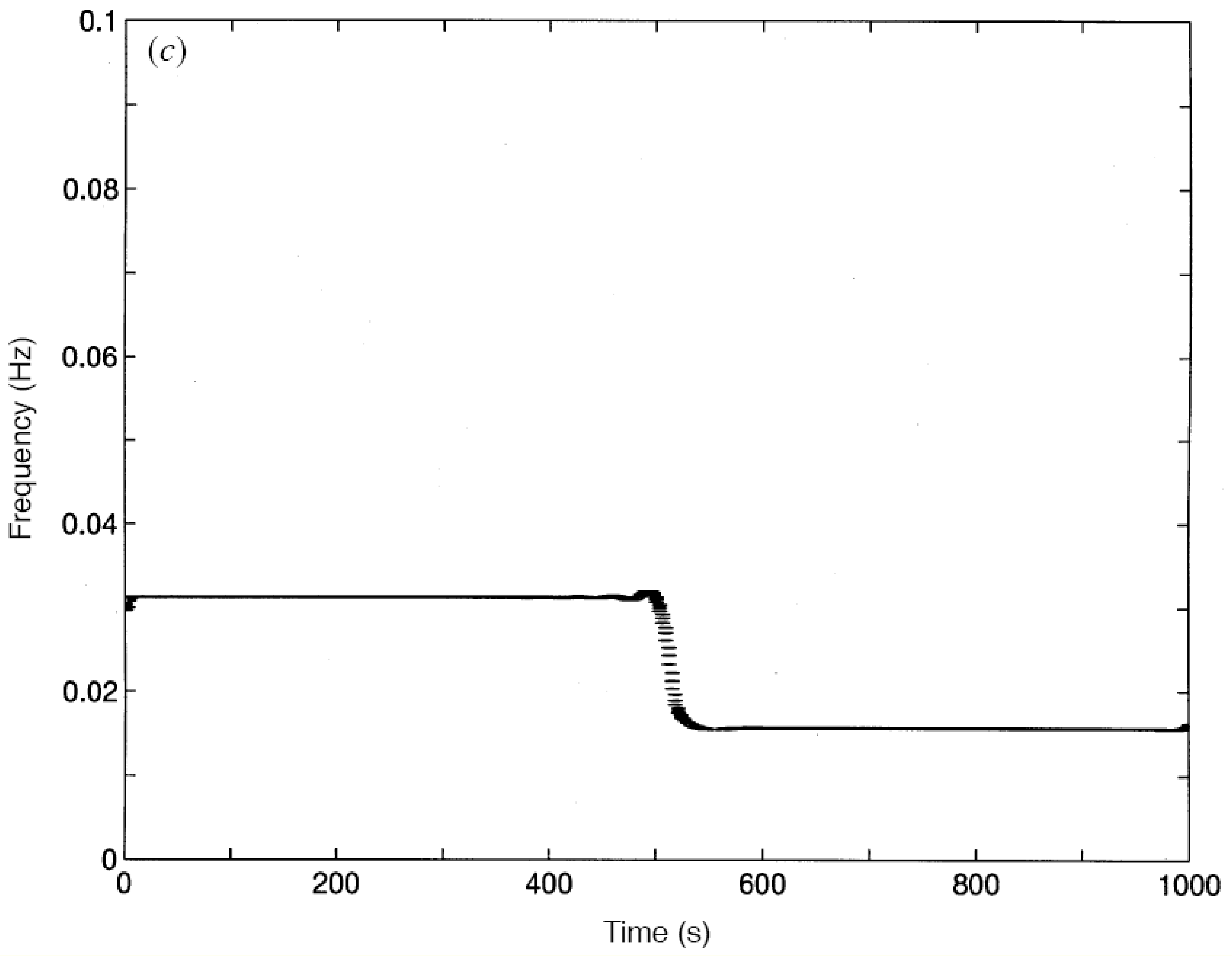
Input data



Wavelet Transform



Hilbert-Huang Transform



Not always happy!

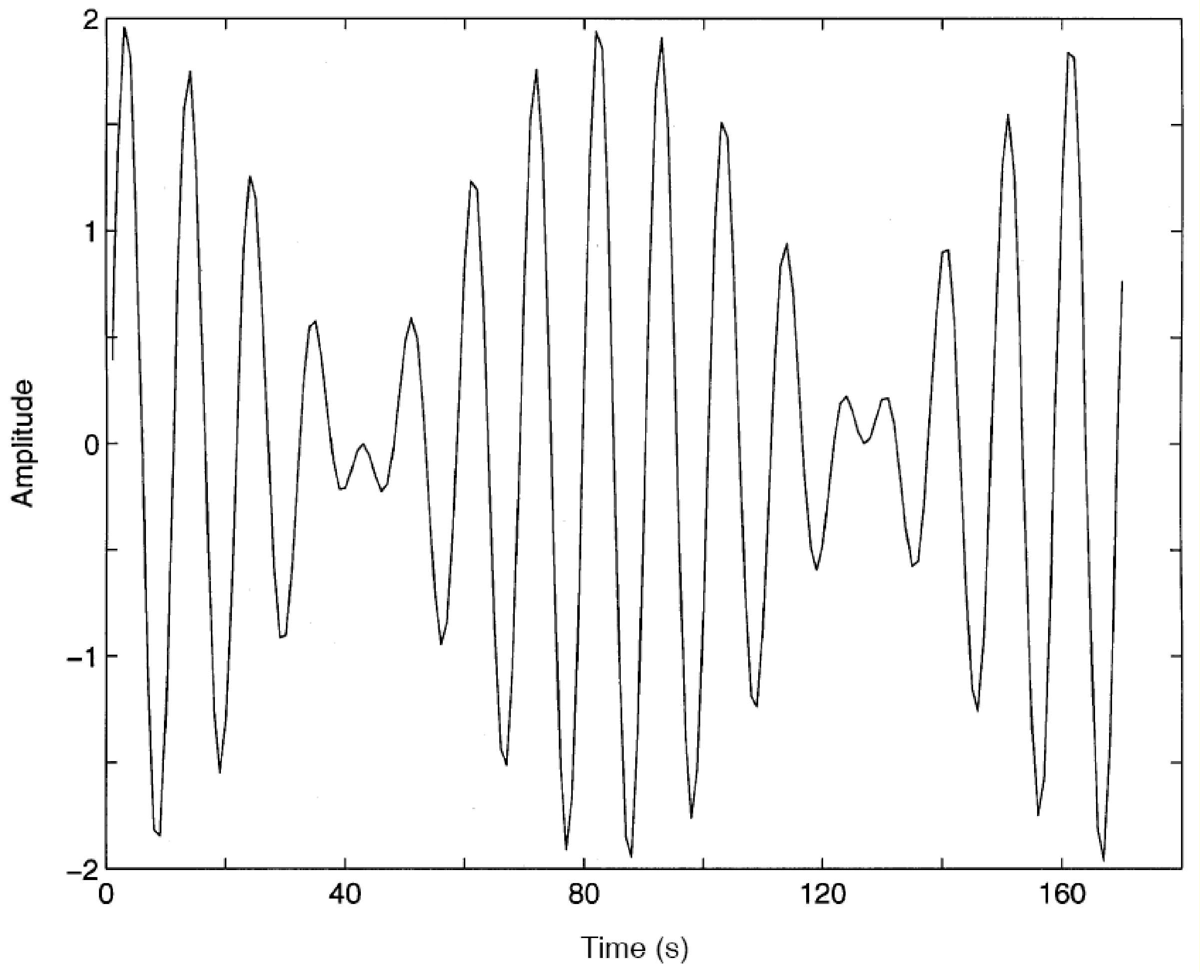
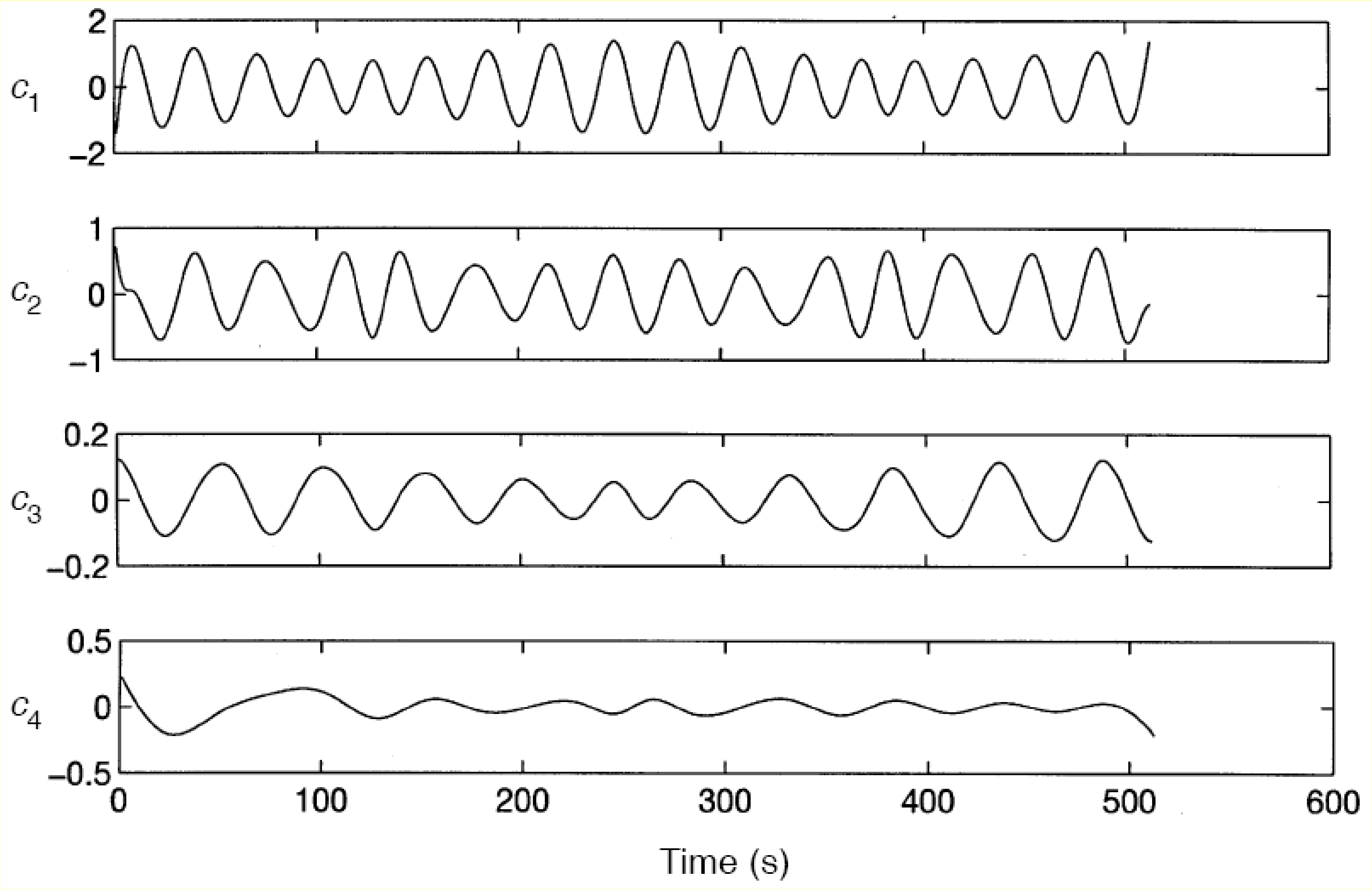
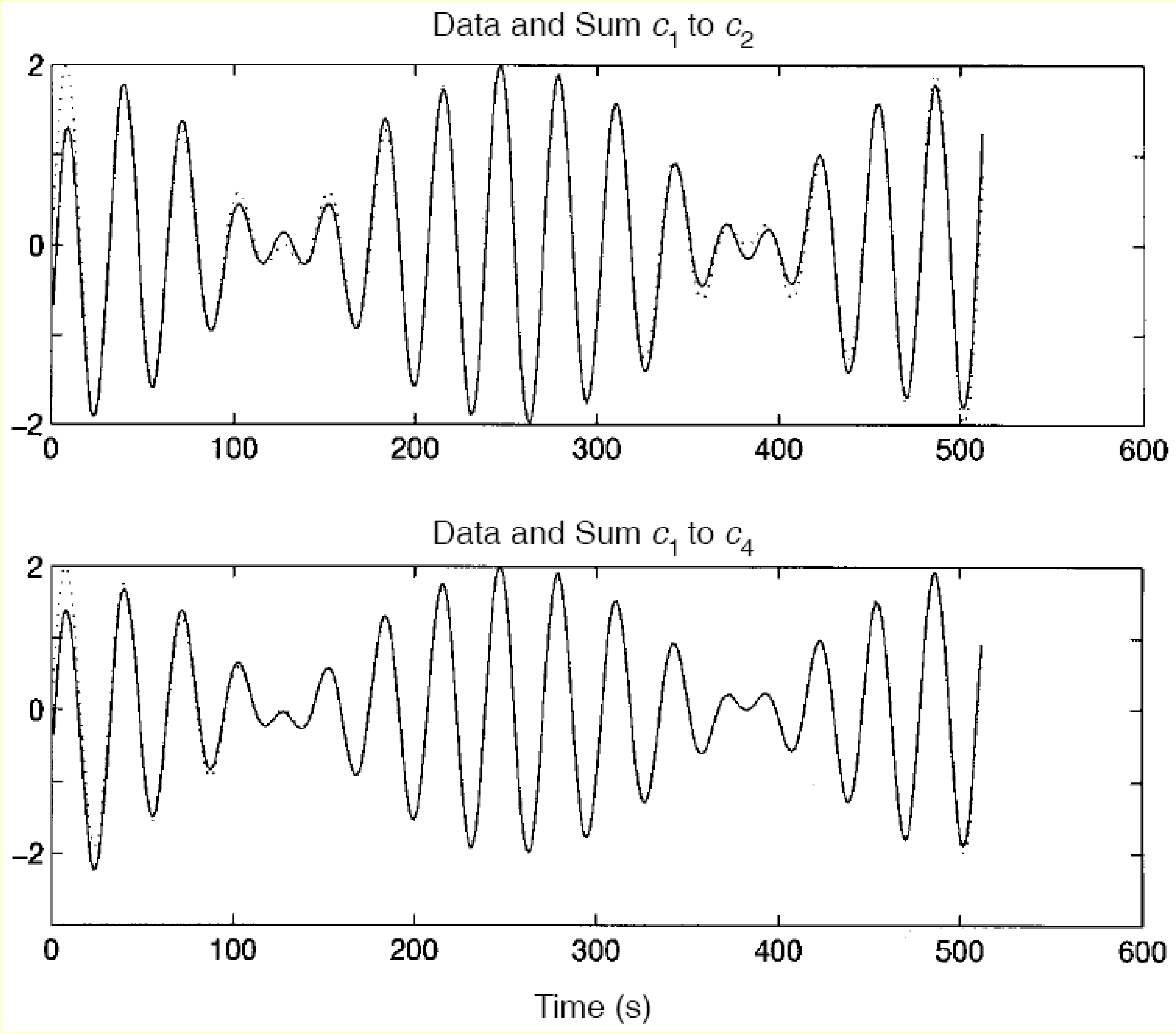


Figure 25. Data of the linear superimposed cosine waves given in equation (8.10): $\cos(\frac{2}{30}\pi t) + \cos(\frac{2}{34}\pi t)$. The data show the expected beats. Although the wave form is asymmetric, the envelopes should be symmetric.

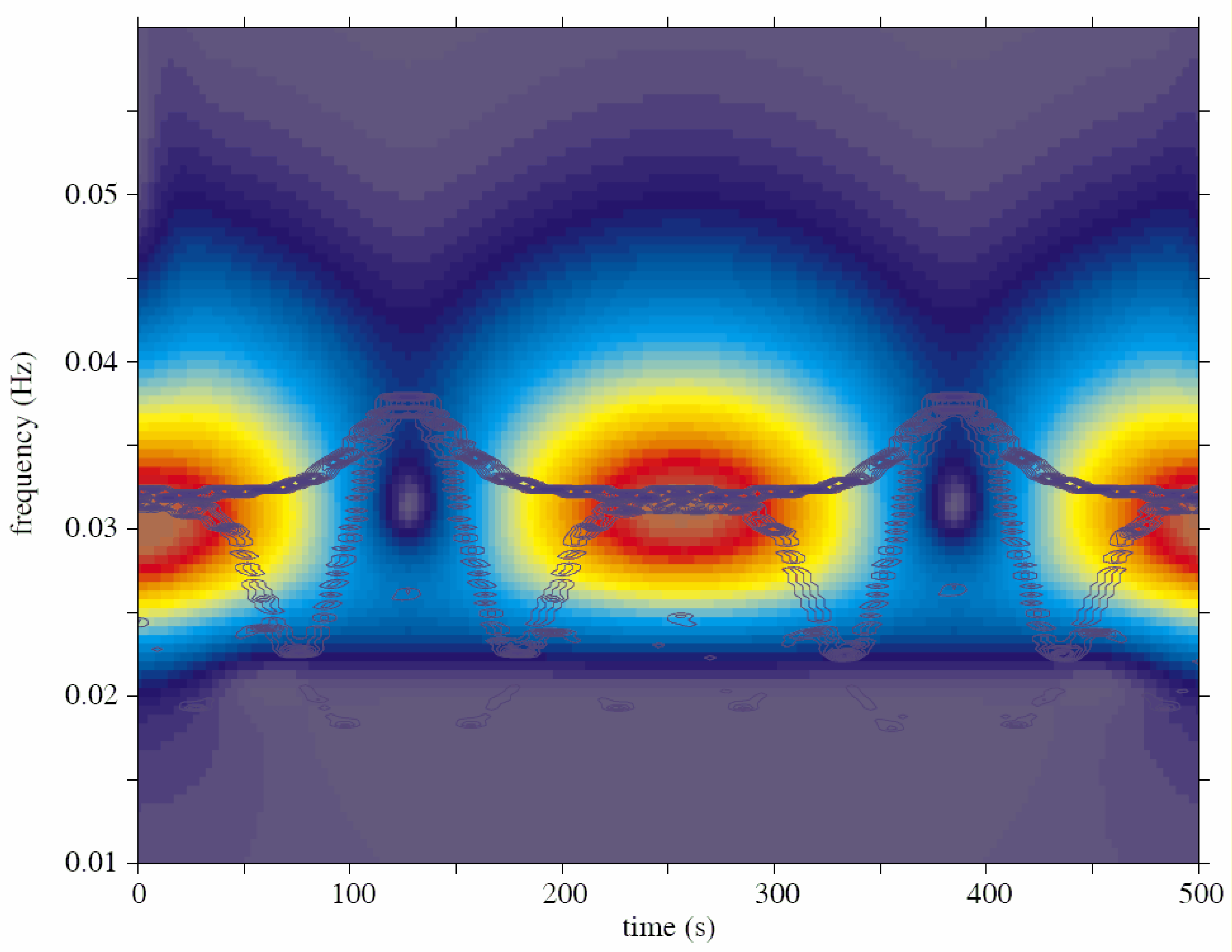
Ooohhh… no, four modes!



FOK – formally is all OK

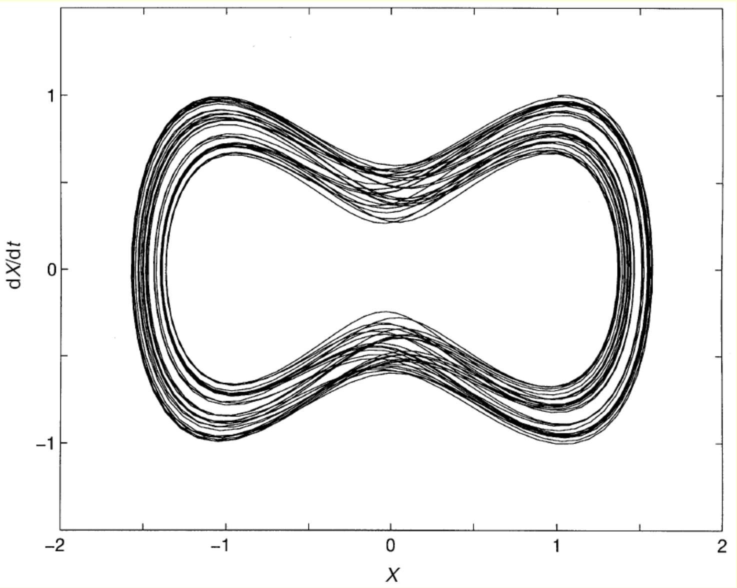


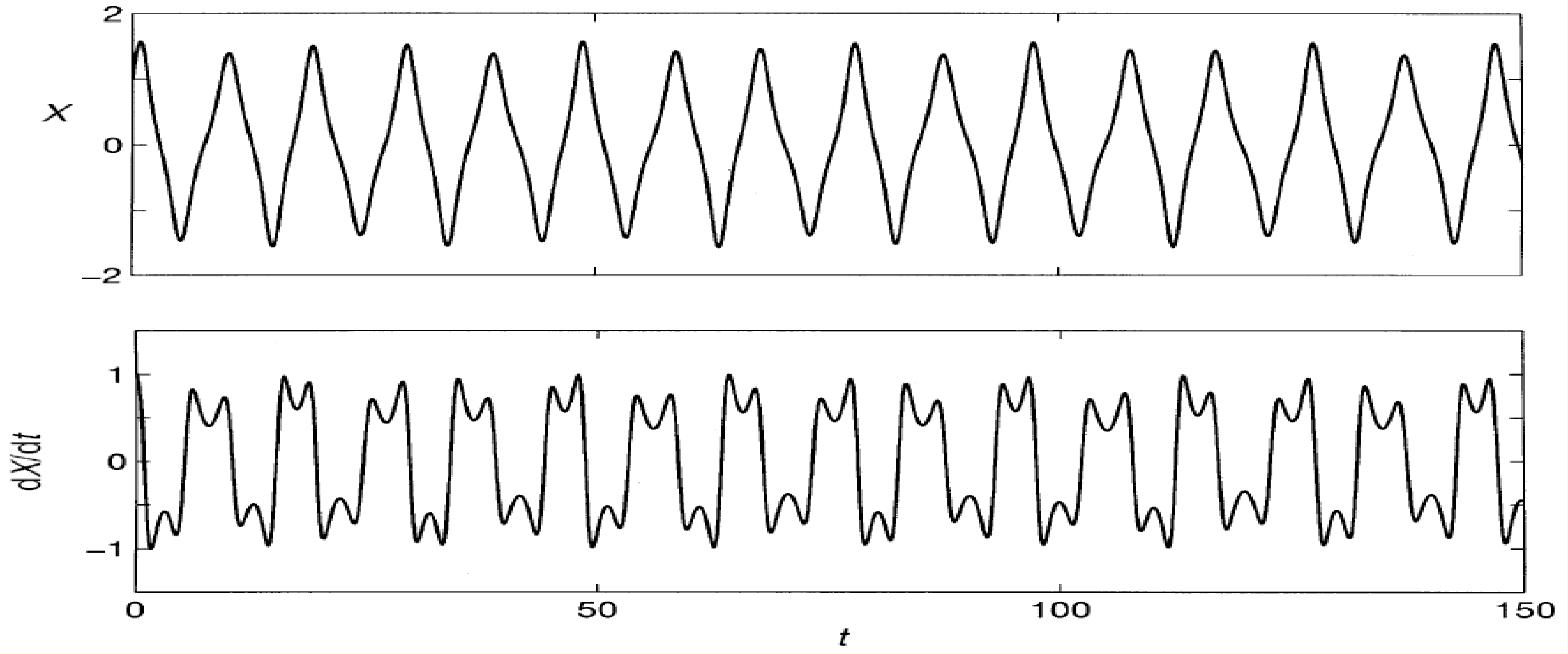
But wavelets are even worse!

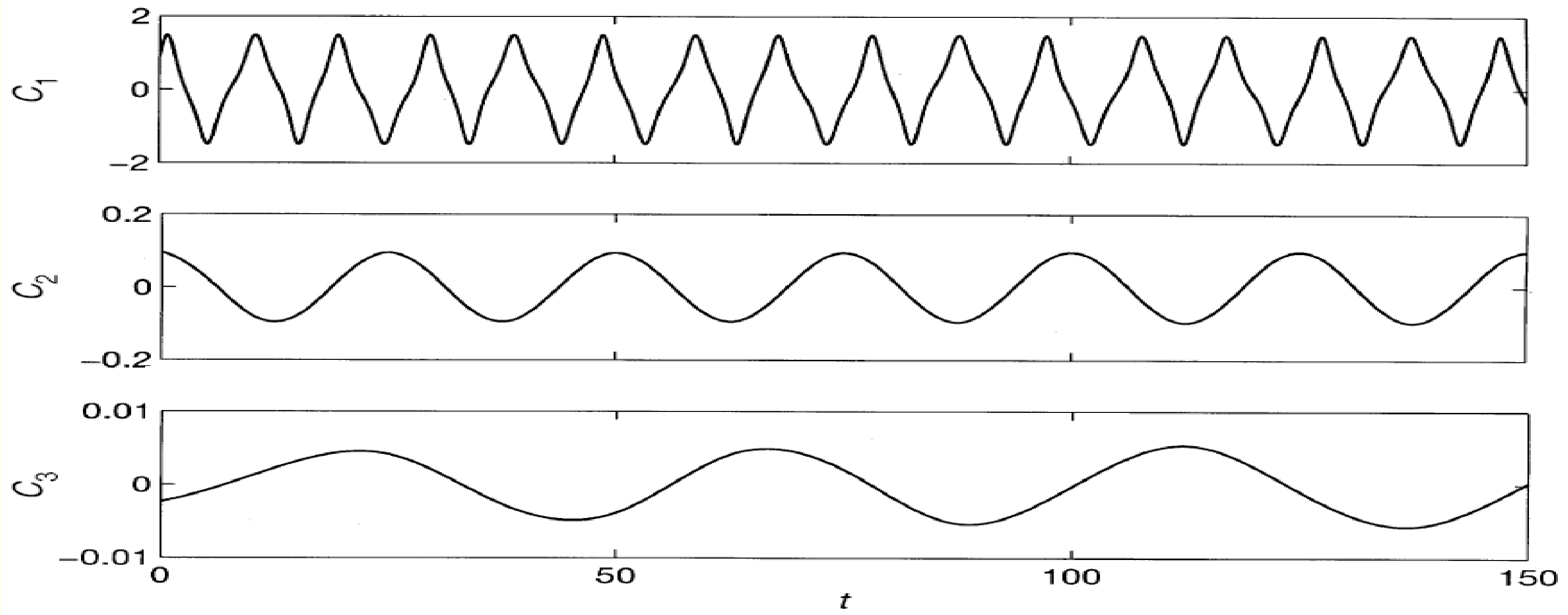


Duffing equation

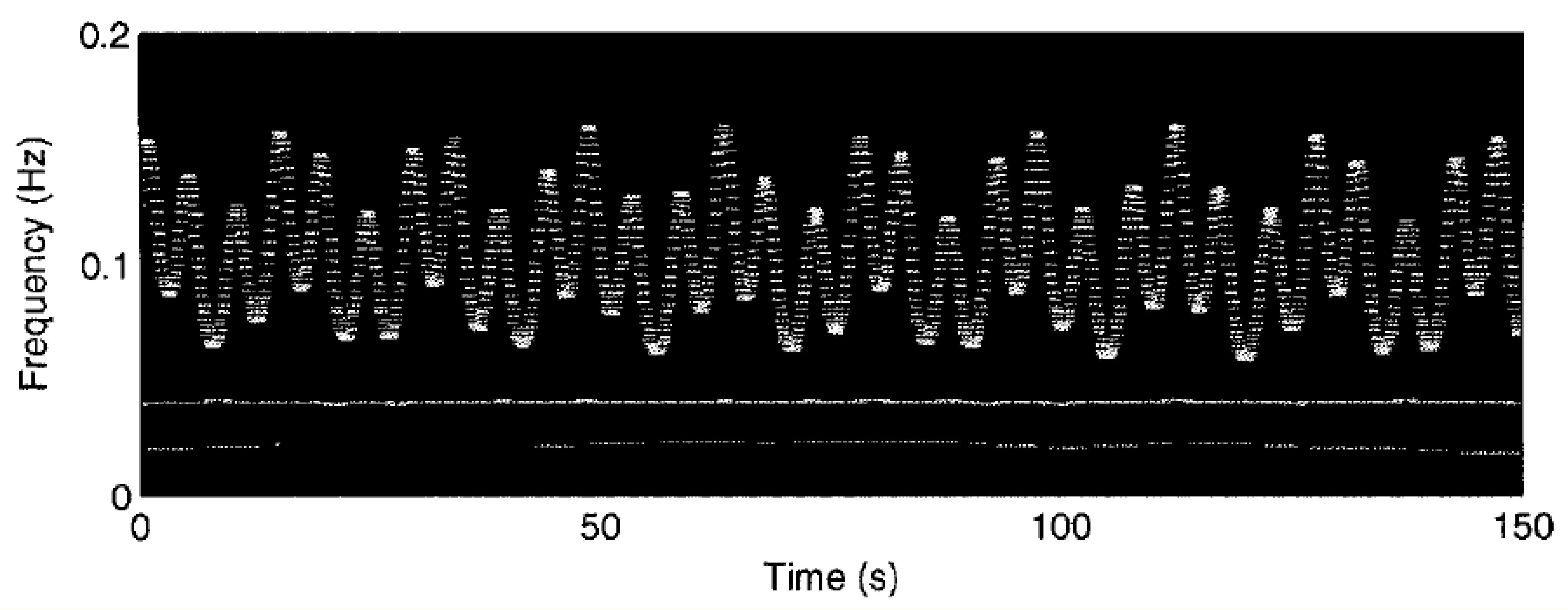
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + x + \epsilon x^3 = \gamma \cos \omega t$$







2
0
−2
C_1
0.2
0
−0.2
C_2
0.01
0
−0.01
C_3
0
50
100
150
t



0.2
0.1
0
Frequency (Hz)
0
50
100
150
Time (s)

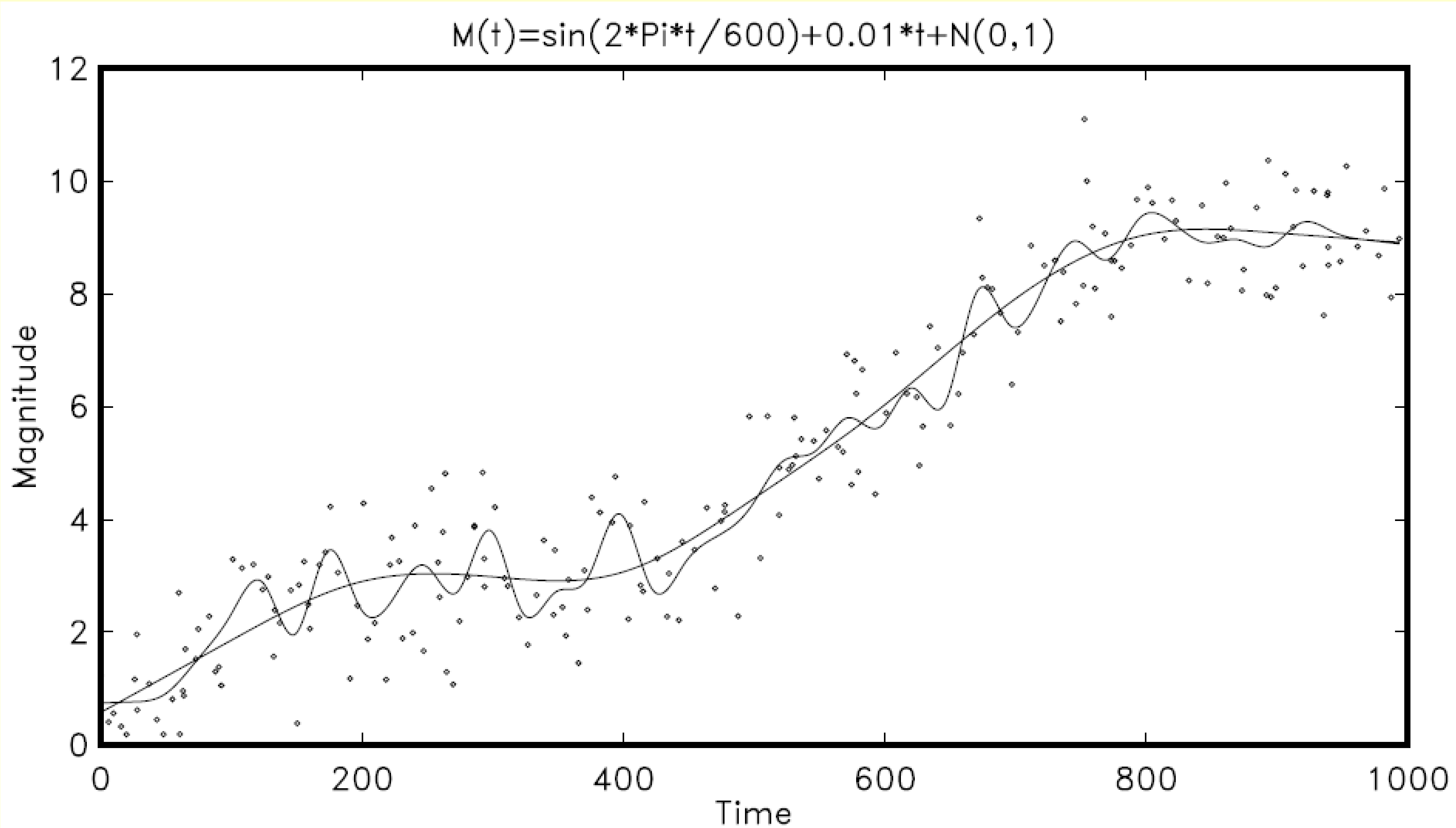
Splines

- for each of the intervals $(a, t_1), (t_1, t_2), \ldots, (t_n, b)$, f is a cubic polynomial:

$$d_i(t-t_i)^3 + c_i(t-t_i)^2 + b_i(t-t_i) + a_i, \text{for } , t_i \leq t \leq t_{i+1}.$$

- $f(t)$ is continuous
- first derivatives of the f are continuous,
- second derivatives of the f are continuous.

Spline fit


M(t)=sin(2*Pi*t/600)+0.01*t+N(0,1)
Magnitude
Time
0
2
4
6
8
10
12
0
200
400
600
800
1000

Carrier fit

Splines

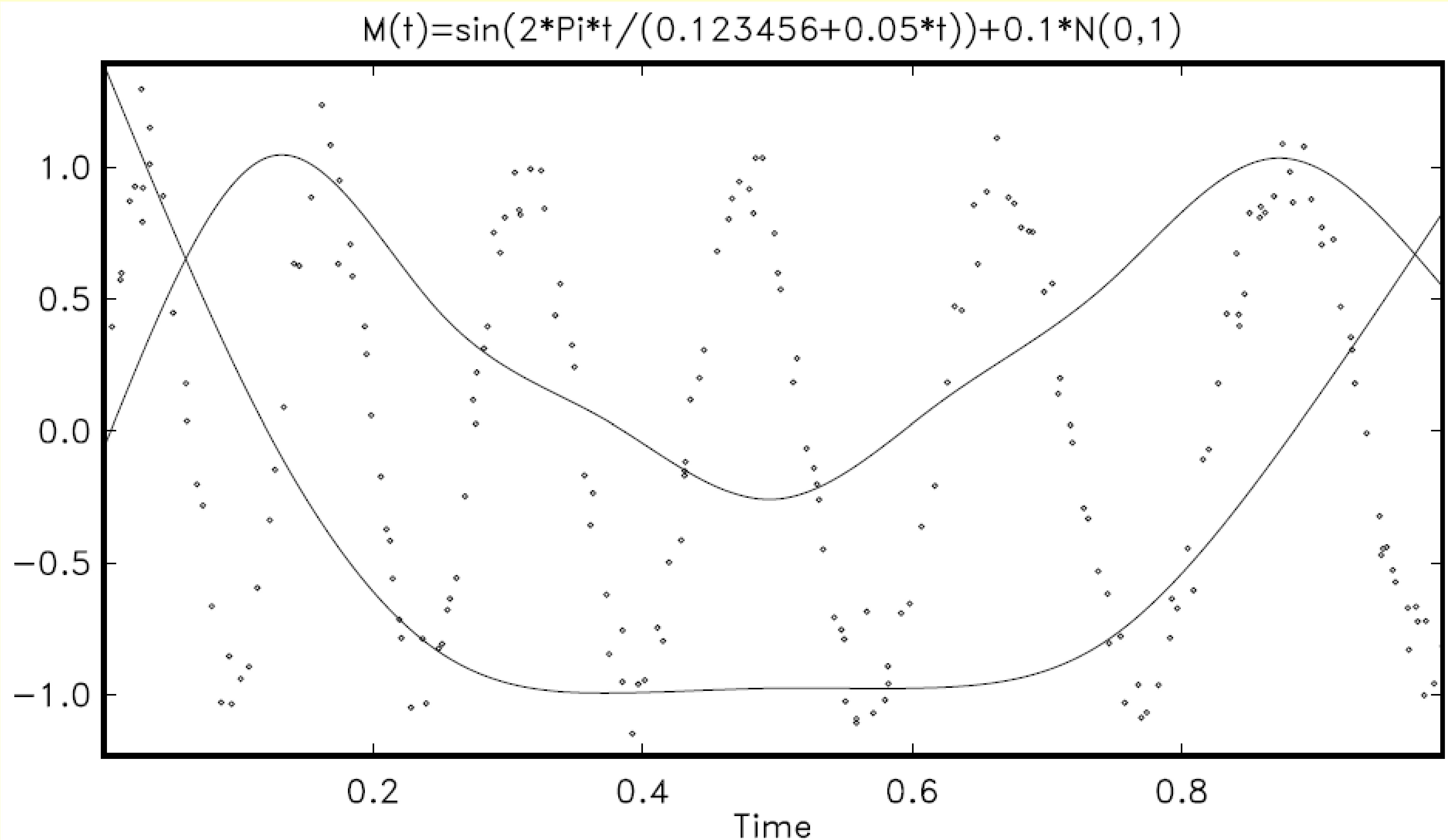
Carrier frequency

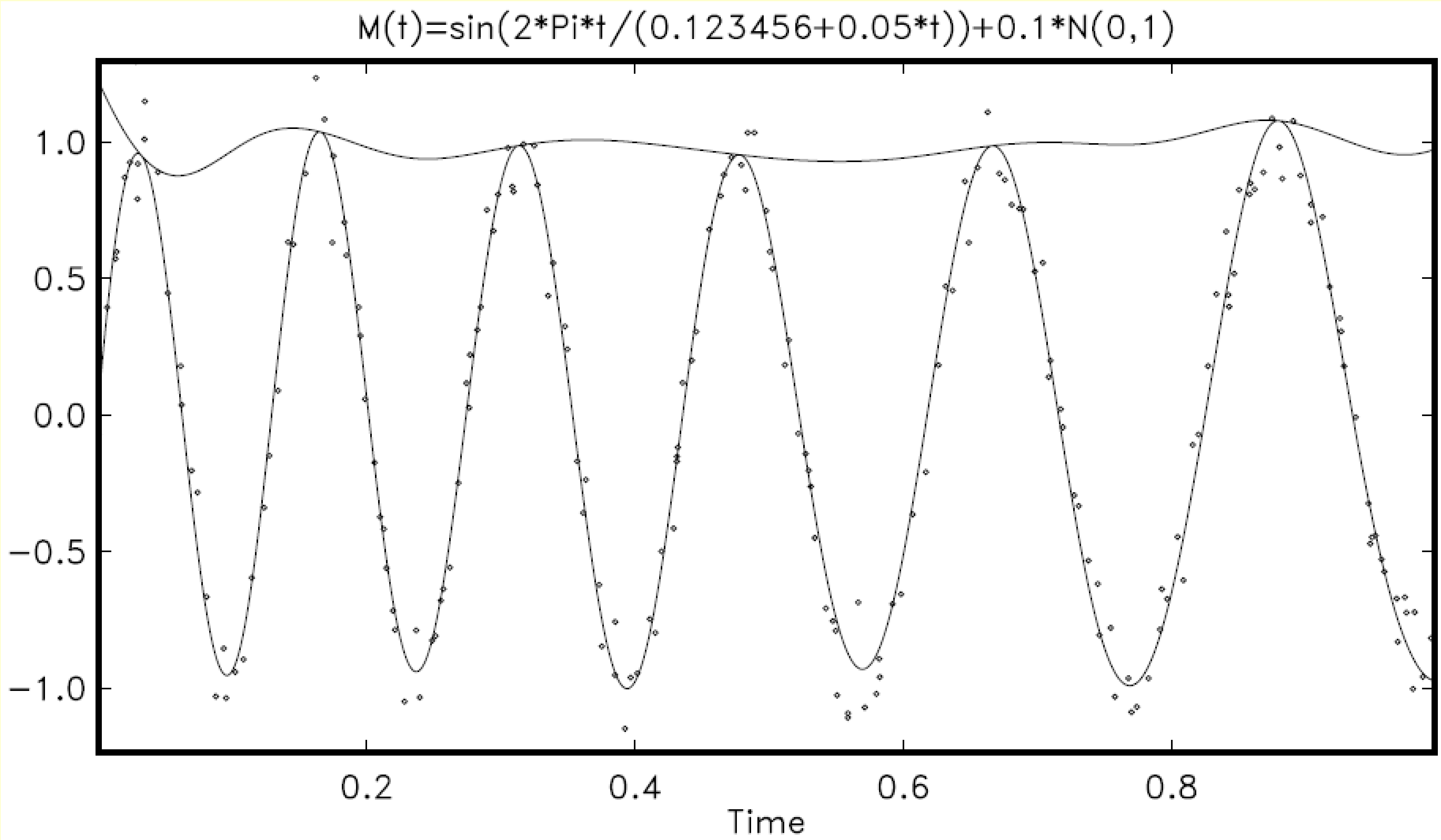
$$f(t) = A(t)\cos(2\pi\nu_0 t) + B(t)\sin(2\pi\nu_0 t)$$

$$f(t) = \sum_{k=1}^{K} A_k(t)\cos(2\pi k\nu_0 t) + B_k(t)\sin(2\pi k\nu_0 t)$$

Hilbert Transform using Bedrosian's theorem

$$H(f(t)) = A(t)\sin(2pn_0 t) - B(t)\cos(2pn_0 t)$$

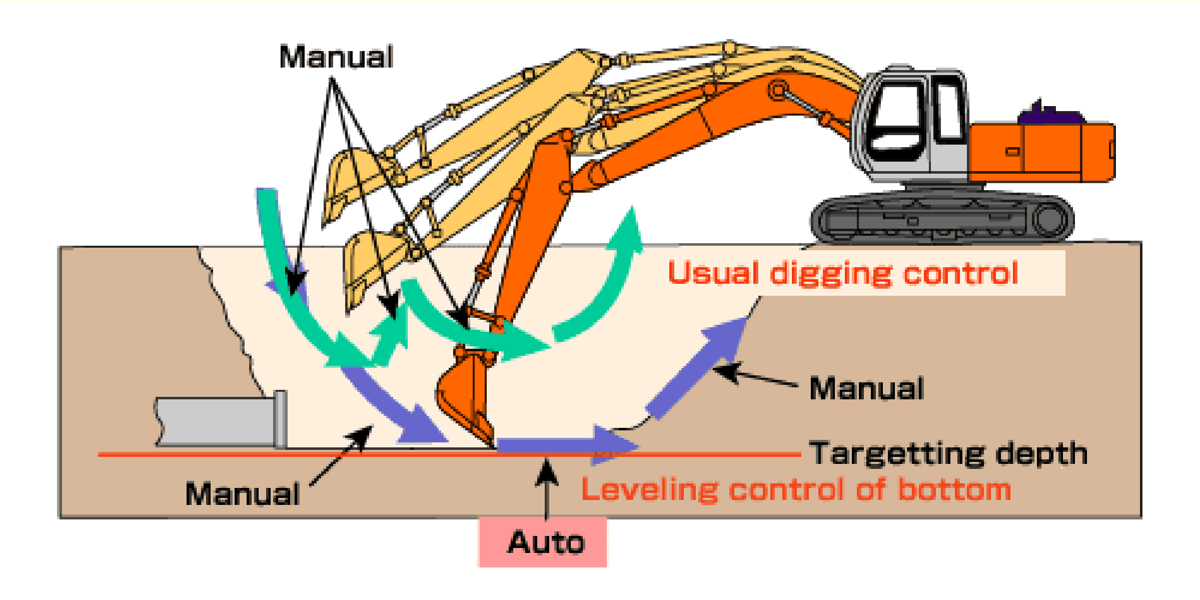

M(t)=sin(2*Pi*t/(0.123456+0.05*t))+0.1*N(0,1)
1.0
0.5
0.0
−0.5
−1.0
0.2
0.4
0.6
0.8
Time



M(t)=sin(2*Pi*t/(0.123456+0.05*t))+0.1*N(0,1)
1.0
0.5
0.0
−0.5
−1.0
0.2
0.4
0.6
0.8
Time

Empirical modes and carrier fit

- Preprocessing – interpolation using carrier fit (?)
- Empirical mode decomposition
- Carrier fit into computed modes (?)



Jaan Pelt
Nigul Olspert

Kas keegi tahab veel?