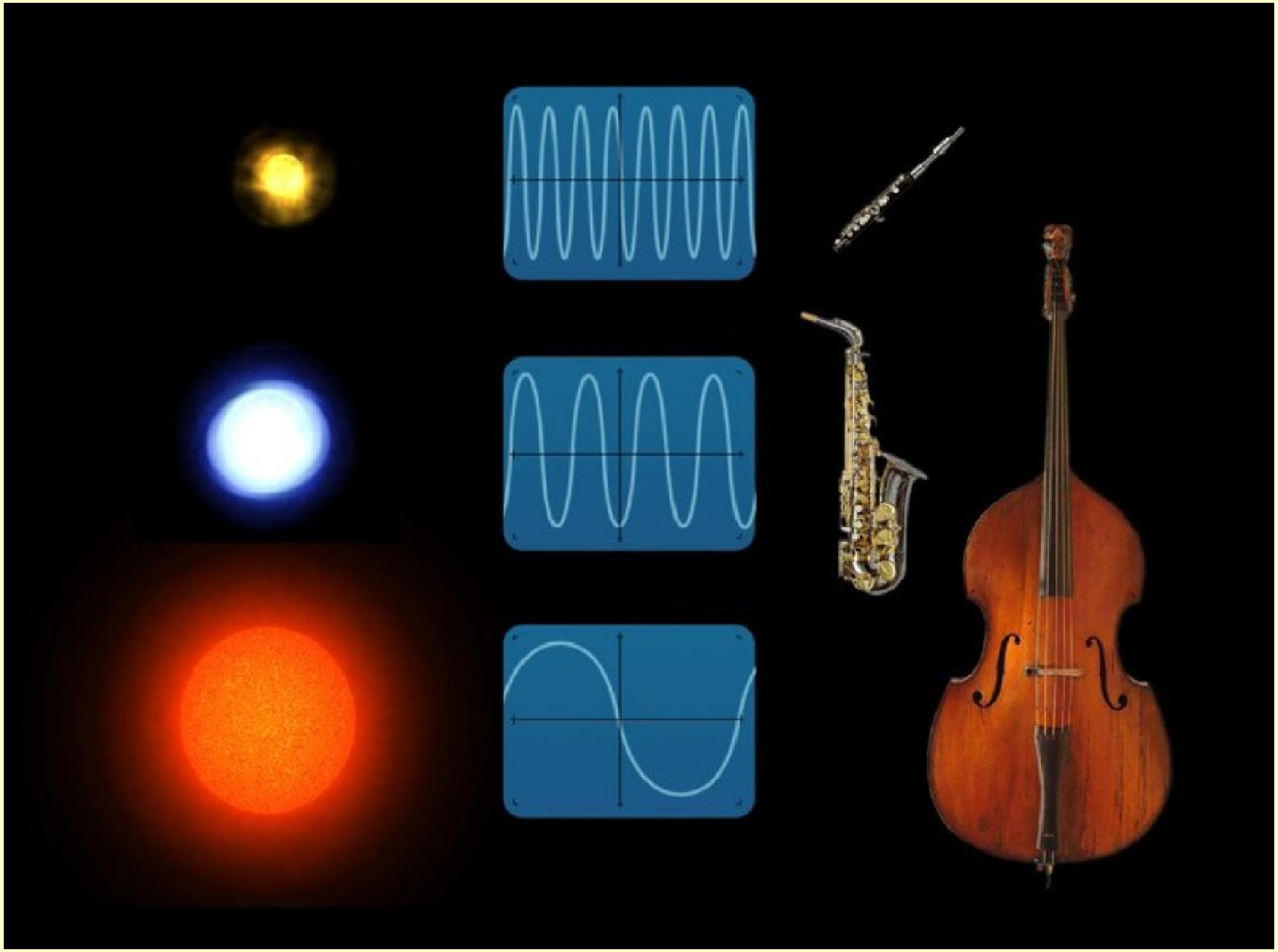
The Finnish Graduate School in Astronomy and Space Physics Summer School 2007:

Time Series Analysis

Summary

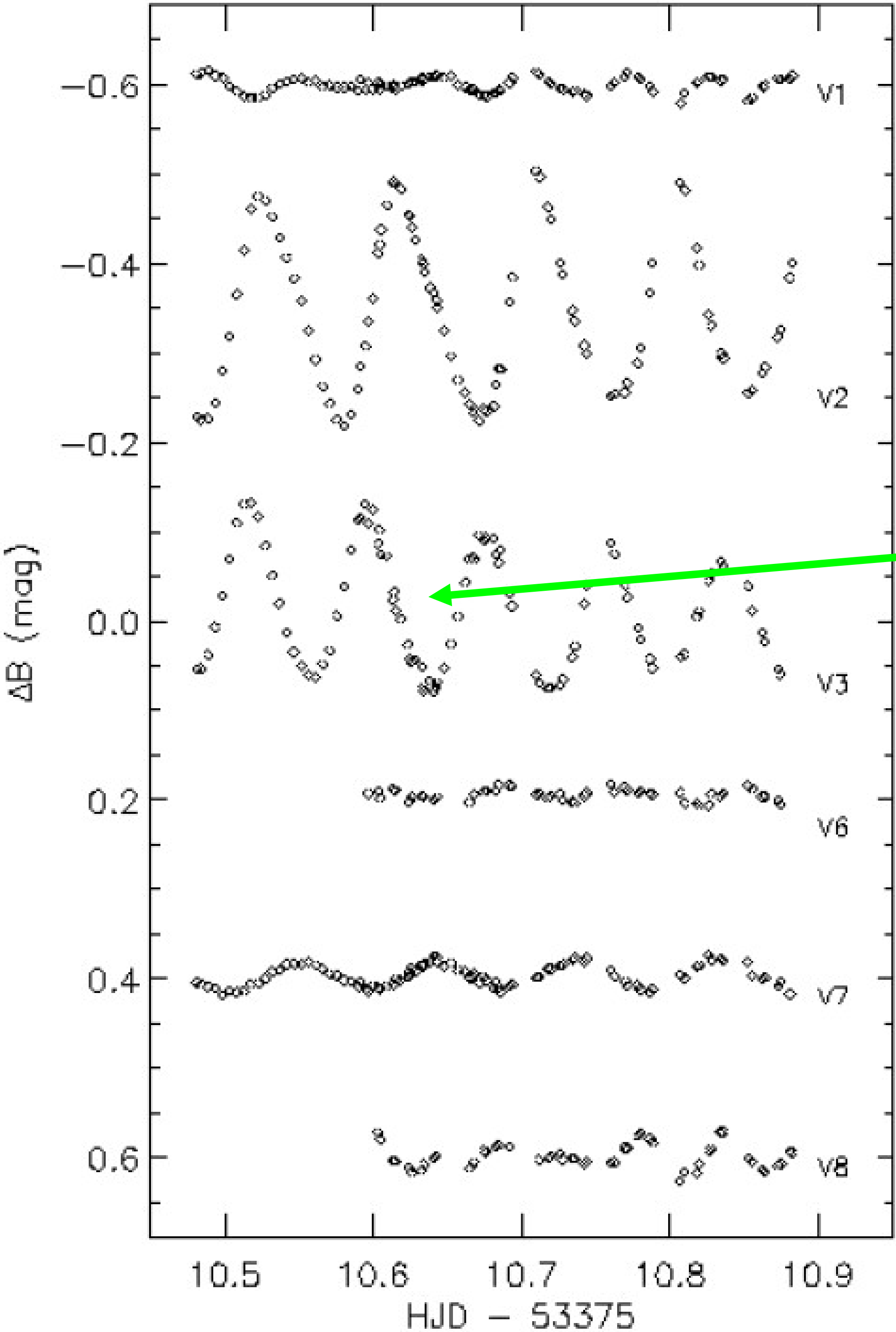
- Different data
- Different ideas
- Different methods
- Different results

Everything starts from harmonics



But reality is more complex

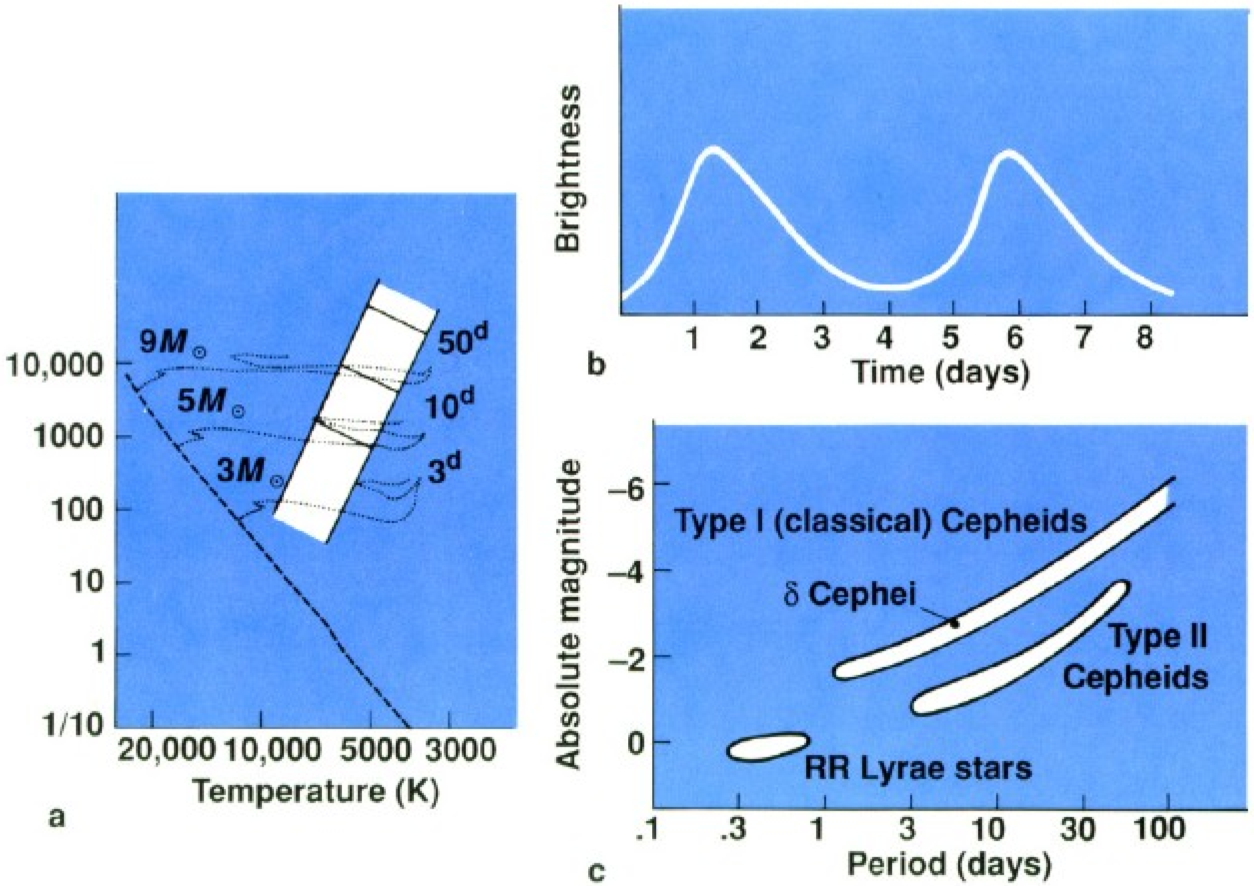
Delta Scuti stars

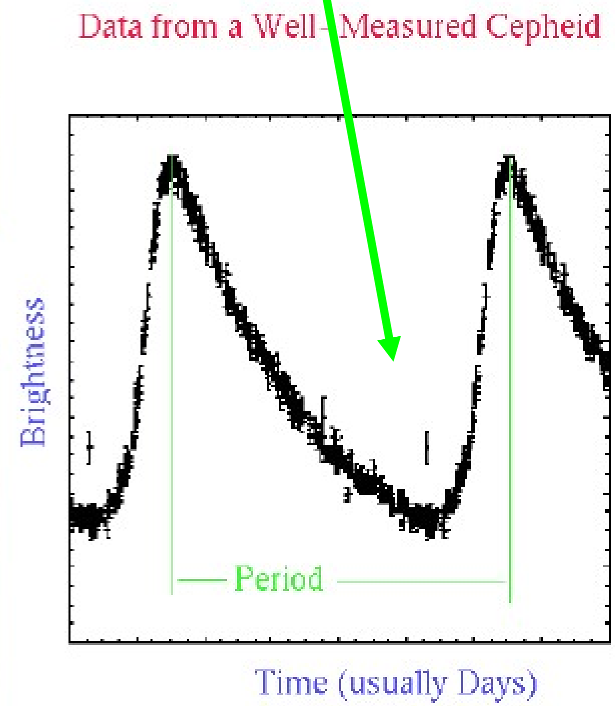


Single or multiple harmonics

Nonharmonic stuff

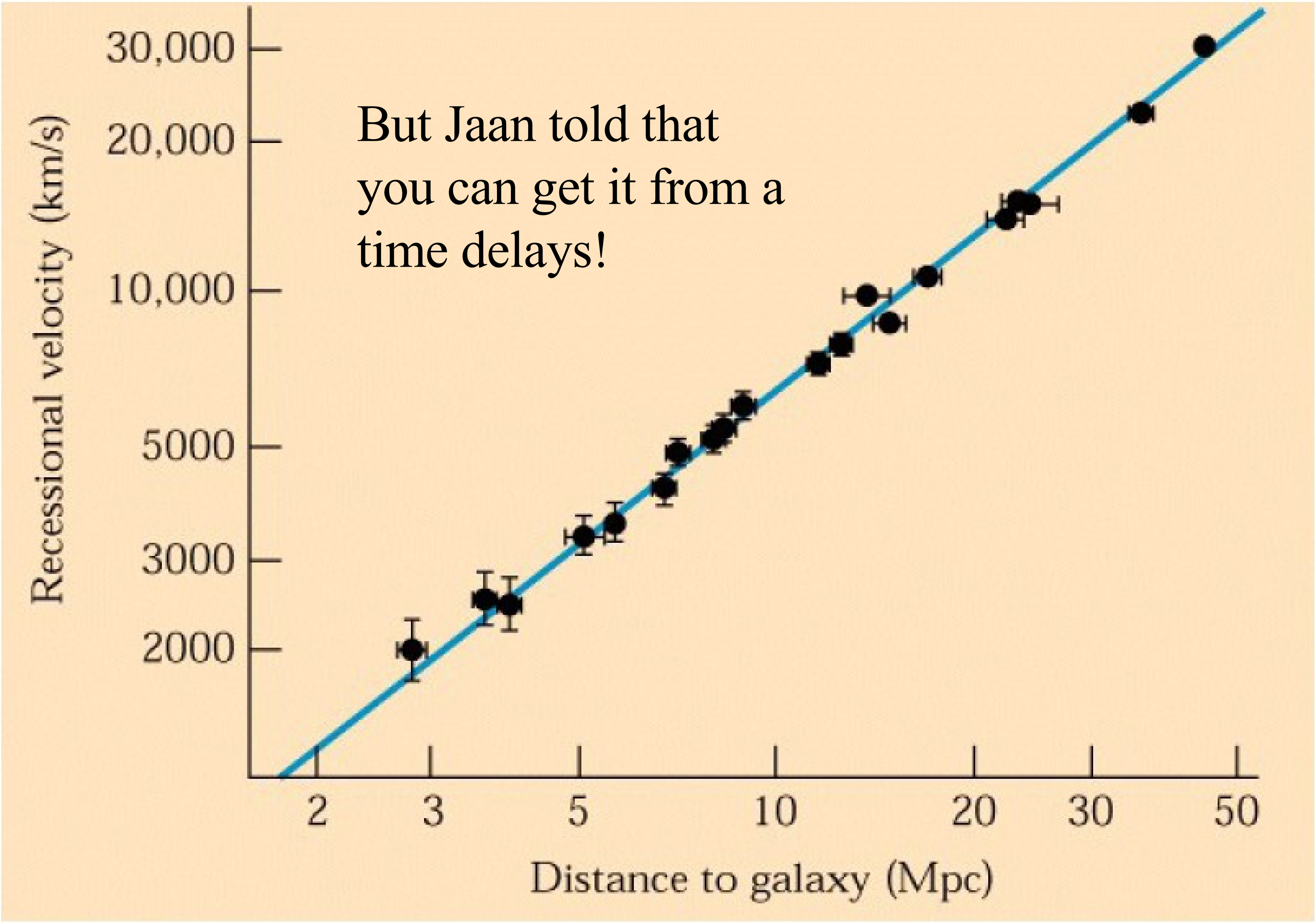
Cepheids



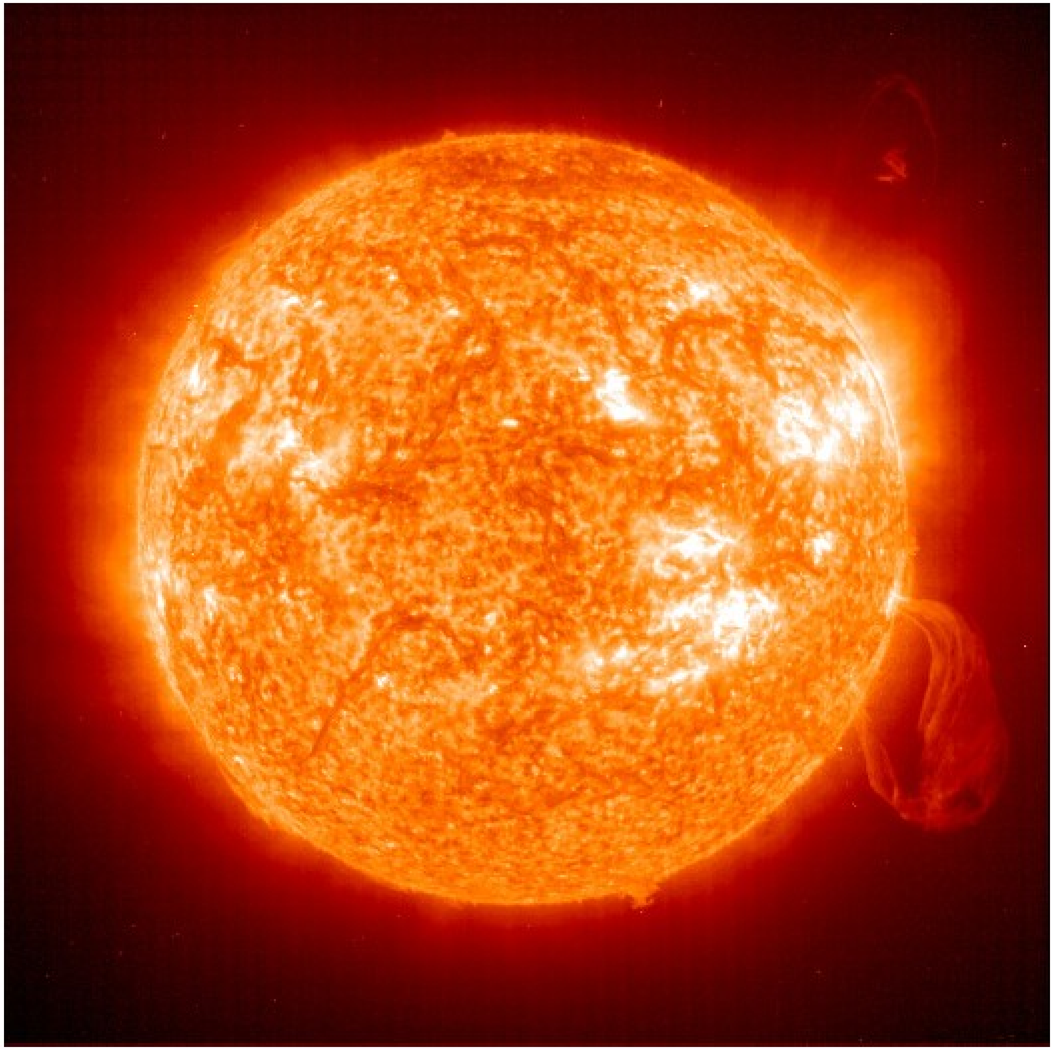


$$\langle M_V \rangle = -2.76\log(P) - 1.40$$

Hubble's law: *velocities* from redshifts – distance from Cepheids

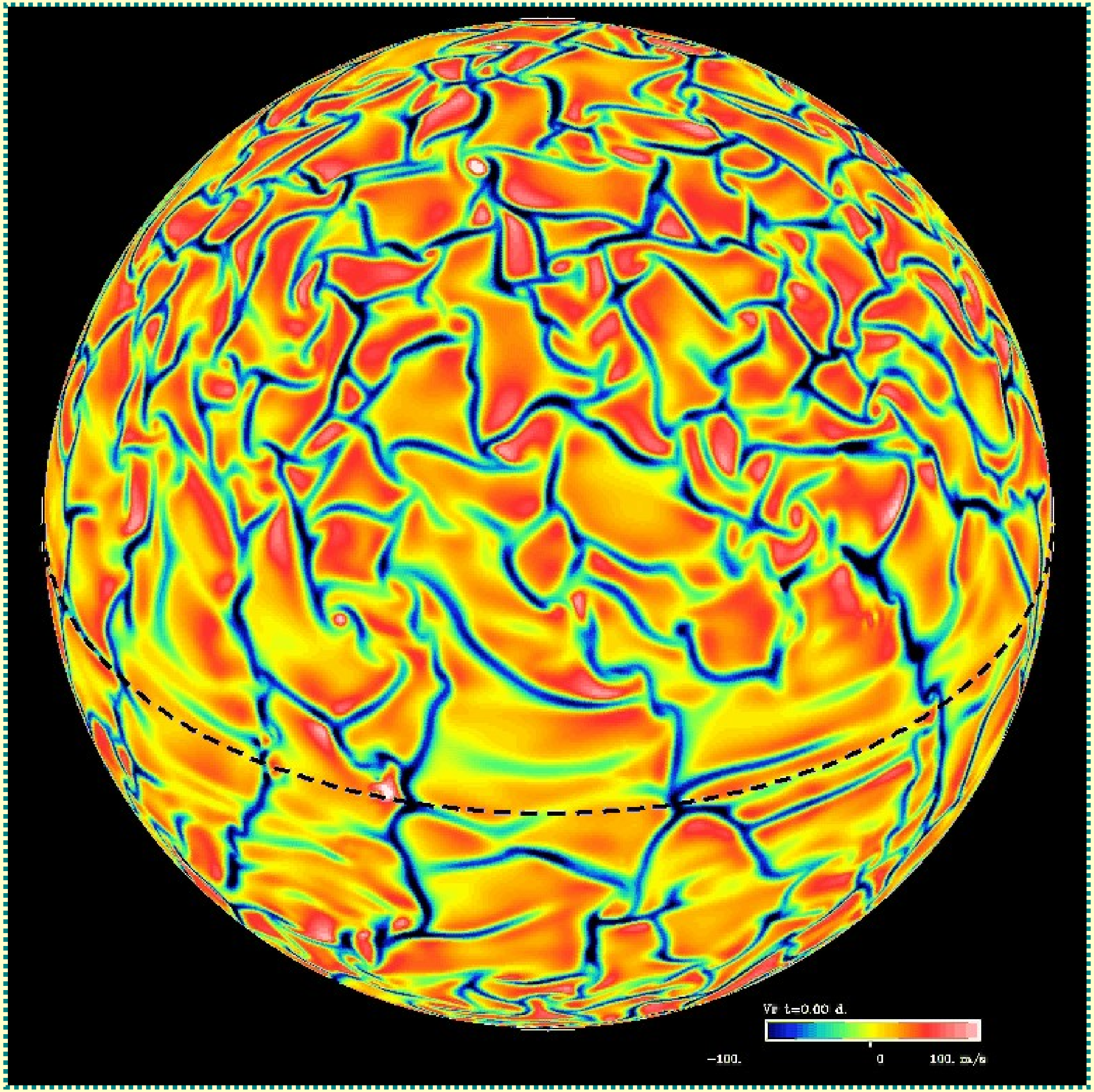


The Sun is oscillating in about 10 million different modes with Periods of 3 – 15 minutes



You can never find them all with ISDA software!

Evolution in Radial Velocity (0.96 R)

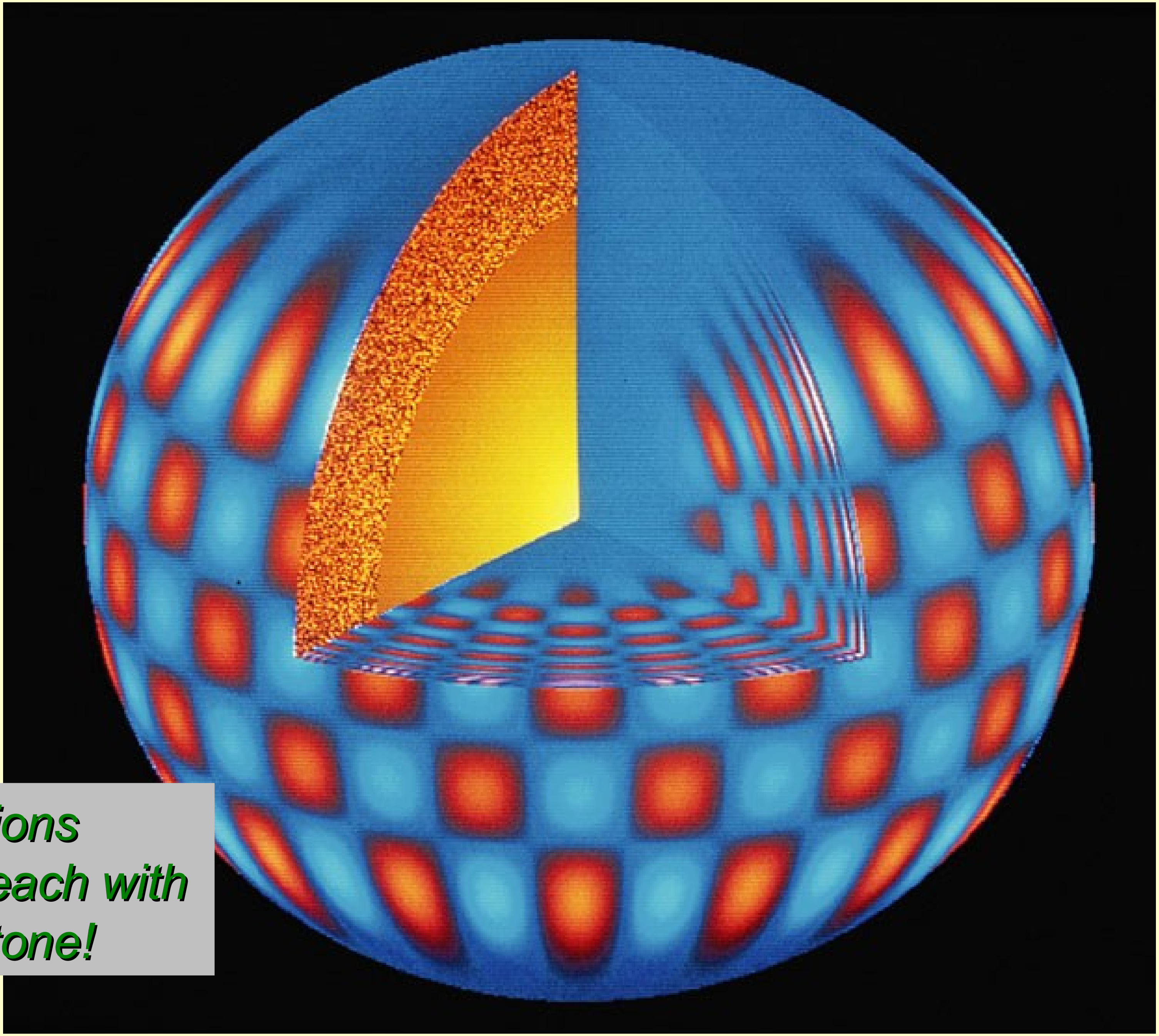


NEAR TOP OF LAYER

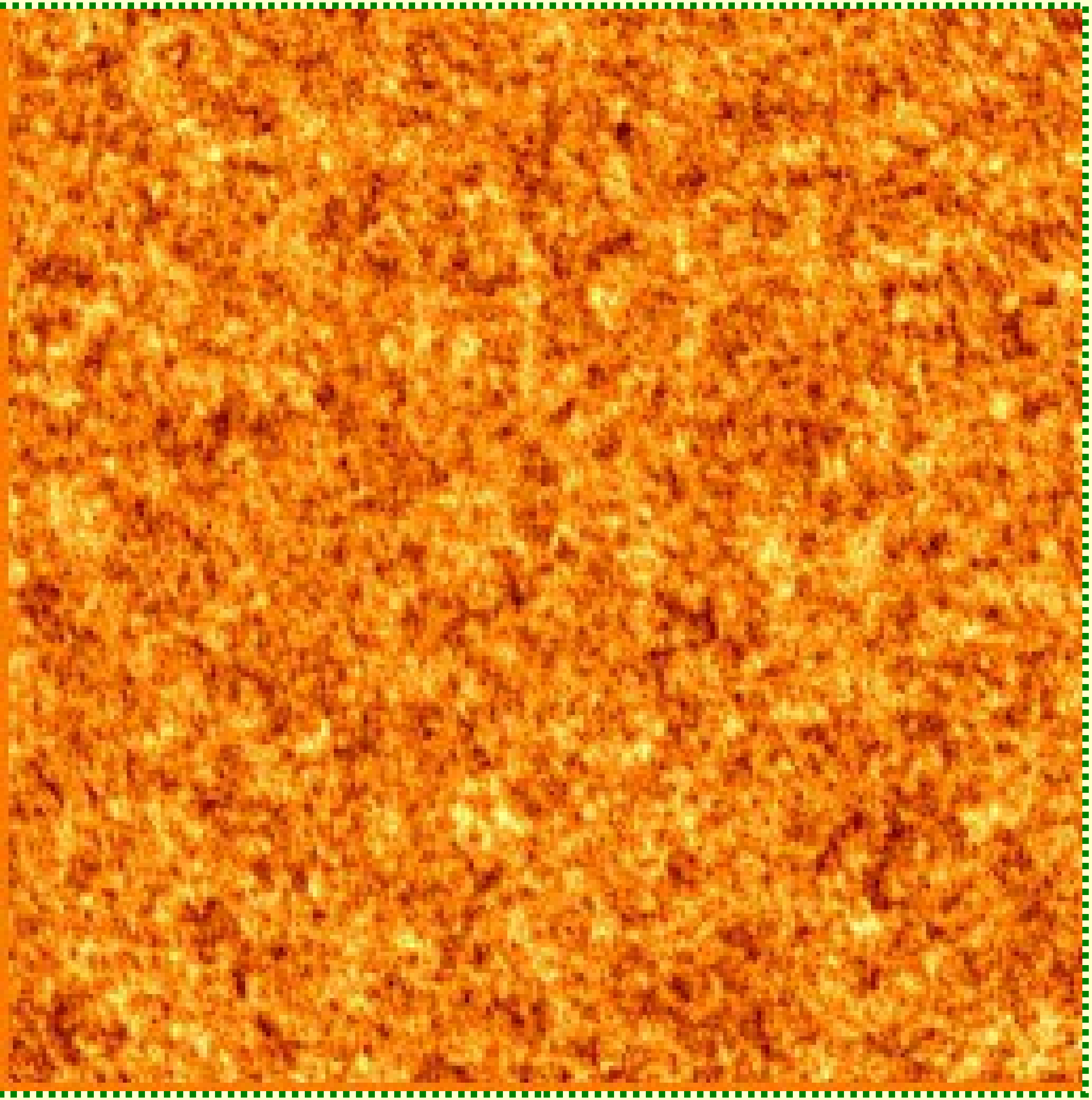
VIEWED IN `CARRINGTON' FRAME

Case E Brun, Miesch & Toomre

Solar Oscillation Mode



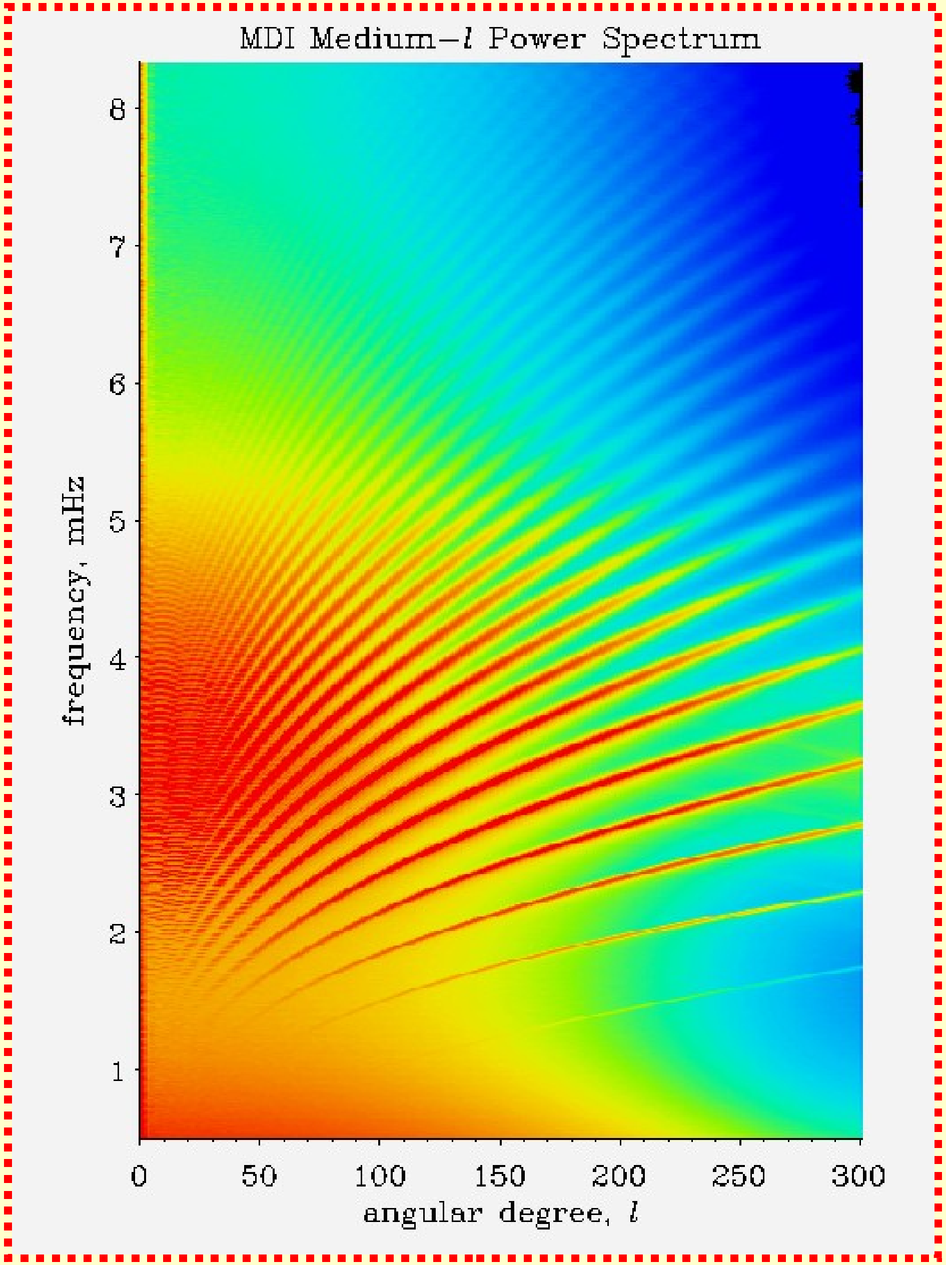
One of millions of modes, each with a different tone!

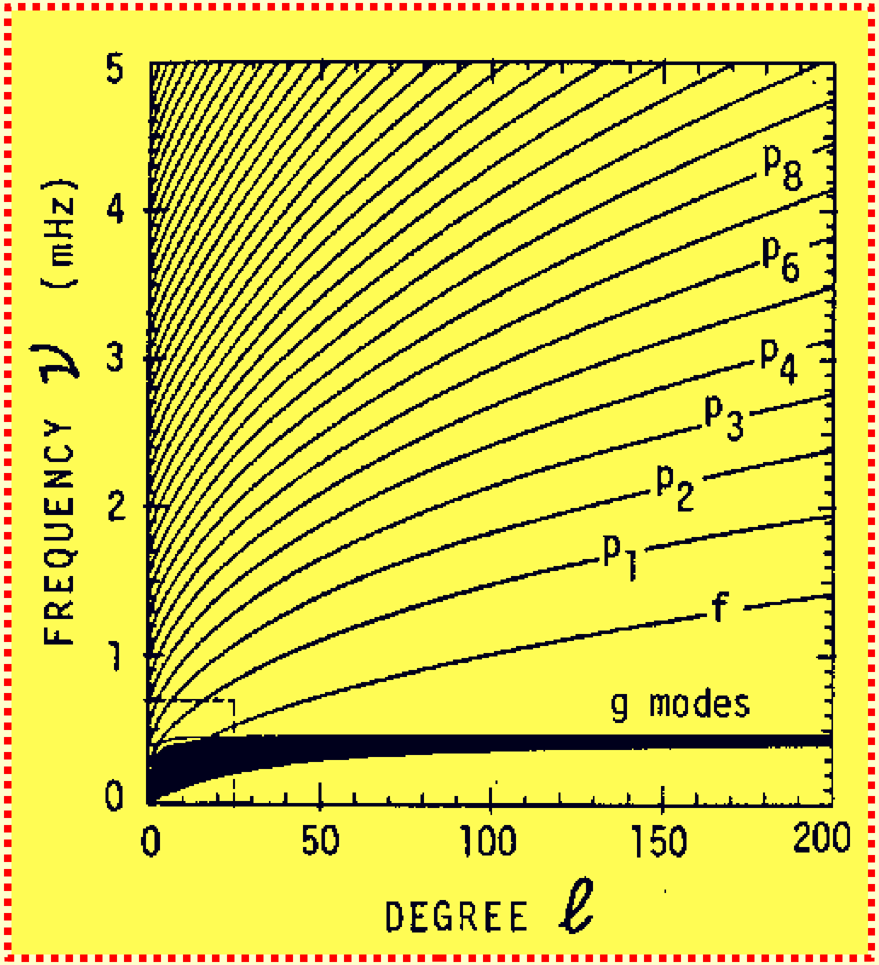


Radial Velocity
from Michelson Doppler Imager

~ 20^o across

Power Spectrum of Global Modes



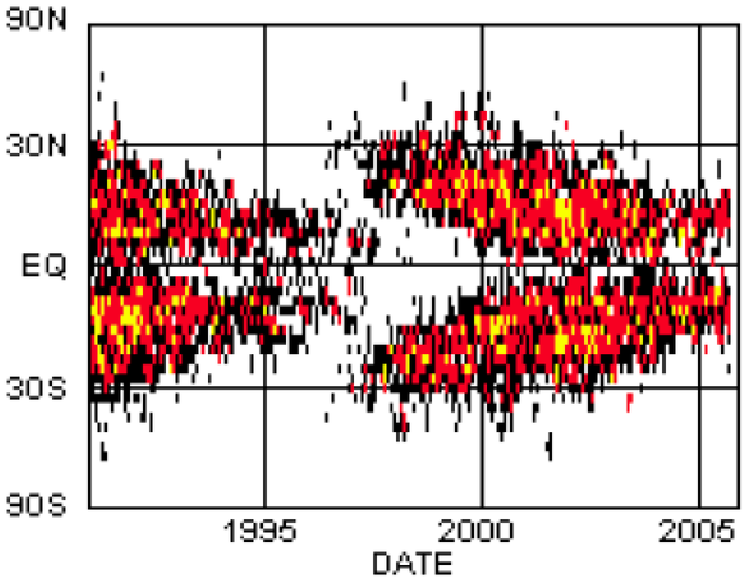


THEORY

MDI on SOHO

Butterfly diagram

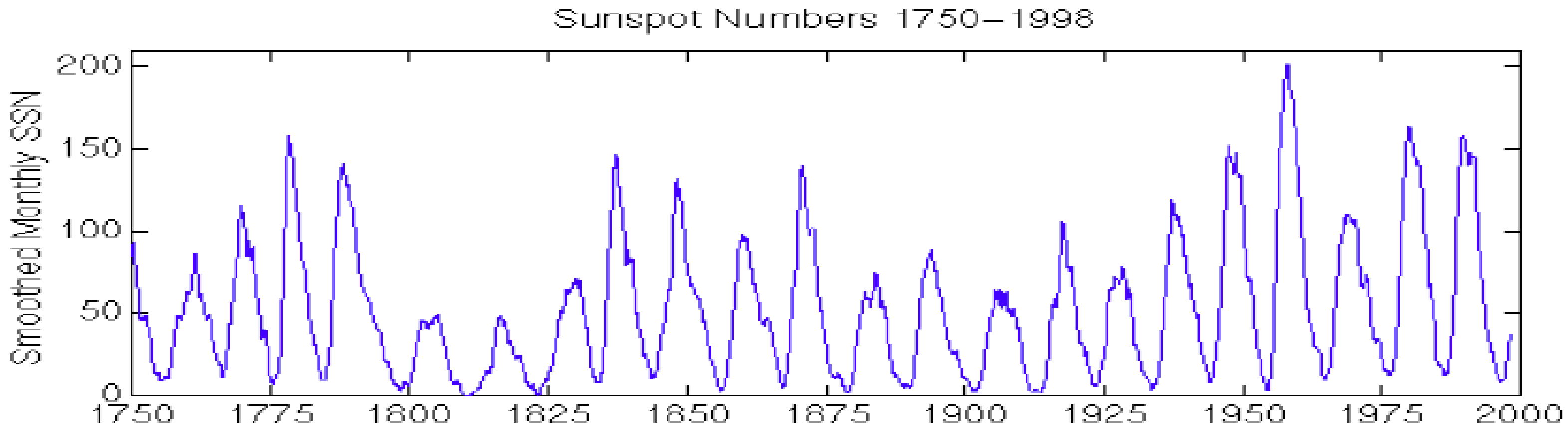
- First spots of the new cycle at latitude ~30-35°
- Last spots of the cycle within ±10 ° of the equator



⇒ Butterfly diagram

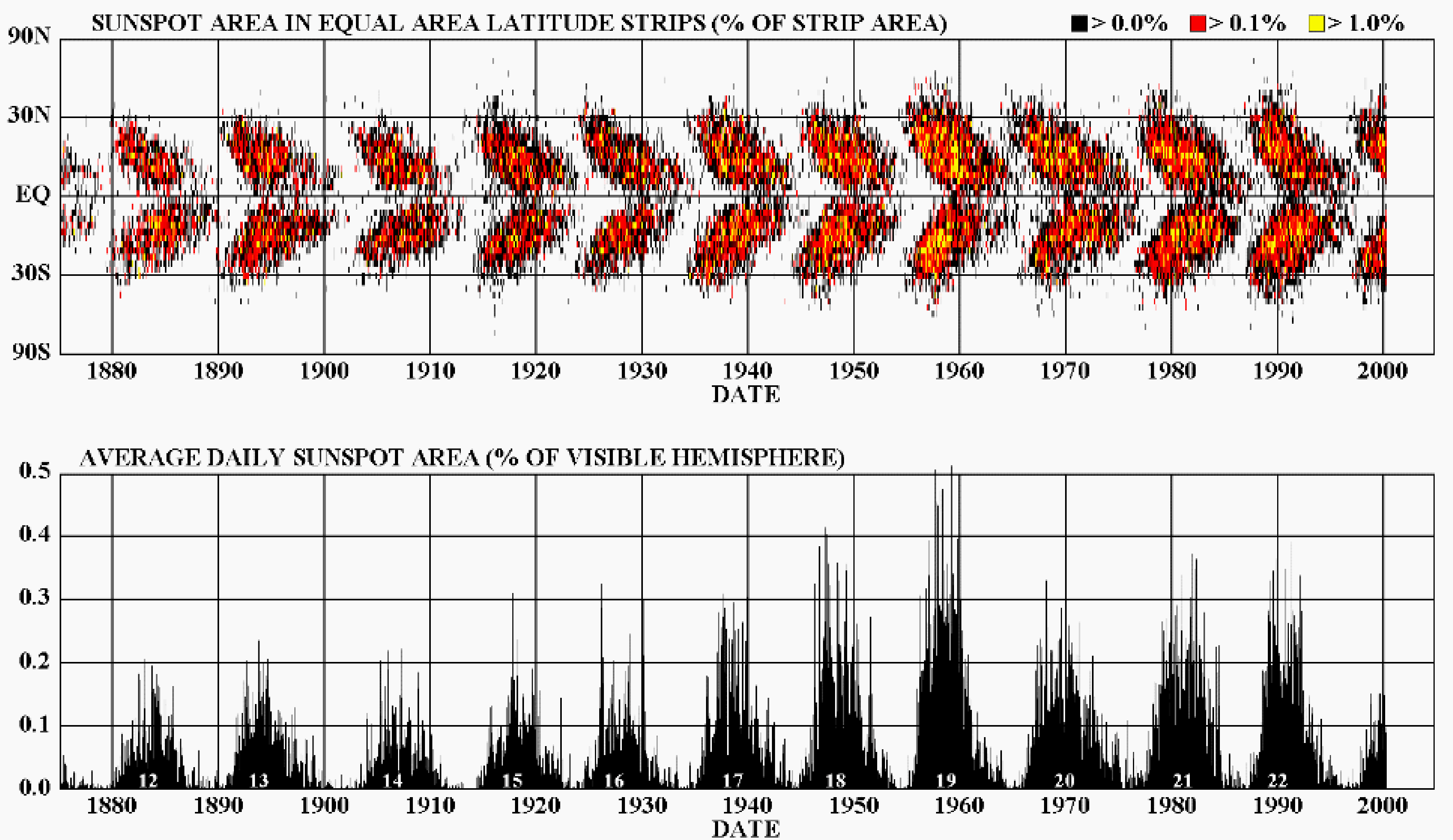
But flip-flop? .... Ooooh...nooo...

Solar cycle



- Solar activity has on average 11 year cycle
- Cycle length changes between 9 and 13 years

11-year Solar Cycles
familiar butterflies… yet profound



Data transformations

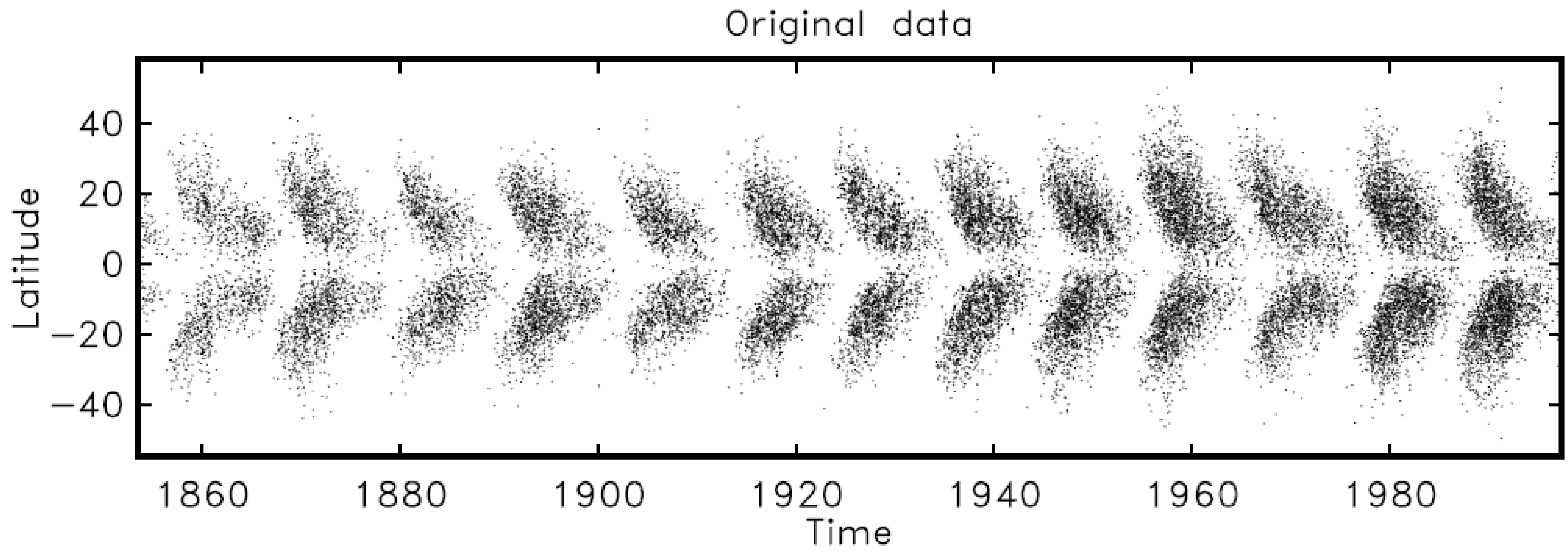
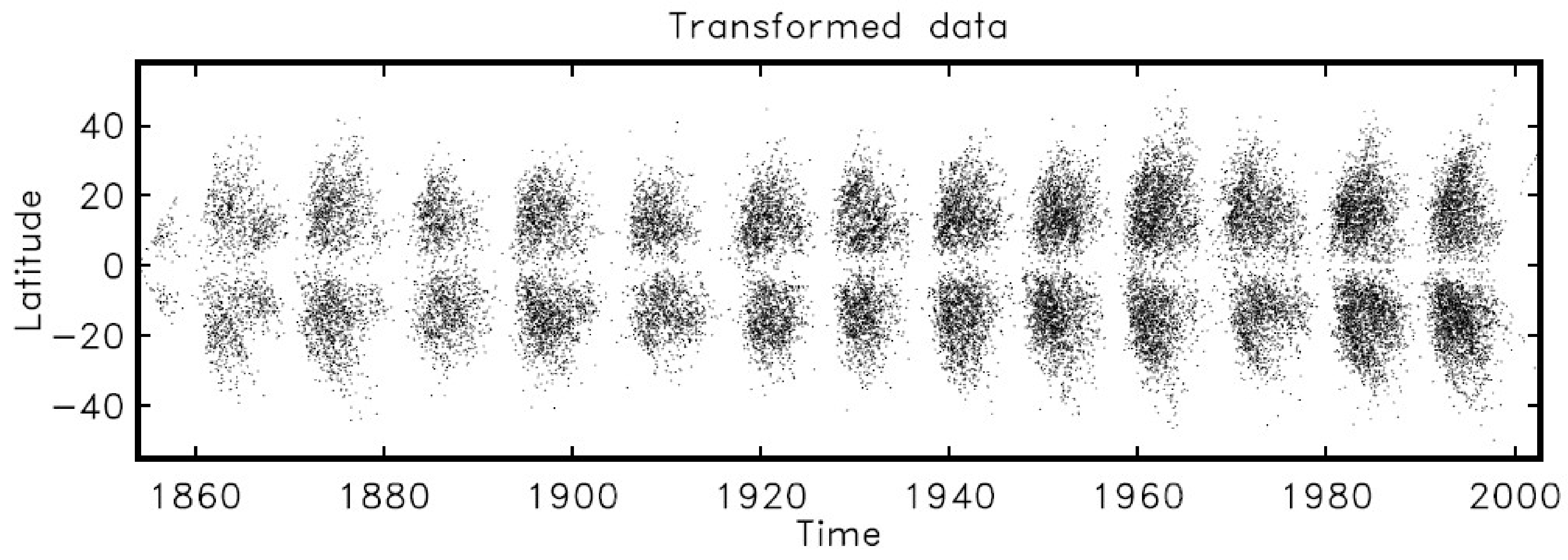


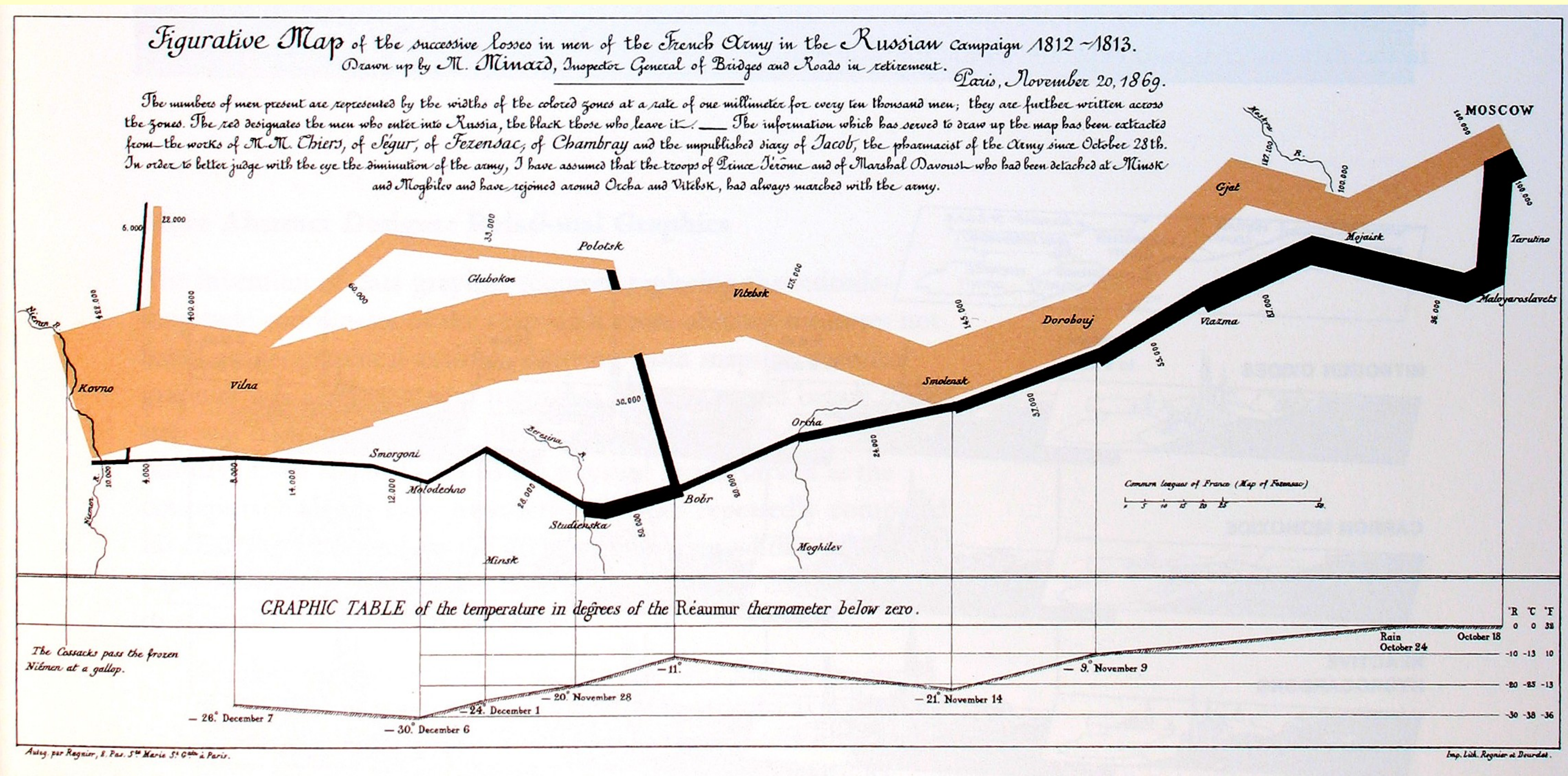
Fig. 2. A butterly diagram showing latitude θ vs time (1853–1996). The cycles are separated by clearly defined gaps.

Transformed data



Charles Joseph Minard

March 27, 1781 in Dijon – October 24, 1870 in Bordeaux



From where to start?

- Time
- Phase
- Movement
- Frequency

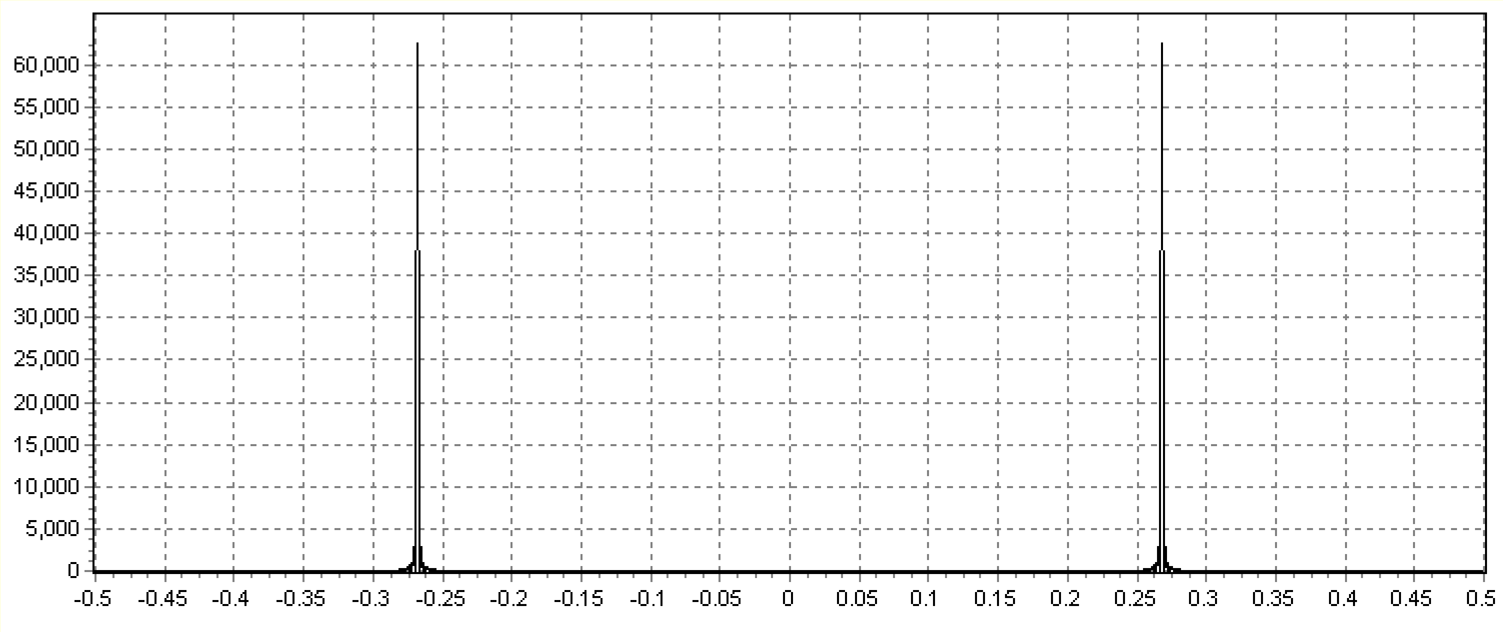
$$\text{Frequency} = \frac{1}{\text{Period}}$$

$$\text{Period} = \frac{1}{\text{Frequency}}$$

Why is there so many peaks in spectra?

- Fourier transform is complex transform
- Data set is finite
- We can compute only approximately
- Data set is not equally spaced
- Signal is not harmonic
- The signal can contain more frequencies
- Signal parameters change in time
- Observations are noisy

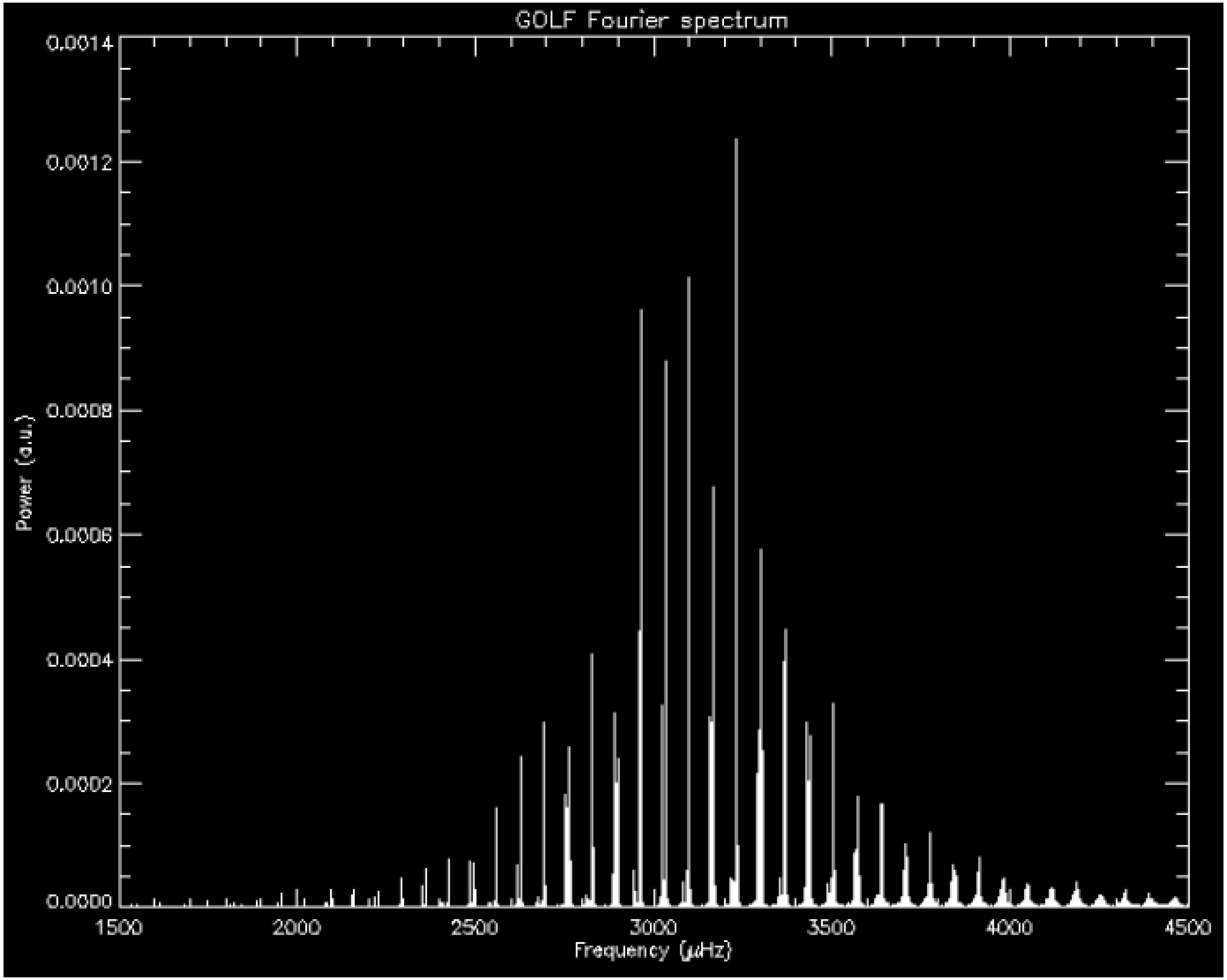
Fourier transform is complex, harmonics are real



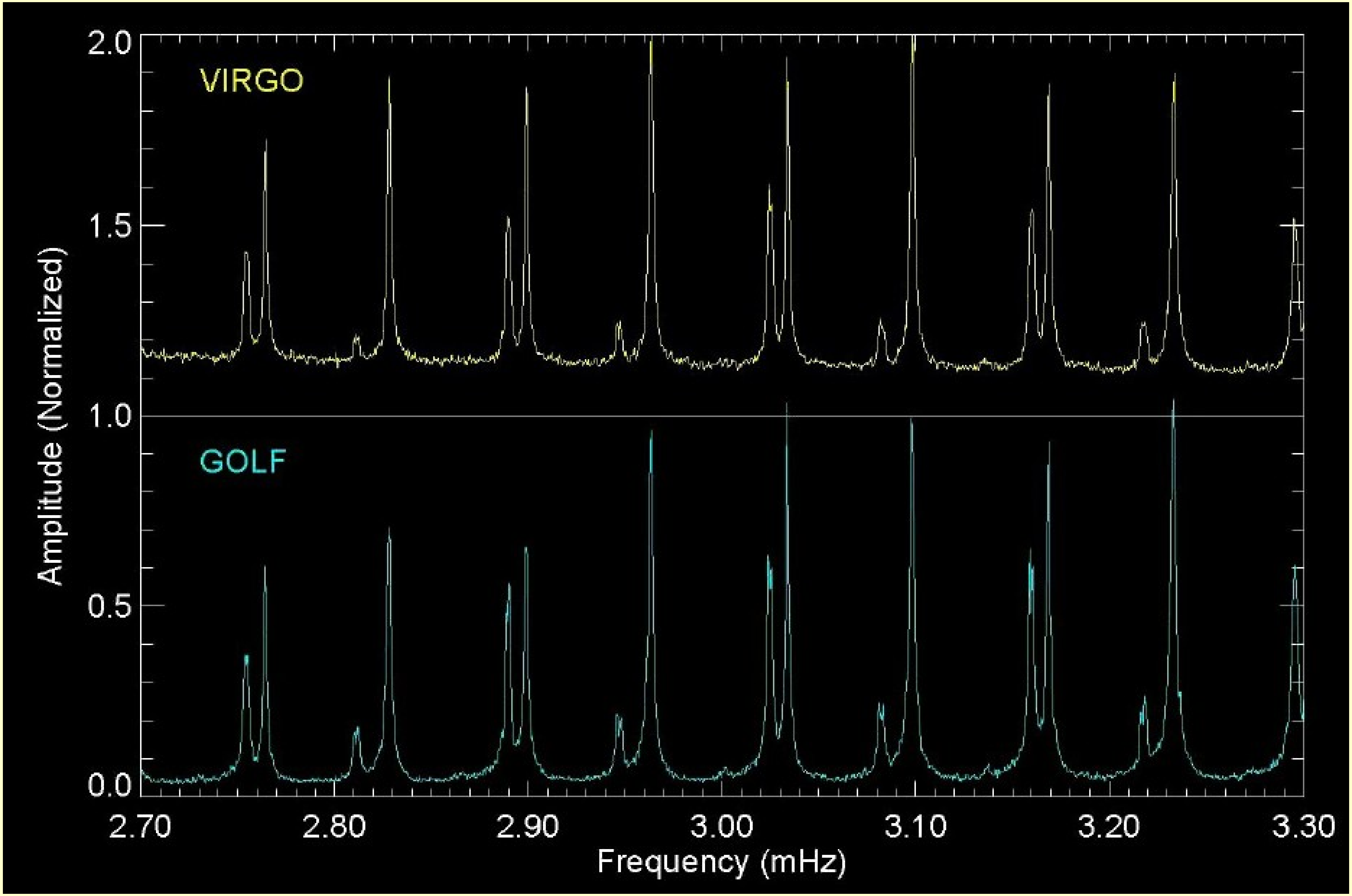
$$\cos(\nu) = \frac{C(\nu) + C(-\nu)}{2}$$

$$f = A_i \sin(2\pi\nu_i t + \varphi_i)$$ → LSQ fit to the time series

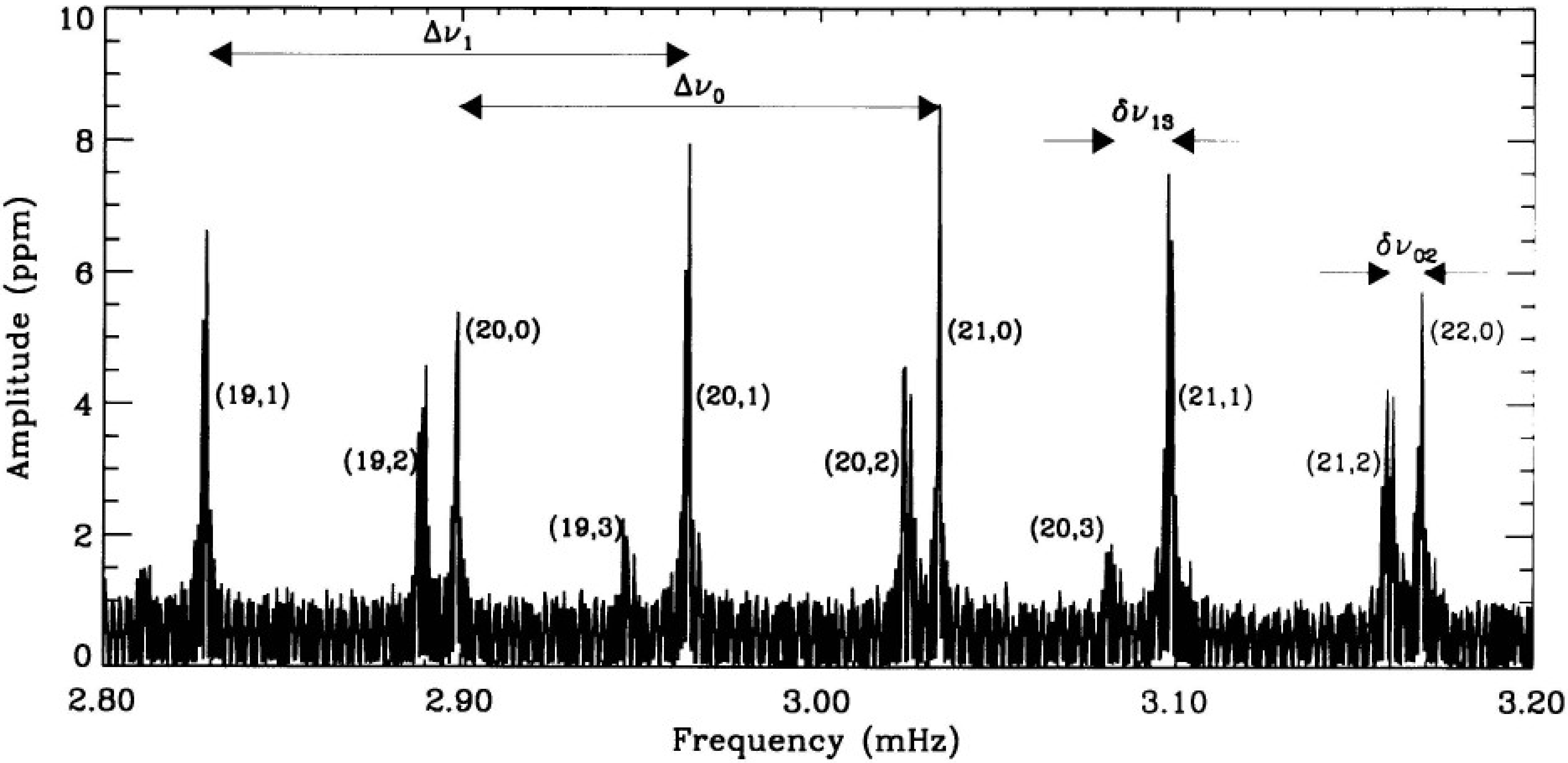
Ampl.



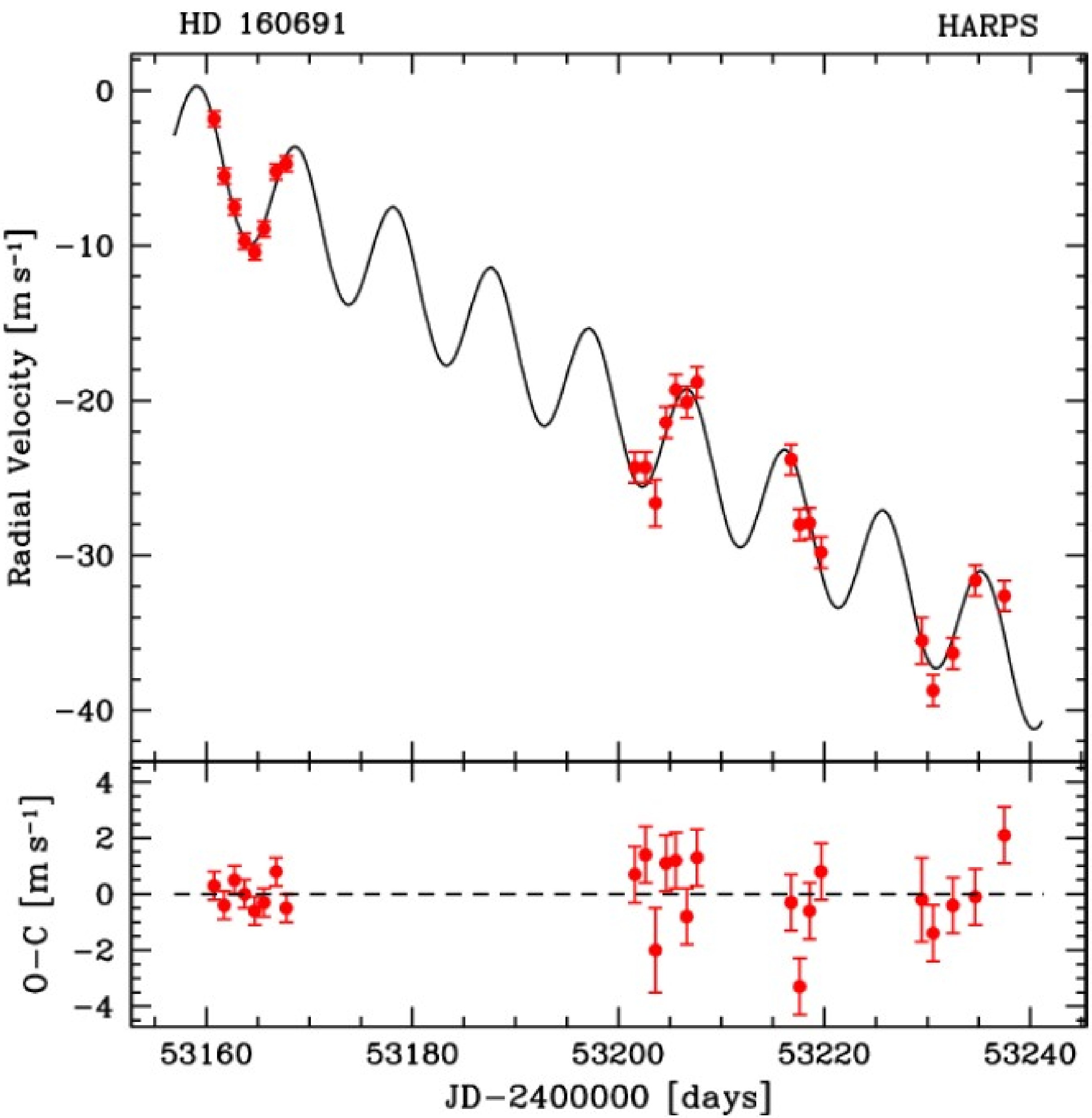
Frequency

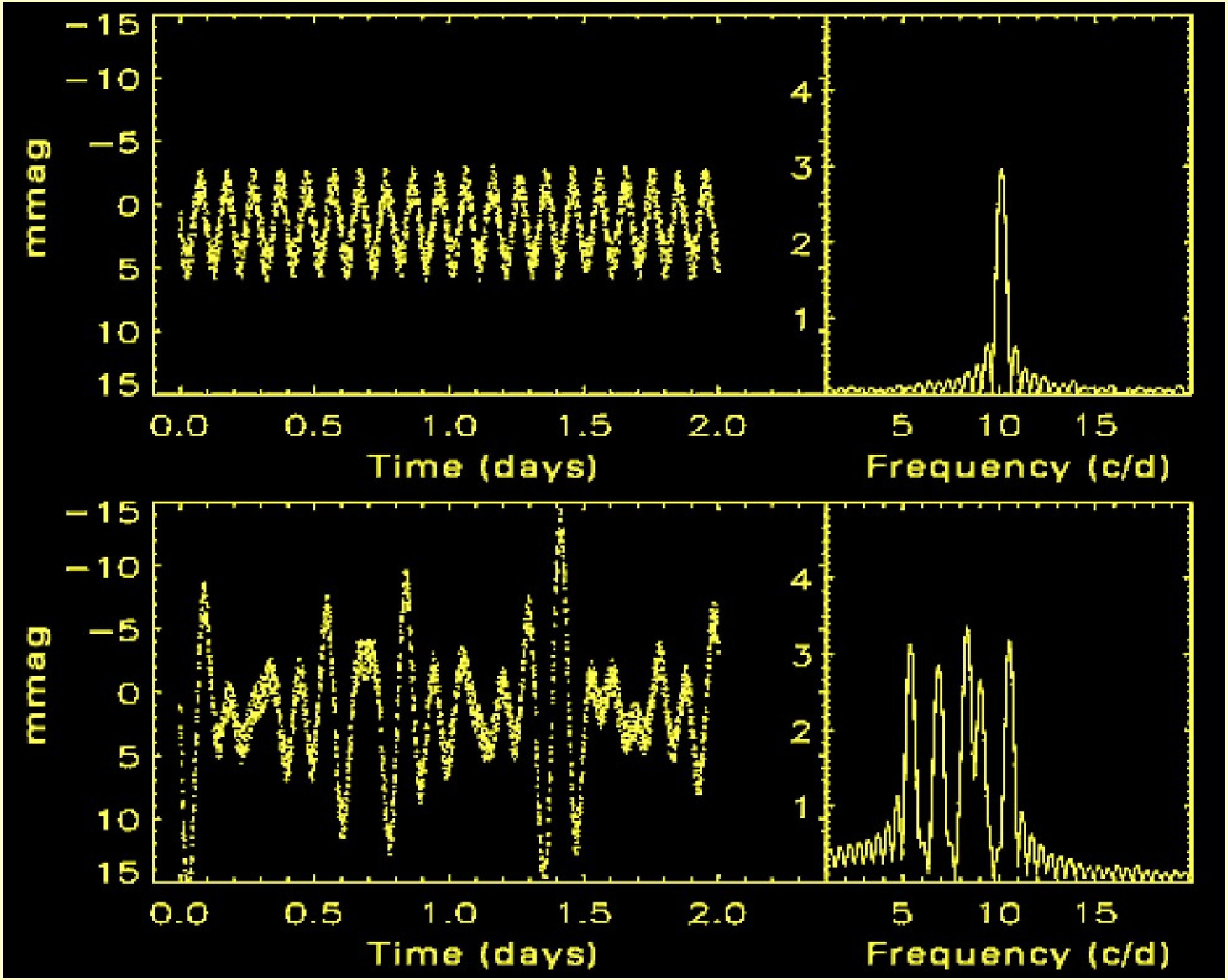

VIRGO
GOLF
Amplitude (Normalized)
2.0
1.5
1.0
0.5
0.0
2.70
2.80
2.90
3.00
3.10
3.20
3.30
Frequency (mHz)

$$\nu_{n,l} = \Delta\nu(n + \frac{1}{2}l + \varepsilon) - l(l+1)D_0$$



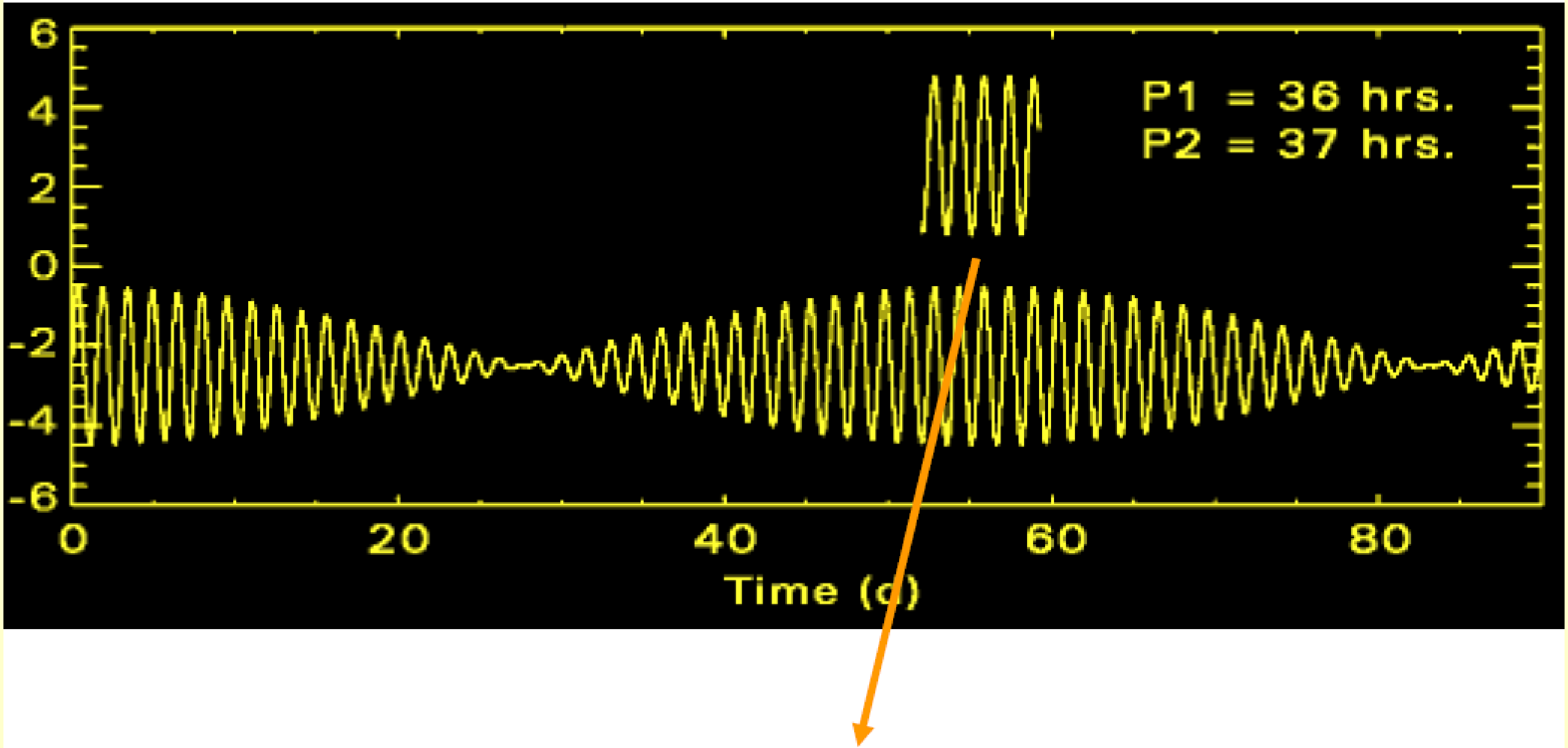
Example: μ Arae – a solar-like star with 3 planets





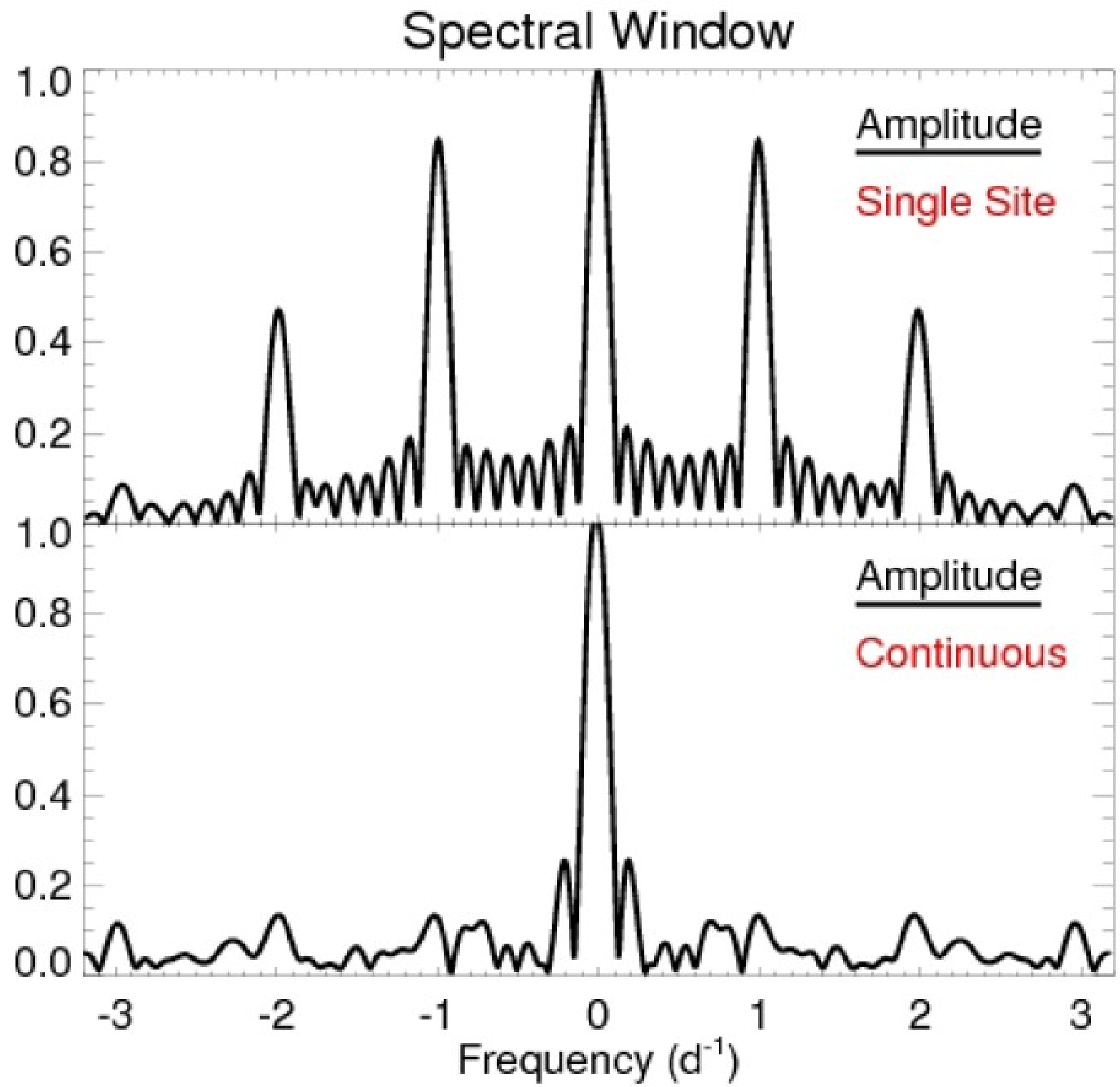
mmag
Time (days)
Frequency (c/d)
mmag
Time (days)
Frequency (c/d)

The observations have to be sufficiently long in order to cover Beat Periods:



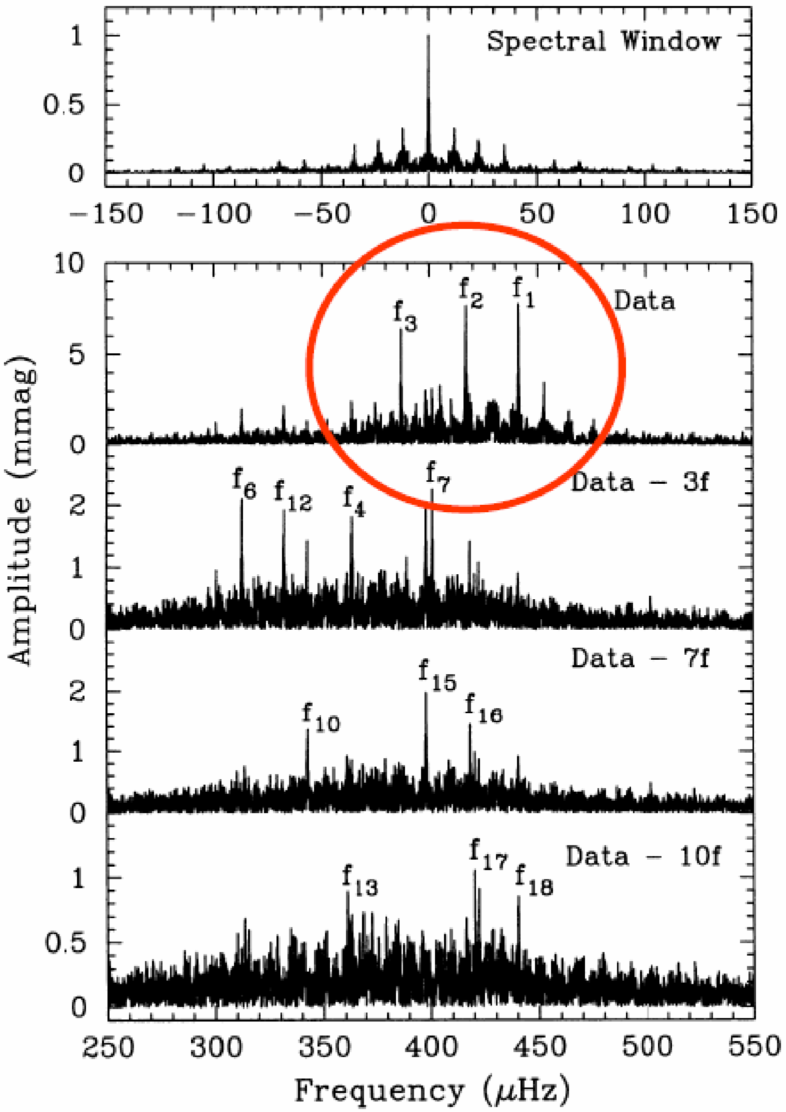
Not even Fourier would help us here…

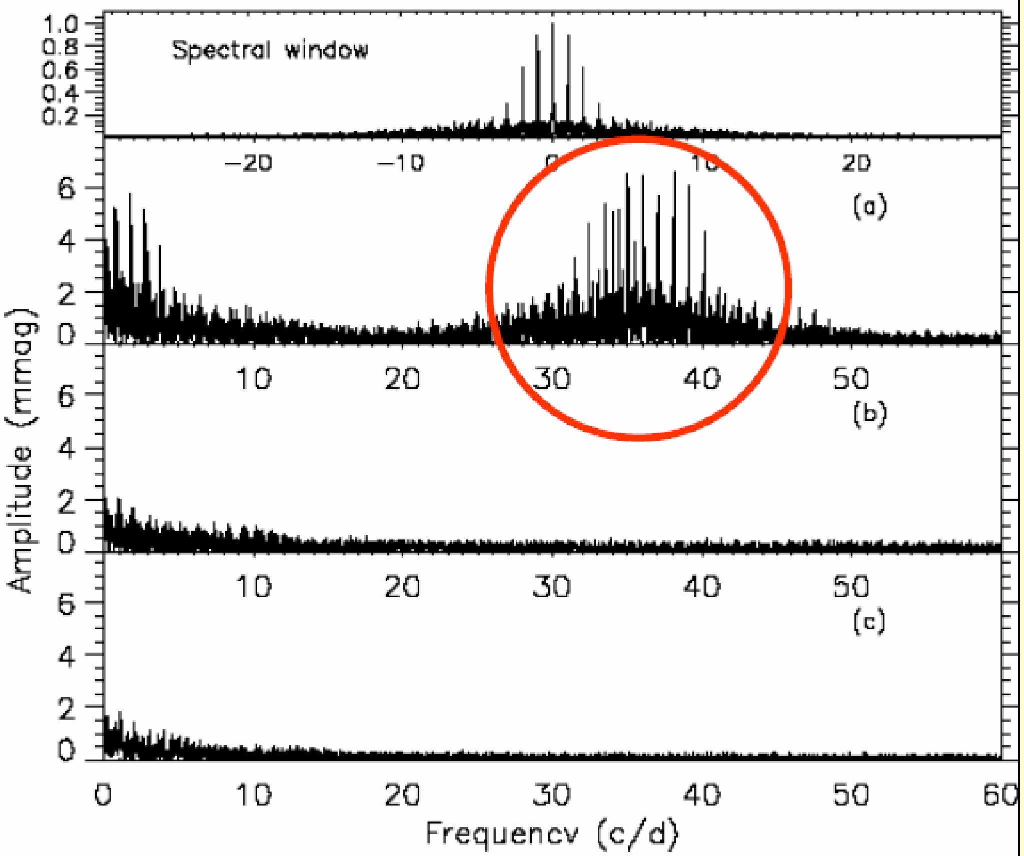
The Spectral Window Function



Multi-site

Single-site





Jaan asks - where is the mean?

Calculating the Power Spectrum

- A set of N observations, $(x_1, t_1), \ldots, (x_N, t_N)$
- Set up a model of the observations at each frequency ν_i

$$x_j = \alpha_i \cdot \cos(\nu_i t_j) + \beta_i \cdot \sin(\nu_i t_j)$$

- Minimize the Sum of Squares

$$R(\nu_i) = \sum_{j=1}^{N} \{x_j - [\alpha_i \cdot \cos(\nu_i t_j) + \beta_i \cdot \sin(\nu_i t_j)]\}^2$$

Calculating the Power Spectrum

- Then the Power Spectrum is calculated as

$$P(\nu) = A_i^2 = \alpha(\nu)^2 + \beta(\nu)^2$$

$$\alpha(\nu) = \frac{s \cdot cc - c \cdot sc}{ss \cdot cc - sc^2}$$

$$\beta(\nu) = \frac{c \cdot ss - s \cdot sc}{ss \cdot cc - sc^2}$$

Can be computed using FFT.
But how?

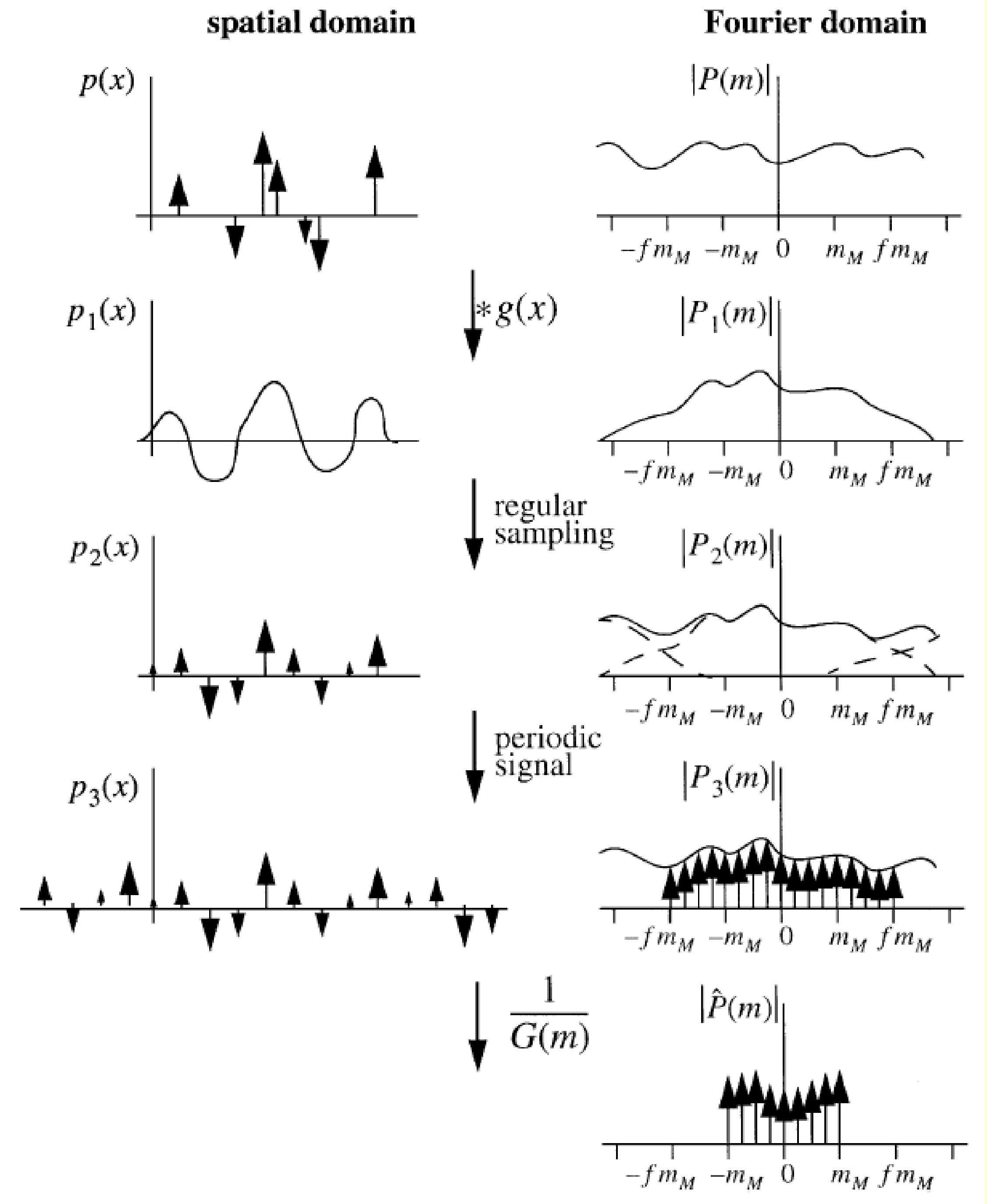
$$s = \sum_{j=1}^{N} x_j \sin(\nu_i t_j)$$

$$c = \sum_{j=1}^{N} x_j \cos(\nu_i t_j)$$

$$ss = \sum_{j=1}^{N} \sin^2(\nu_i t_j)$$

$$cc = \sum_{j=1}^{N} \cos^2(\nu_i t_j)$$

$$sc = \sum_{j=1}^{N} \sin(\nu_i t_j) \cdot \cos(\nu_i t_j)$$

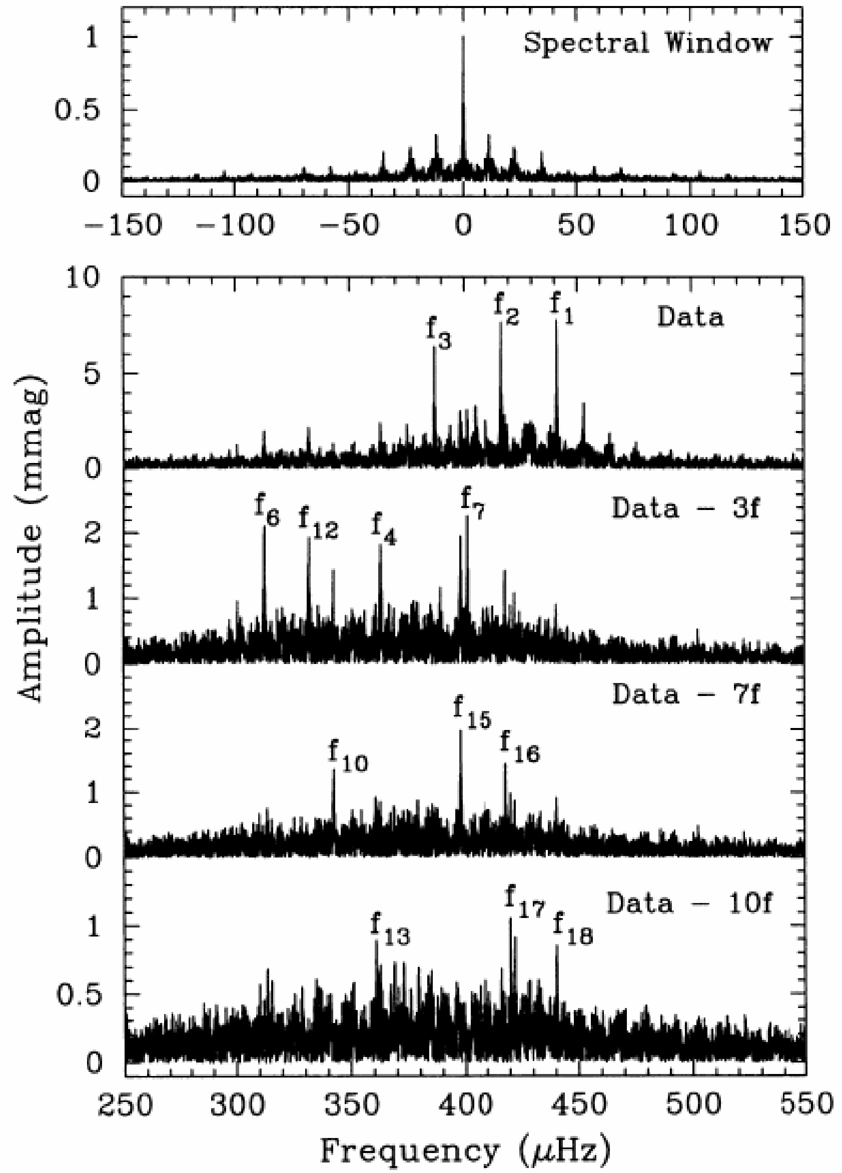


spatial domain
Fourier domain
p(x)
|P(m)|
−fm_M −m_M 0 m_M fm_M
*g(x)
p_1(x)
|P_1(m)|
regular sampling
p_2(x)
|P_2(m)|
periodic signal
p_3(x)
|P_3(m)|
1/G(m)
|P̂(m)|

Removing the oscillation signal

Prewhitening
CLEAN algorithm
Iterative deconvolution
Basis pursuit

Lot of names,
but can be still
misleading



More generally speaking

Fit general model for time series:

$$RSS = \sum_{i=1}^{N} \left[f(t_i) - M(t_i, c_1, c_2, \ldots, c_K) \right]^2$$

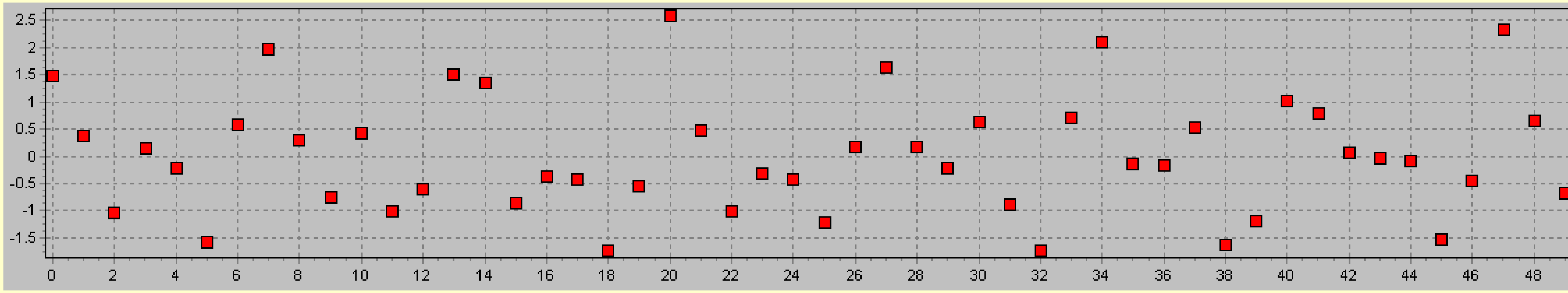
Some parameters are linear, some not:

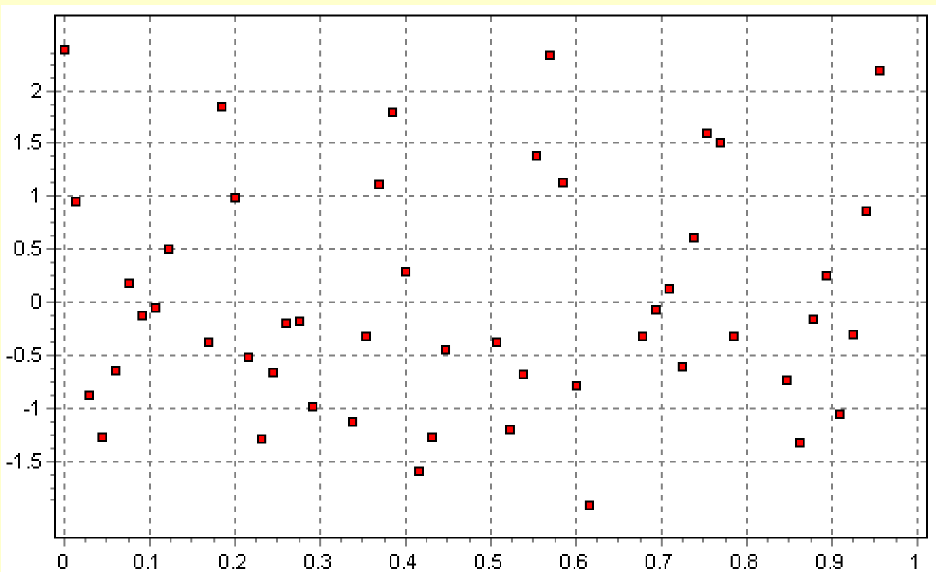
$$M(t, c_1, \ldots, c_K) = \sum_{r=1}^{R} c_r M_r(t, c_{R+1}, \ldots, c_K)$$

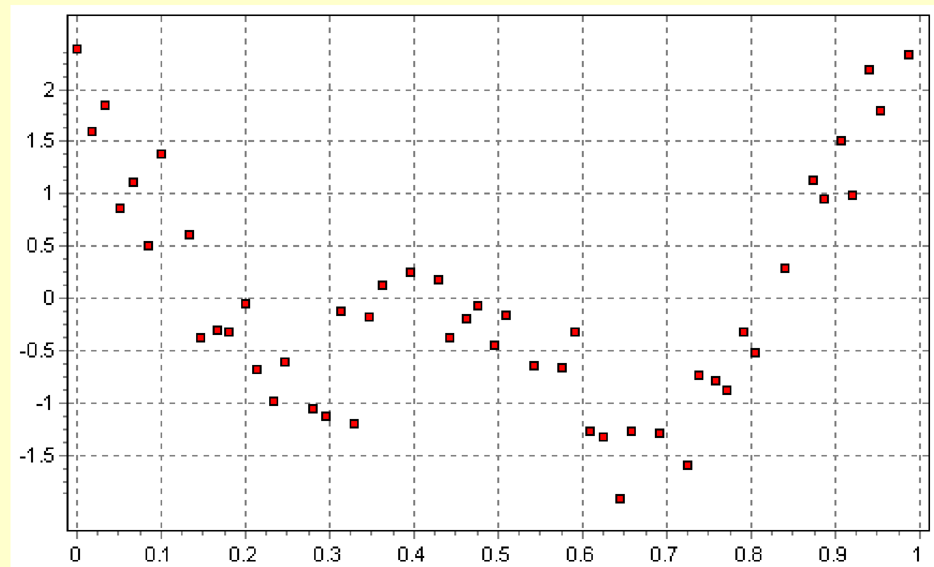
Using general linear least-squares fit package reduce task to the nonlinear optimization problem.

$$LSS(c_{R+1}, \ldots, c_K) = \sum_{i=1}^{N} \left(f(t_i) - M(t_i, \hat{c}_1, \hat{c}_2, \ldots, \hat{c}_R, c_{R+1}, \ldots, c_K) \right)^2$$

Phase-process diagram (folding)







$$t_i, f(t_i), i = 1, 2, \ldots, N$$

$$\varphi_P(t_i) = \mathrm{Frac}(t_i P^{-1})$$

Estimation of dispersion through subsampling

For a set of observations y_i, $i = 1, 2, \ldots, N$ we can observe that

$$\frac{1}{2N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}(y_i - y_j)^2 =$$

$$= \frac{1}{2N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left((y_i-\mu)-(y_j-\mu)\right)^2 =$$

$$= \frac{1}{N}\sum_{i=1}^{N}(y_i-\mu)^2 - \frac{1}{N^2}\Big(\sum_{i=1}^{N}(y_i-\mu)\Big)^2.$$

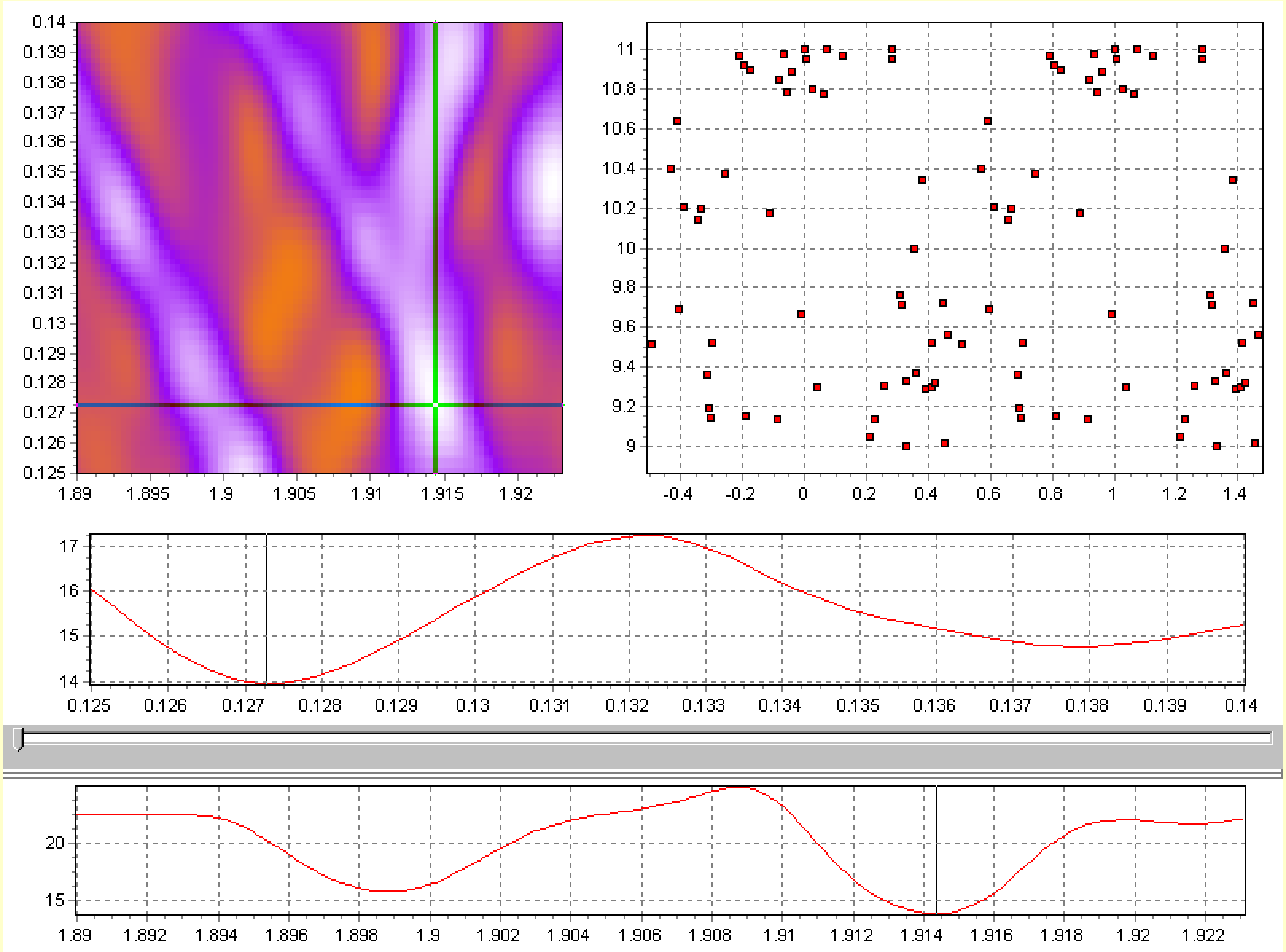
If we additionally set $\mu = \frac{1}{N}\sum_{i=1}^{N} y_i$

$$S^2 = \frac{1}{N(N-1)}\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}(y_i-y_j)^2 =$$

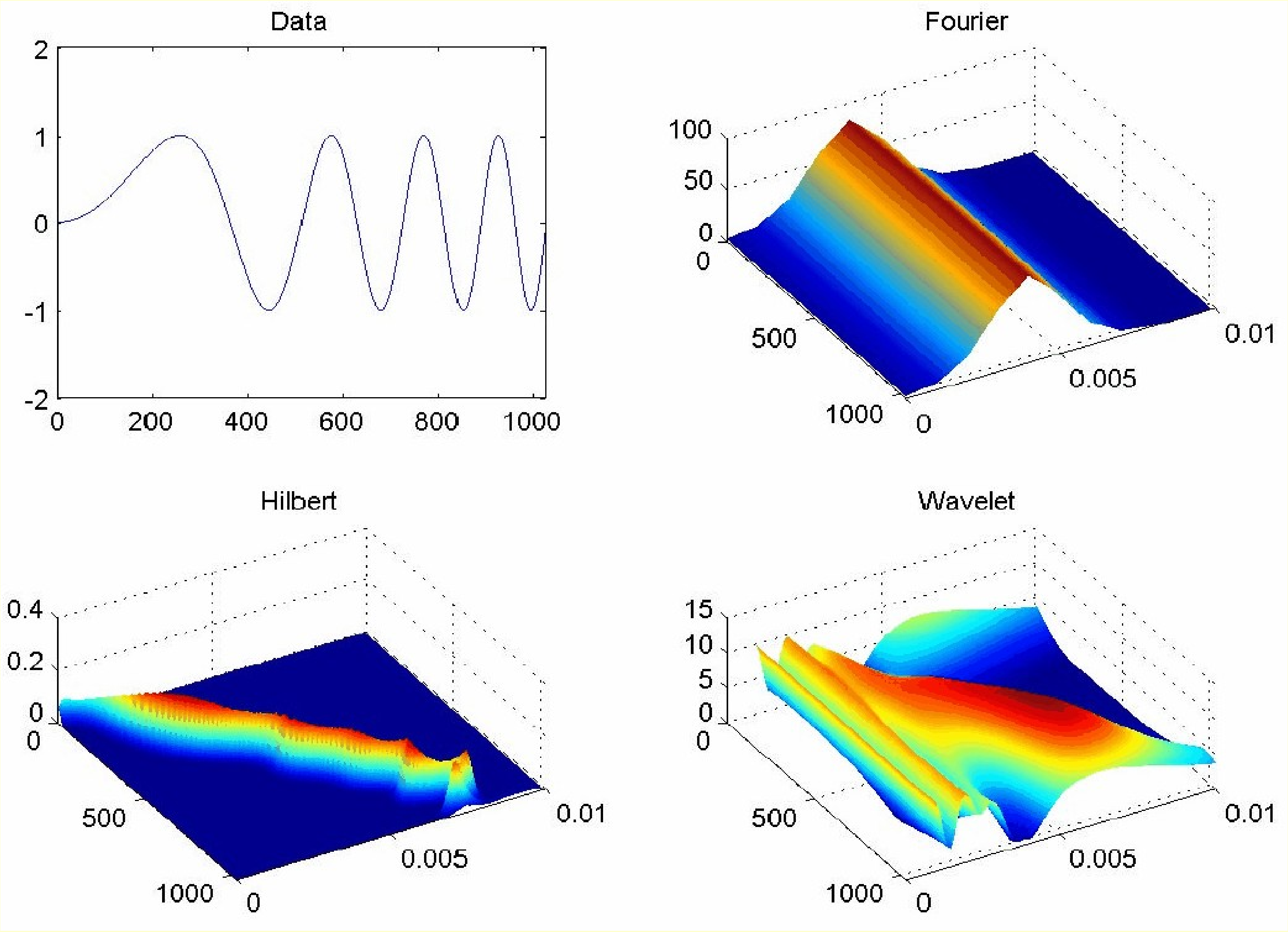
$$= \frac{1}{N-1}\sum_{i=1}^{N}(y_i-\mu)^2.$$

Observed and counted sample

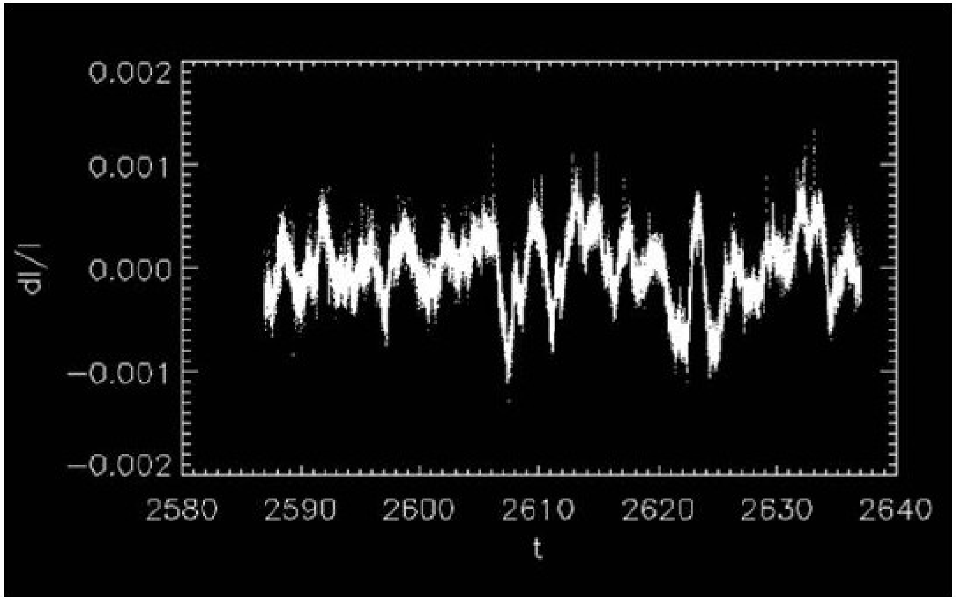
Two frequencies in one shot

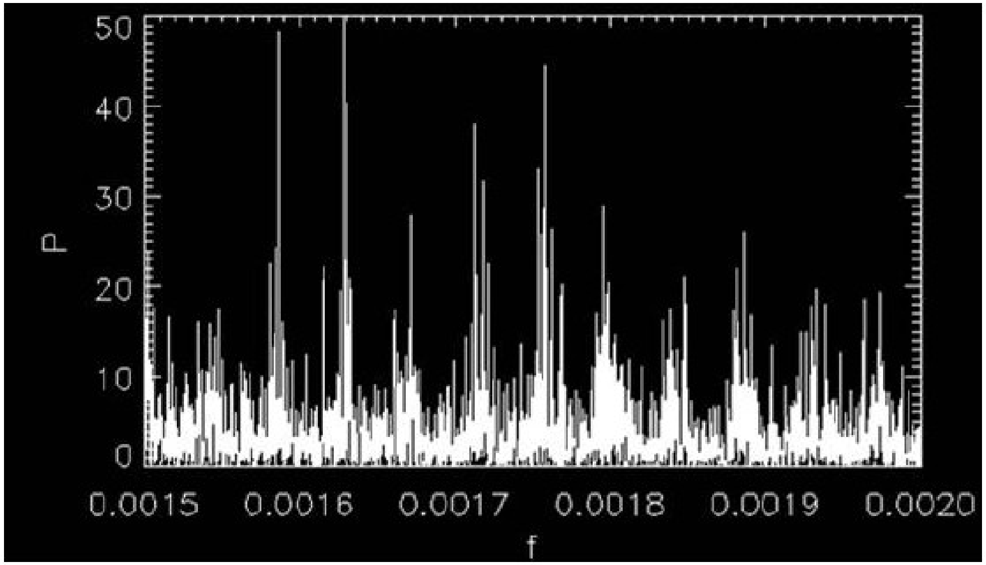


Comparison of the Hilbert-Huang Transform with others

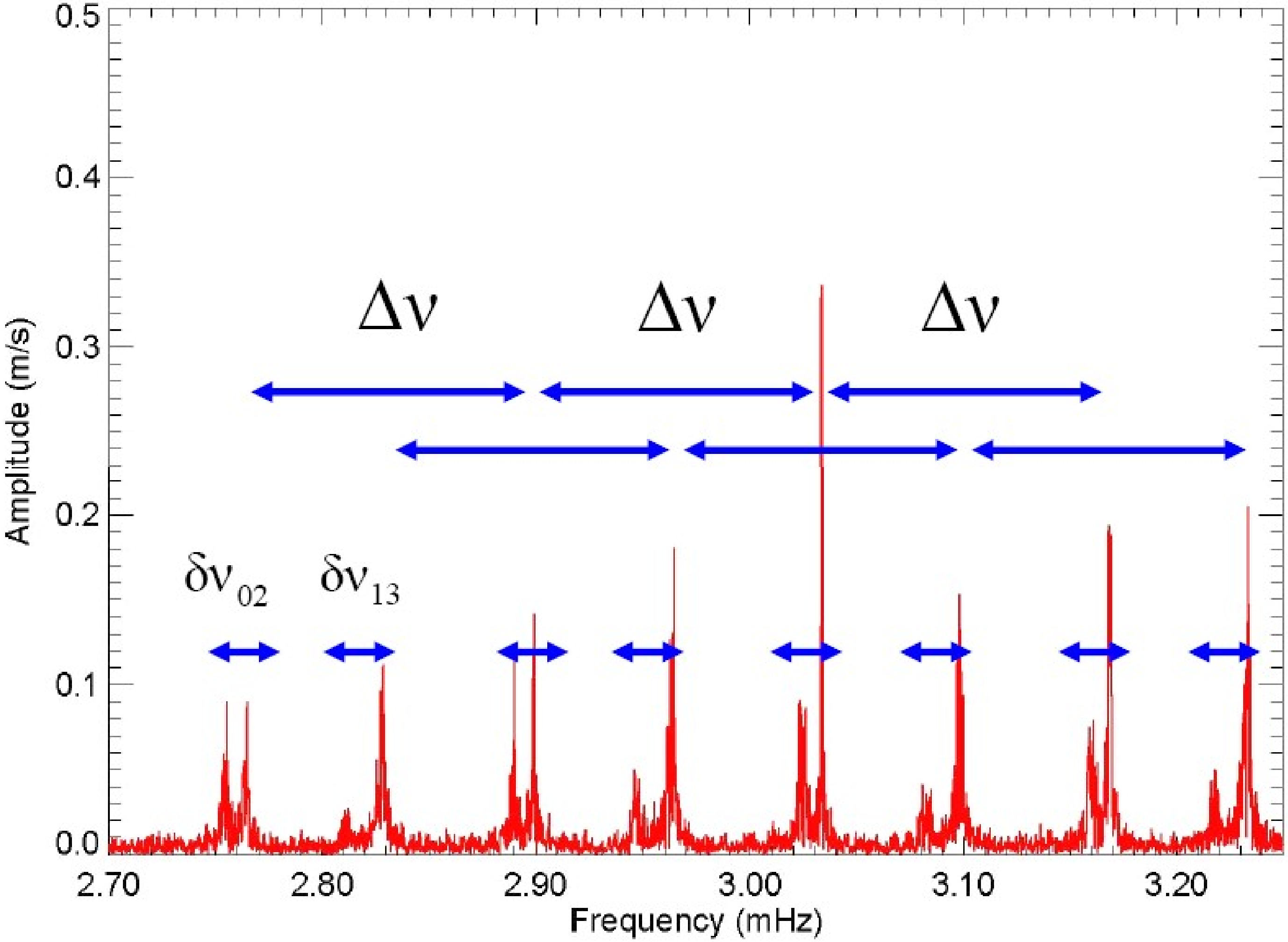


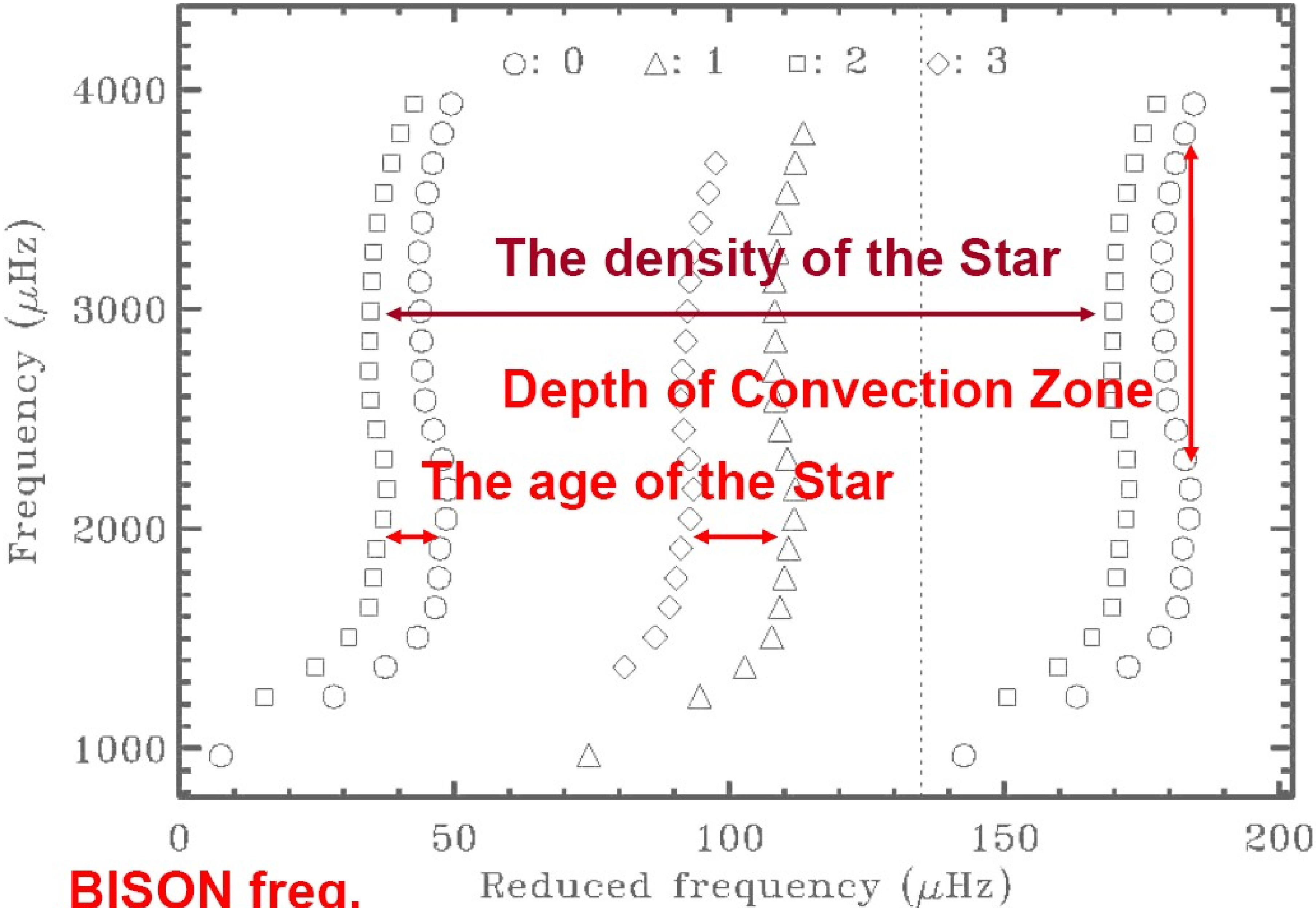
Activity and Oscillations in a Solar-like star



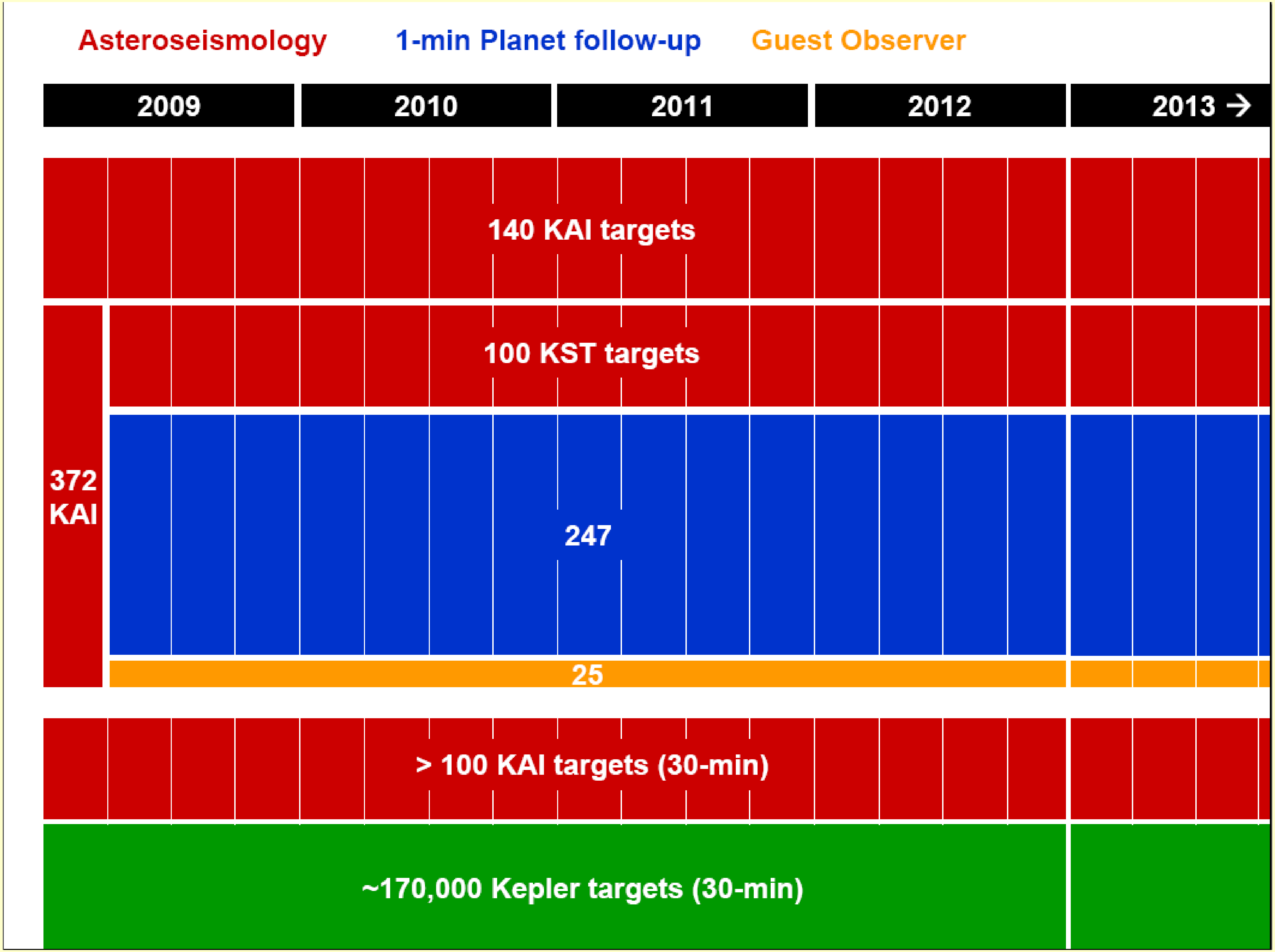


Determine the large Separation

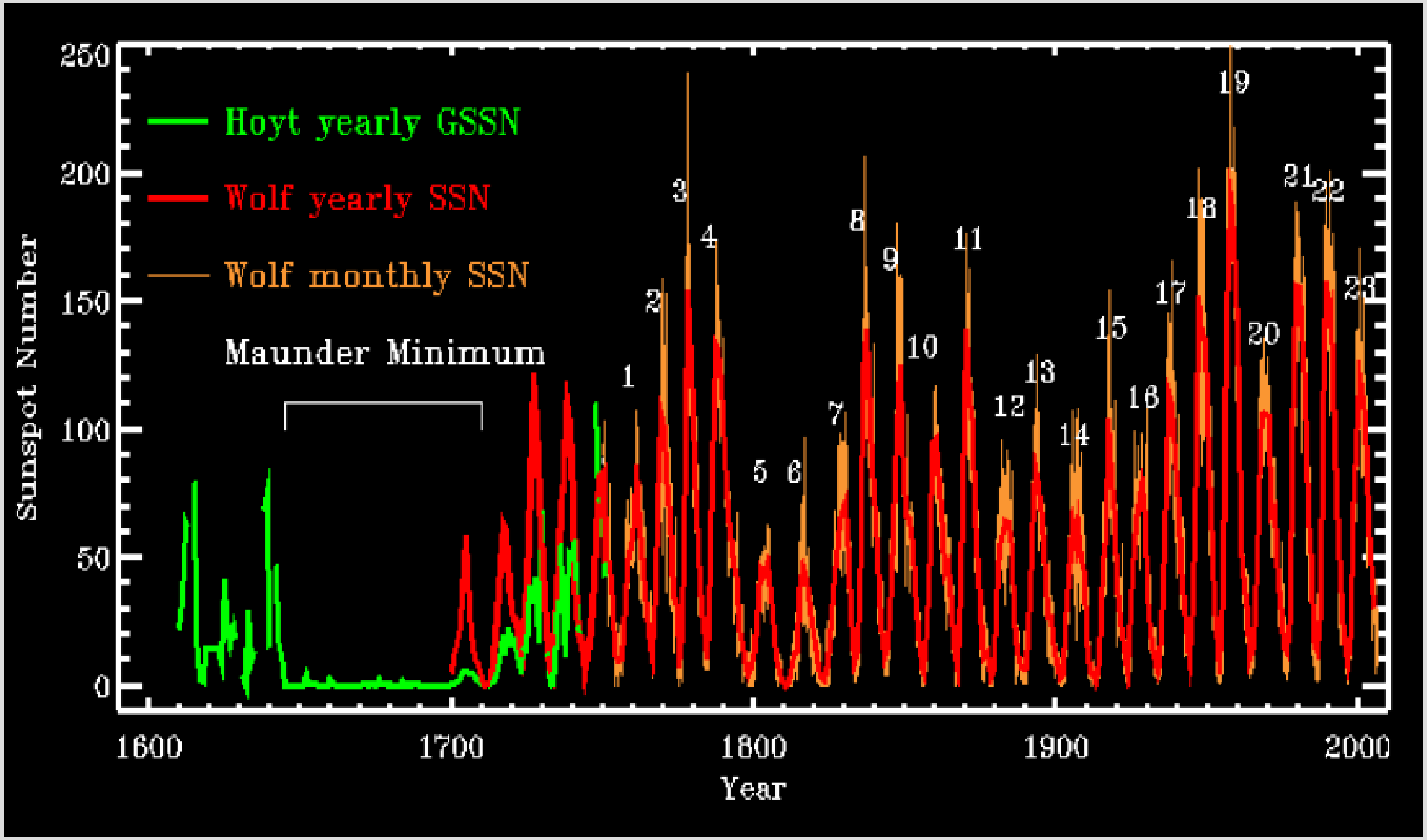



○: 0
△: 1
□: 2
◇: 3
4000
3000
2000
1000
Frequency (μHz)
0
50
100
150
200
Reduced frequency (μHz)
The density of the Star
Depth of Convection Zone
The age of the Star

BISON freq.

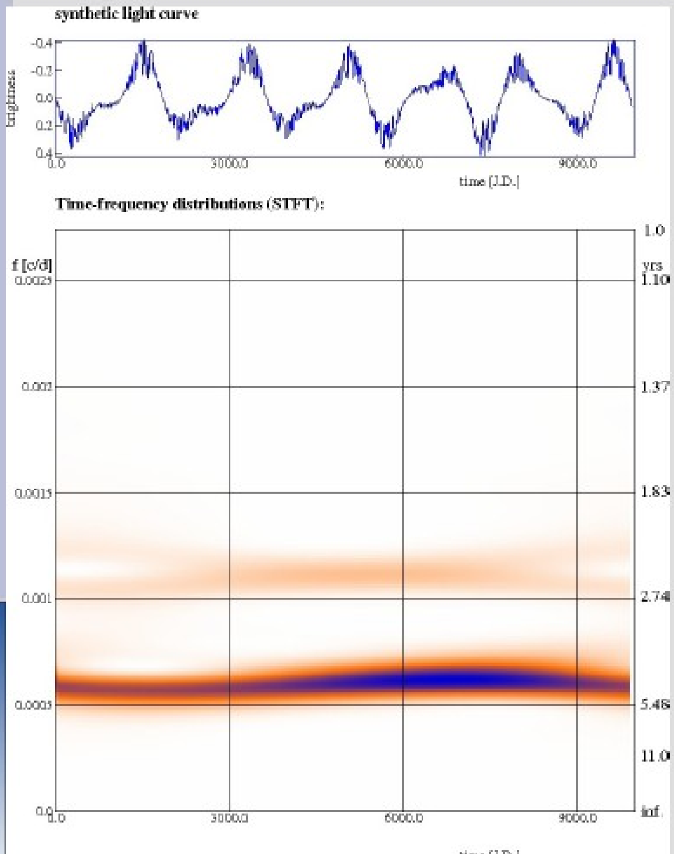

Asteroseismology
1-min Planet follow-up
Guest Observer
2009
2010
2011
2012
2013 →
140 KAI targets
372 KAI
100 KST targets
247
25
> 100 KAI targets (30-min)
~170,000 Kepler targets (30-min)

Long-term observations of the solar cycle



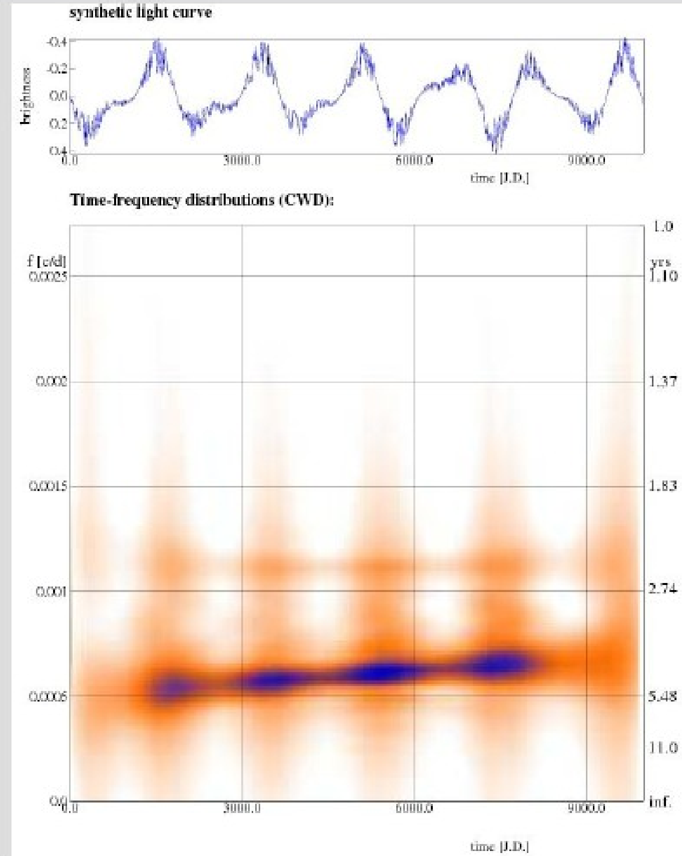
Test cycle lengths: 5.48 yrs (changing)
2.47 yrs (constant)

Short-term Fourier Transformation



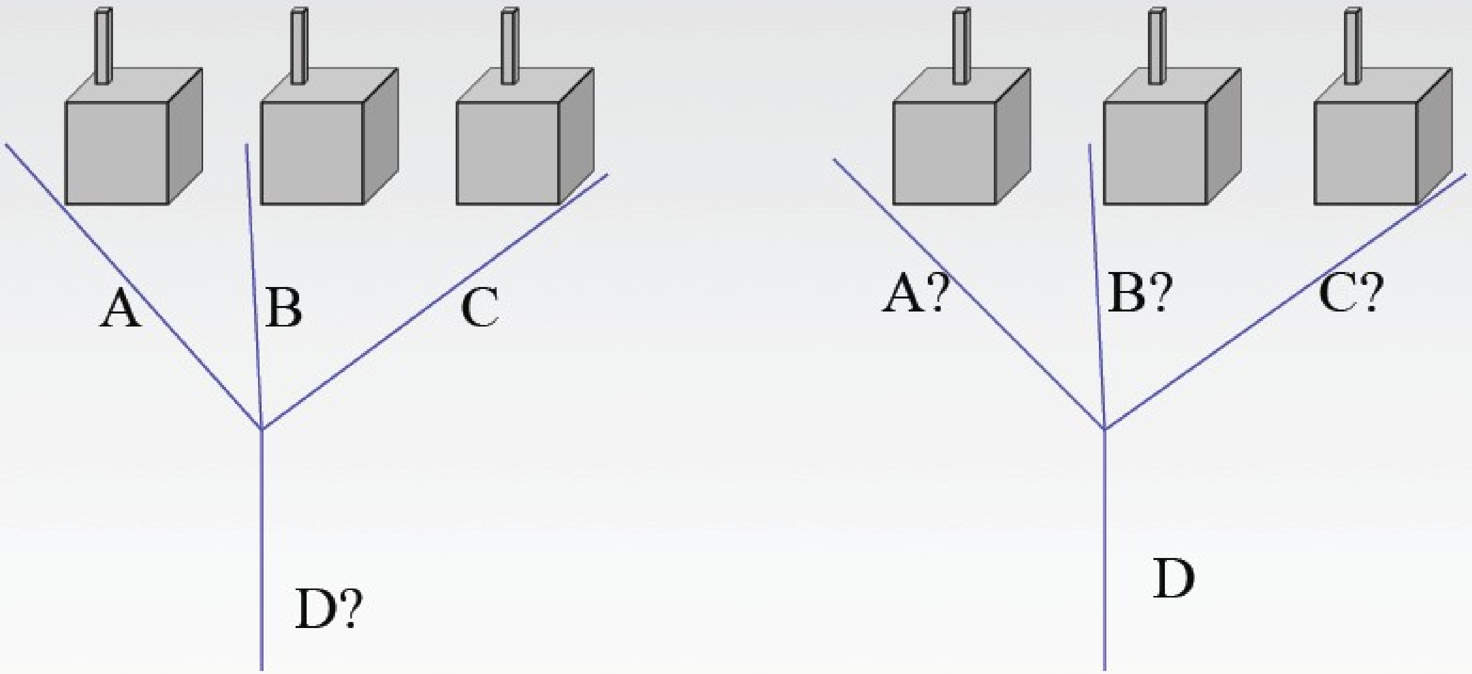
lower resolution,
artificial bend

Choi-Williams distribution



higher resolution

Forward vs. Inverse problem

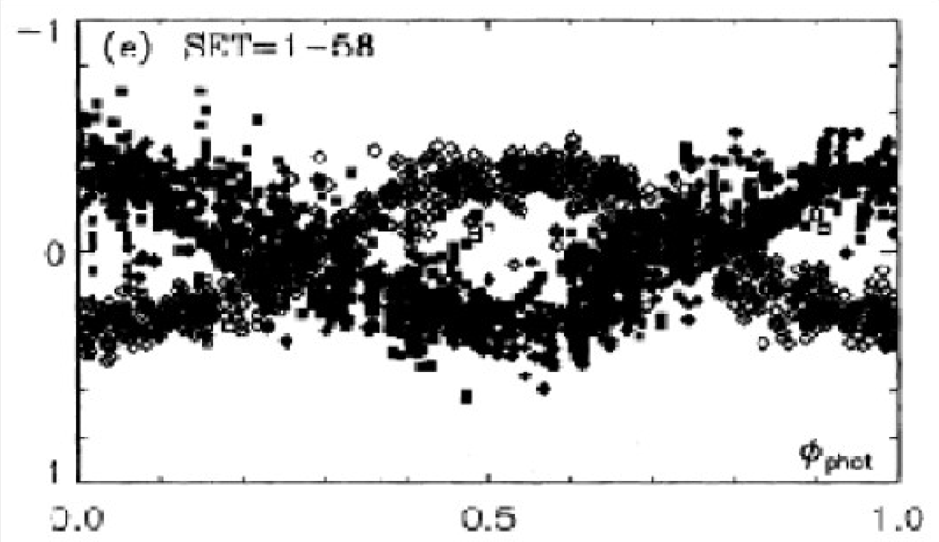


Forward problem

Inverse Problem

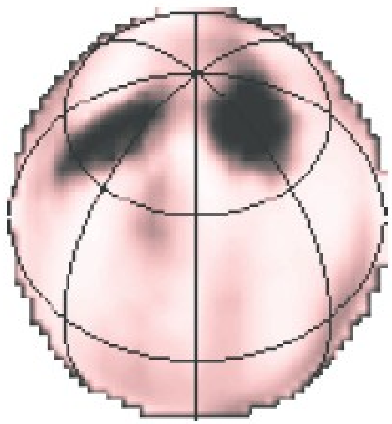
Flip-flop phenomenon

- Discovered on the single giant FK Com in the early 1990's (Jetsu et al. 1993)
- Activity concentrates on two permanent active longitudes and flips between them every few years

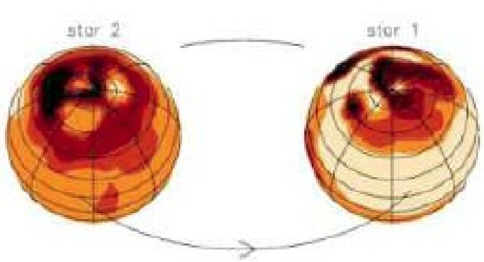


Jetsu et al. 1993

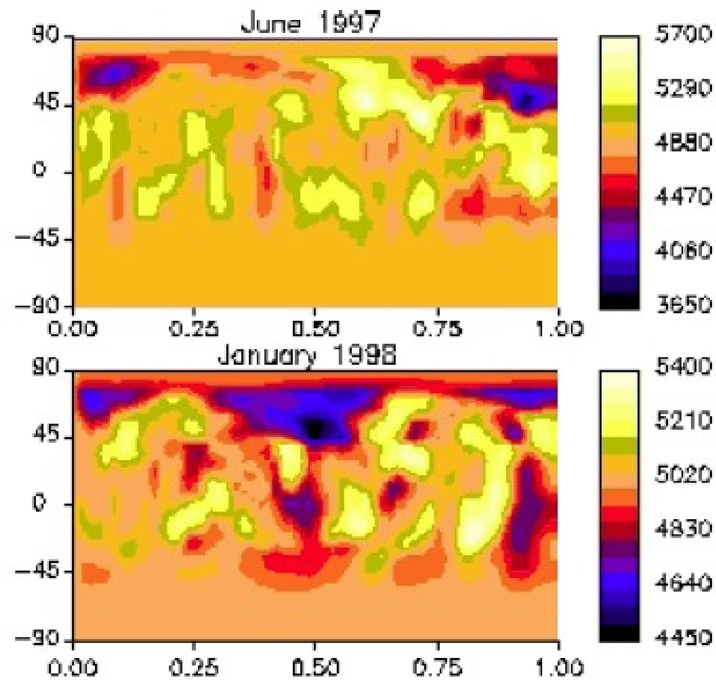
Doppler imaging results



II Peg: Berdyugina et al 1998

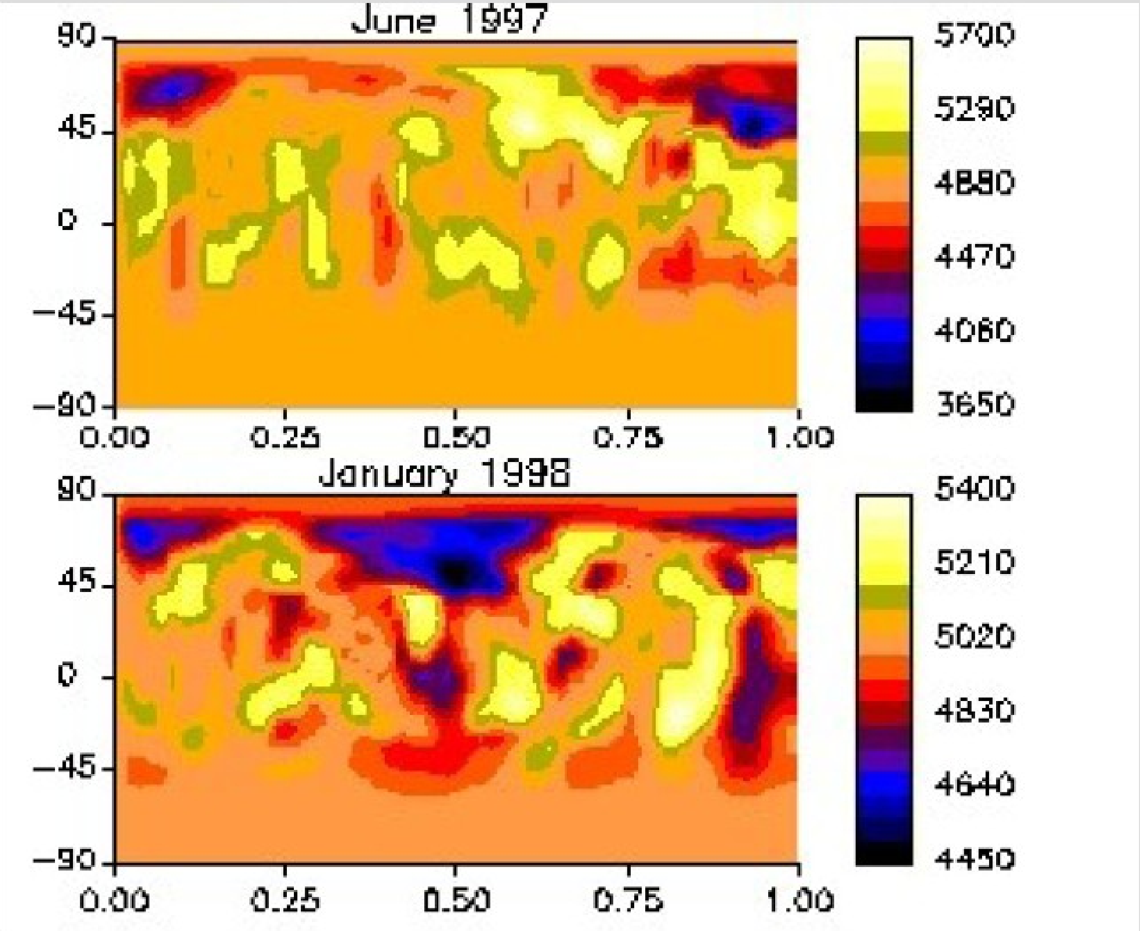


σ² CrB: Strassmeier & Rice 2003

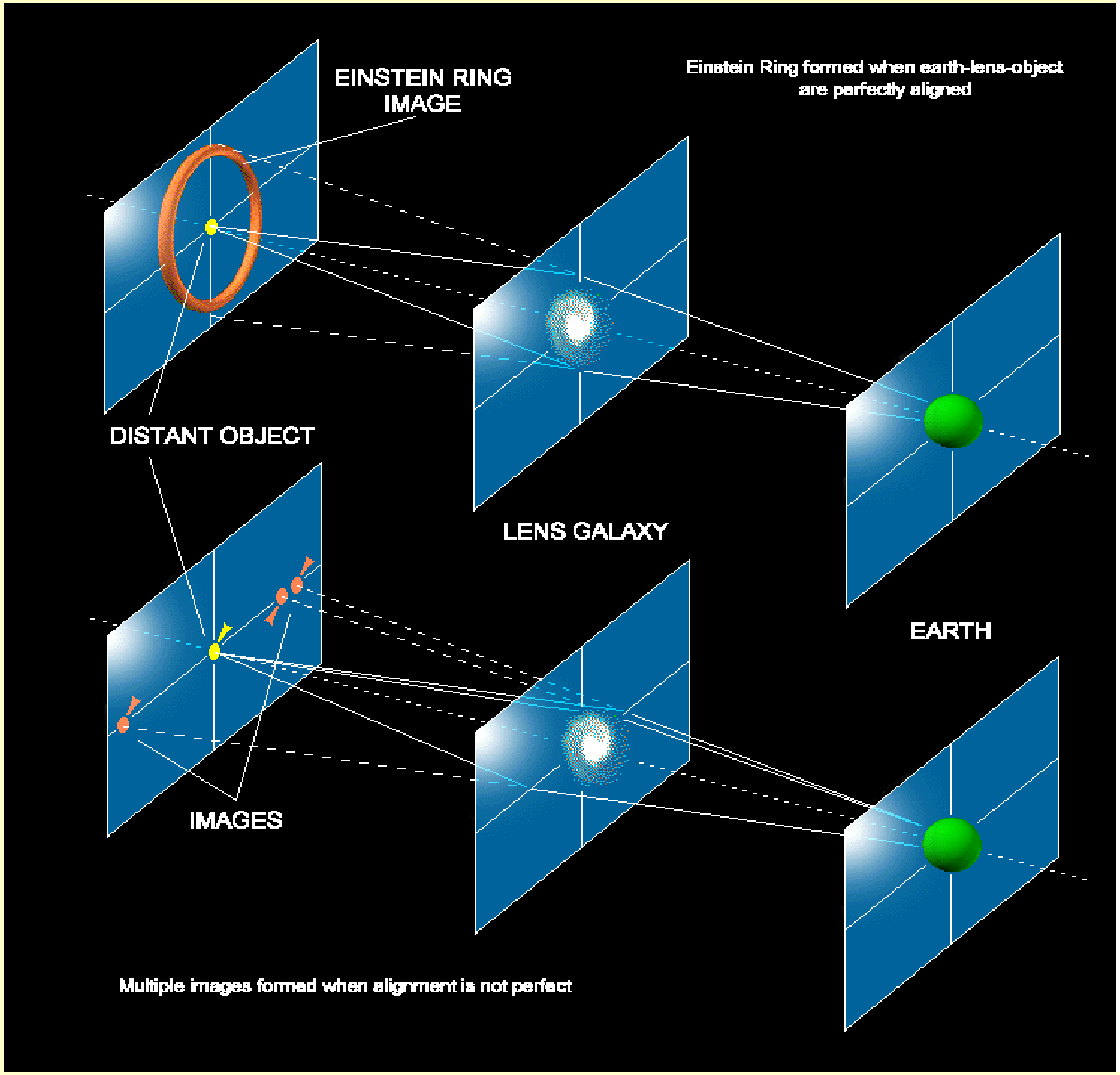


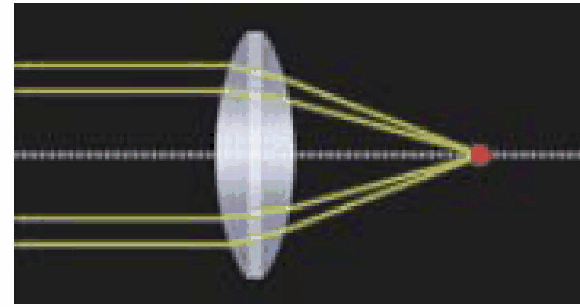
FK Com: Korhonen et al 2001

Flip-flops in detail



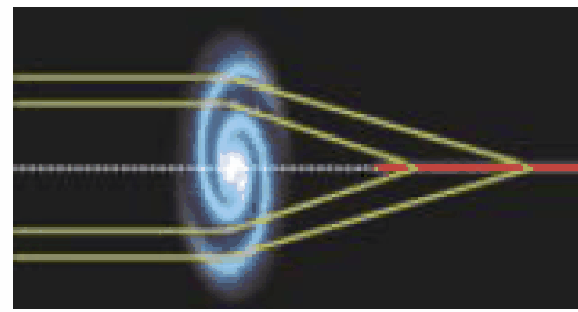
Korhonen et al. 2001





CONVEX GLASS LENS

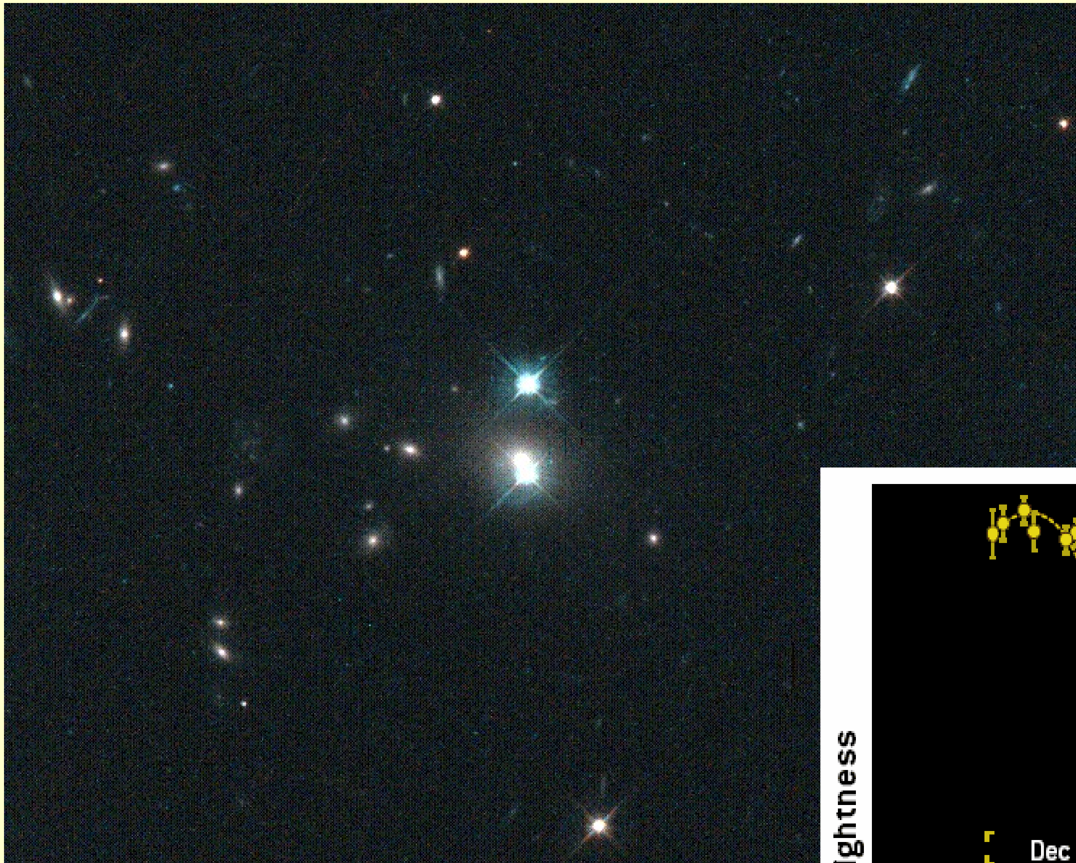
Light near the edge of a glass lens is deflected more than light near the optical axis. Thus, the lens focuses parallel light rays onto a point.

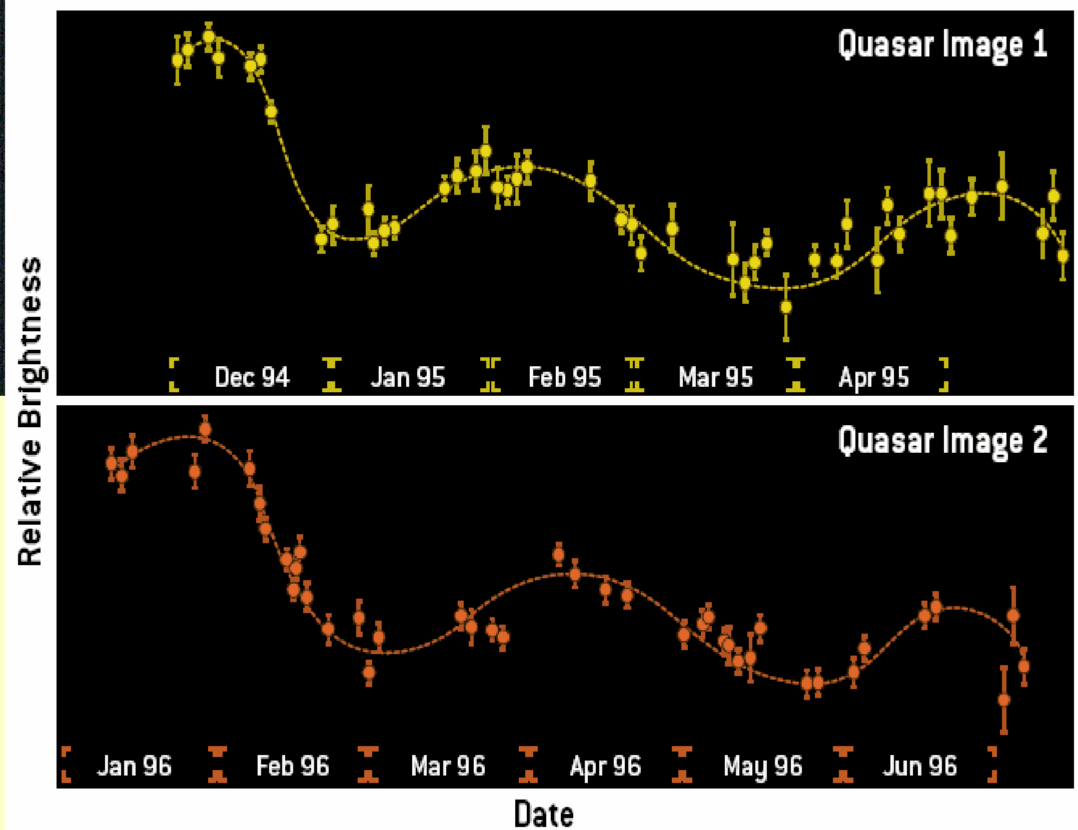


GRAVITATIONAL LENS

Light near the edge of a gravitational lens is deflected less than light near the center. Thus, the lens focuses light onto a line rather than a point.

Glass and gravitational lenses





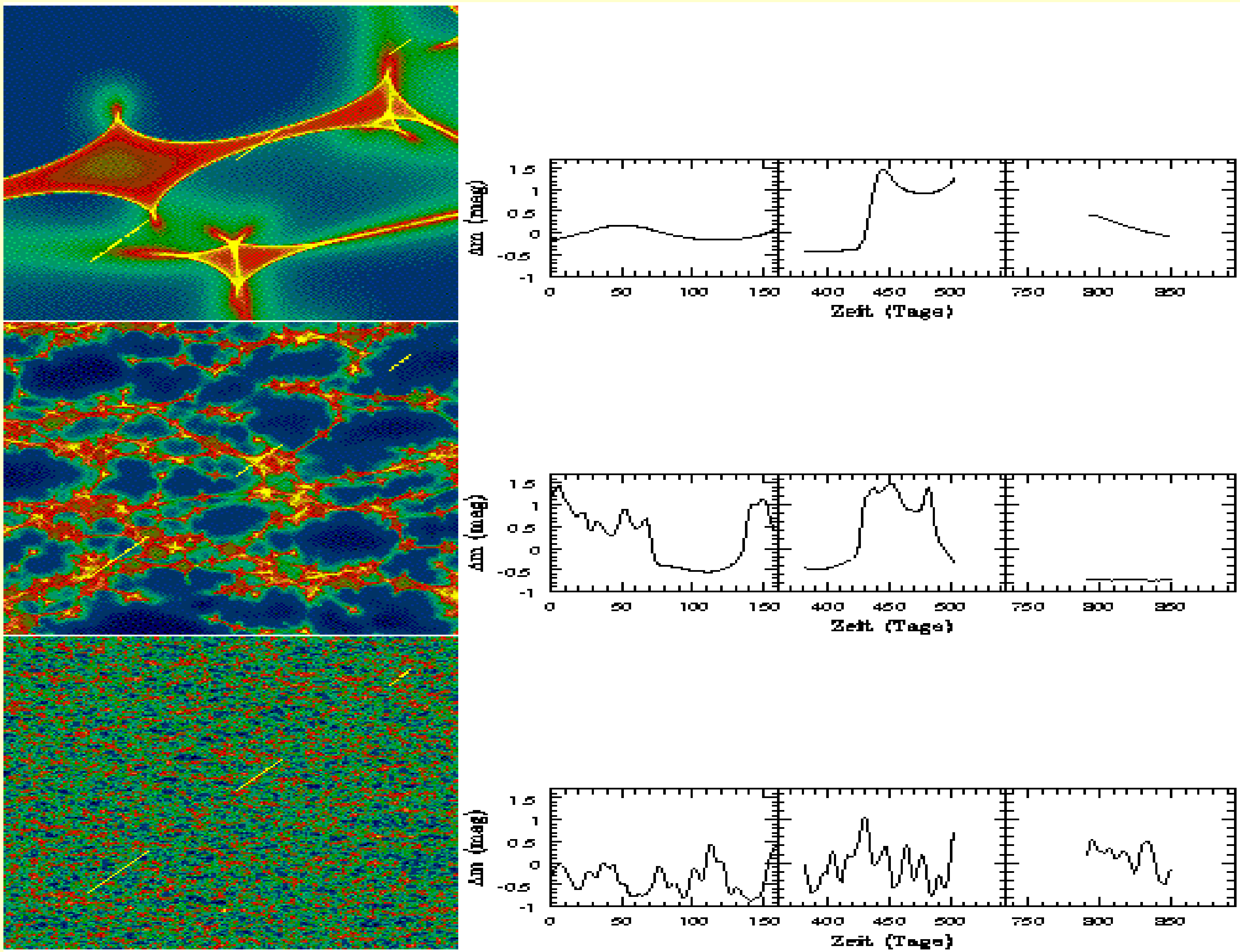
MATCHING BRIGHTNESS VARIATIONS of the double quasar Q0957+561

Time series frequency content depends on masses of halo objects

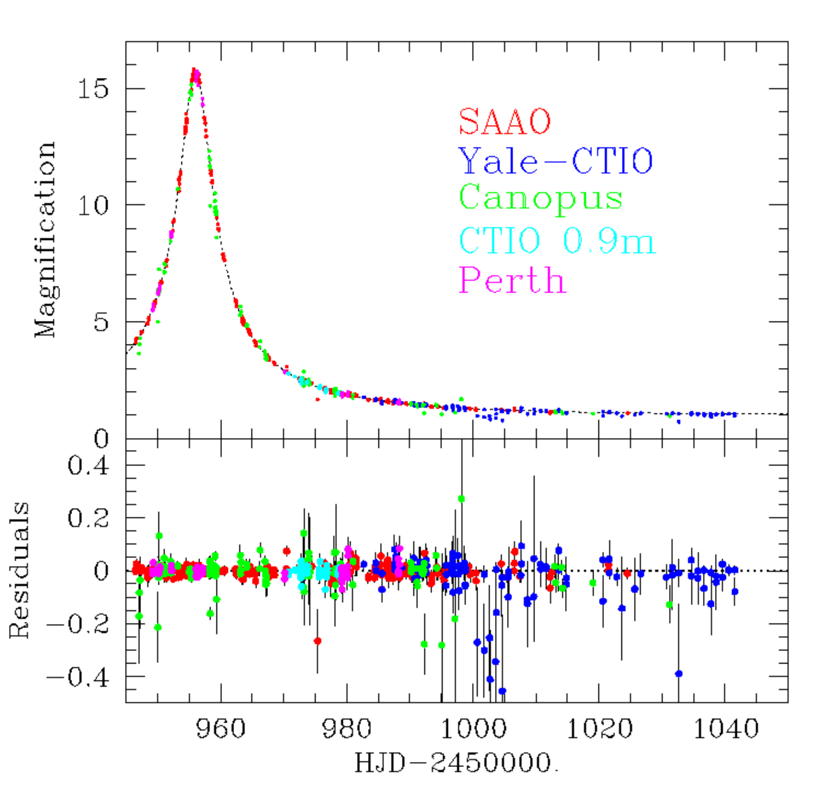
$10^{-1}\ M_\odot$

$10^{-3}\ M_\odot$

$10^{-5}\ M_\odot$



Microlensing in our Galaxy



Galactic structure (OGLE, MACHO, EROS)

Galactic Halo (MACHO, EROS)

Exoplanets (MPS, MOA, MicroFun)

MACHO - MAssive Compact Halo Objects

EROS - Expérience pour la Recherche d'Objets Sombres

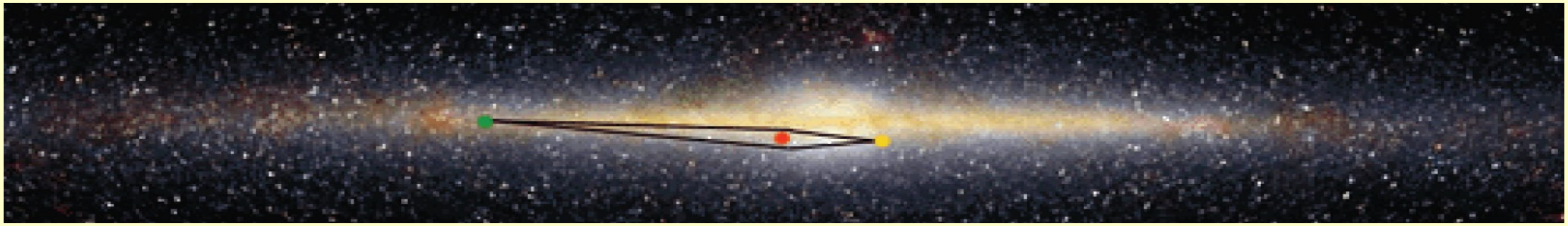
OGLE - the Optical Gravitational Lens Experiment

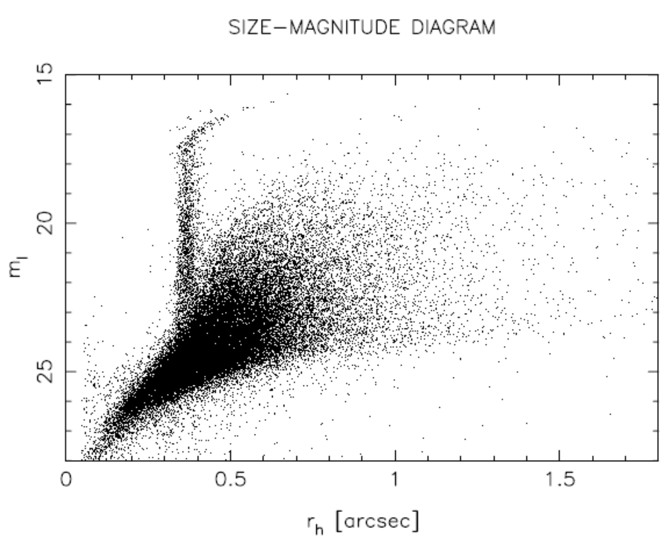
MOA - Microlensing Observations in Astrophysics

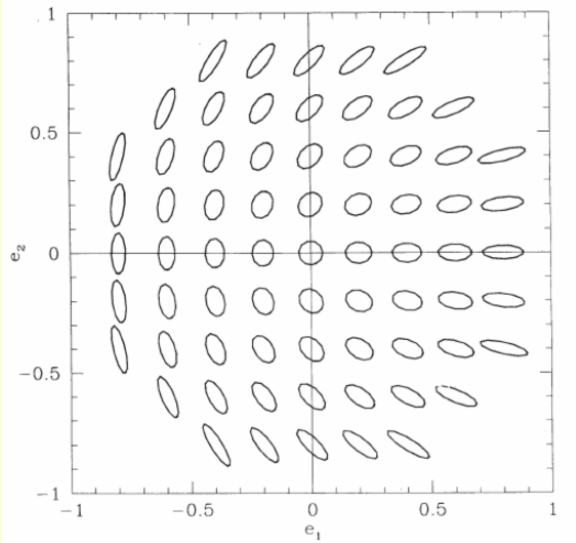
MPS - Microlensing Planet Search Project

MicroFun - Microlensing Follow Up Network

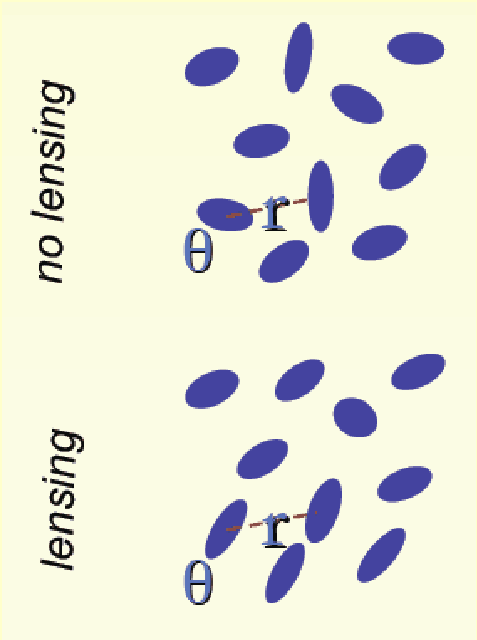
▶ Magnification of the source







Heidi knows how to do that!



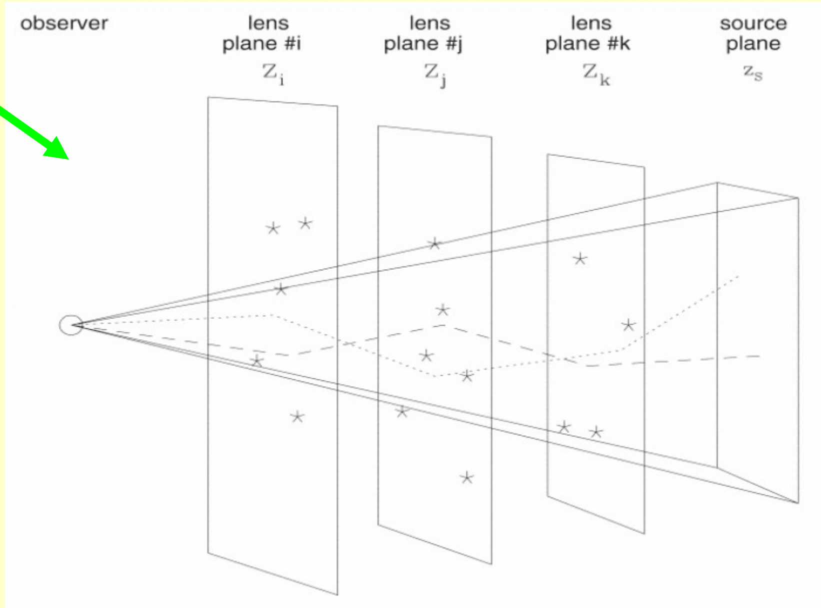
•Stars out

•Search of images

•Averaging of ellipticities

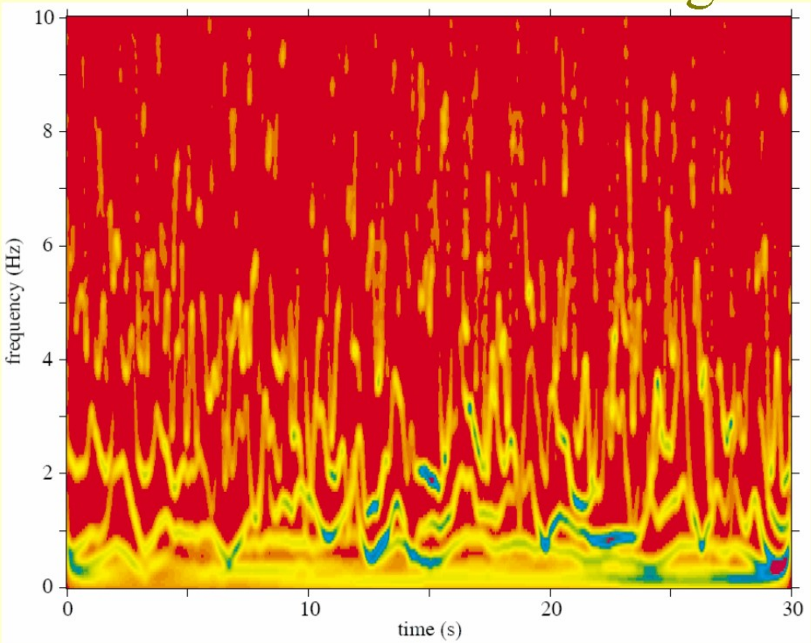
•Sorting by distance

•Inversion



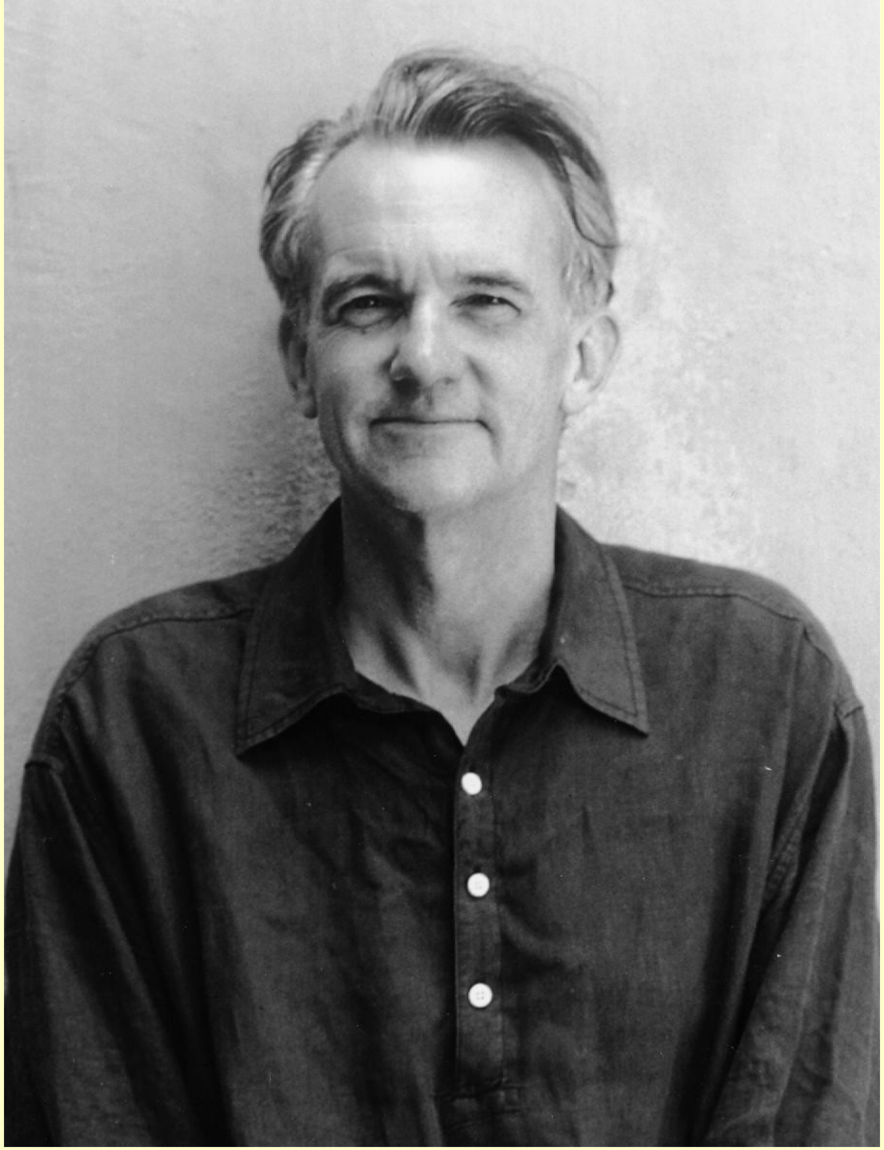
The Finnish Graduate School in Astronomy and Space Physics Summer School 2007

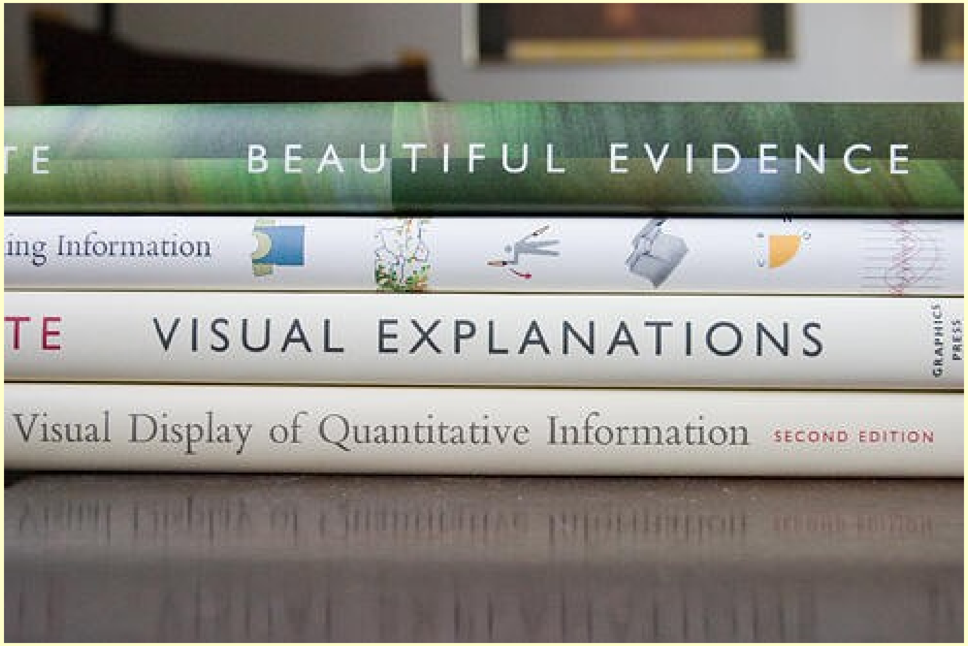
Smoothed Hilbert-Huang



Practical approach to weak wave patterns

Sources





Edward Tufte

http://www.edwardtufte.com/tufte/

Student exercise

I_1	I_4
I_2	I_3

Observations:

$I_1+I_2+I_3+I_4=36$

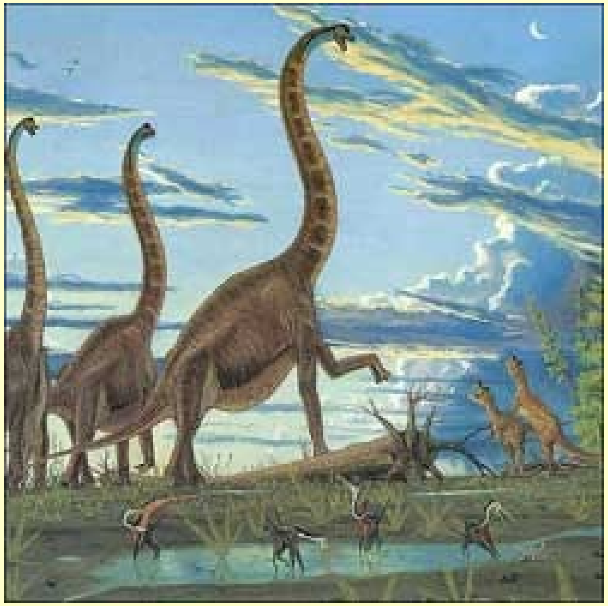
$I_1+I_2=12$

$I_1+I_4=12$

Calculate I_1, I_2, I_3 and I_4 using both Tikhonov regularisation and Maximum Entropy Principle

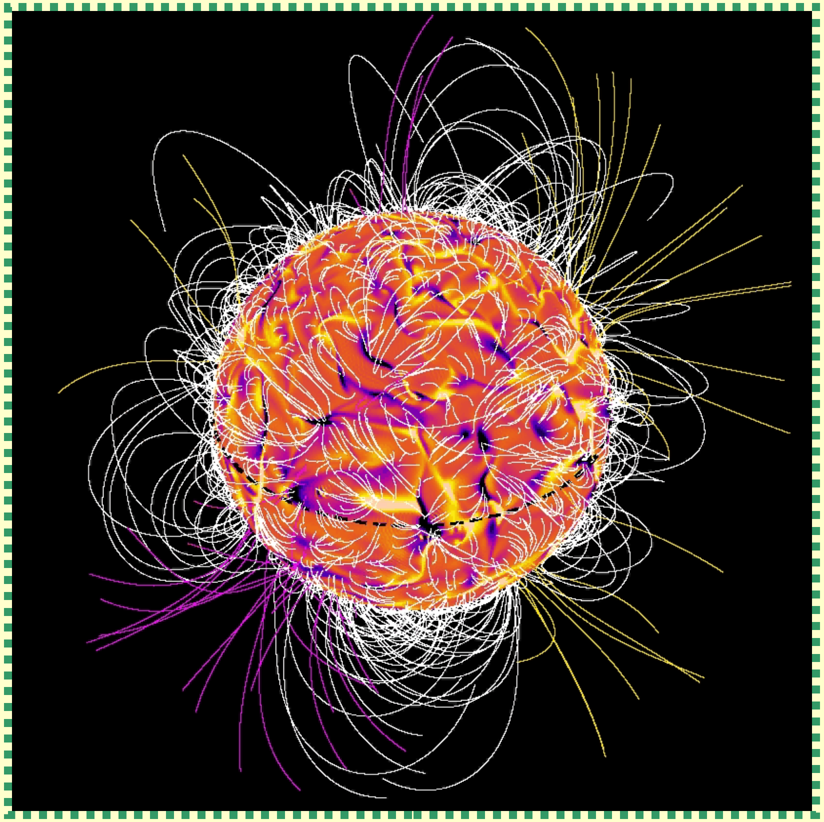
The Teaching Company

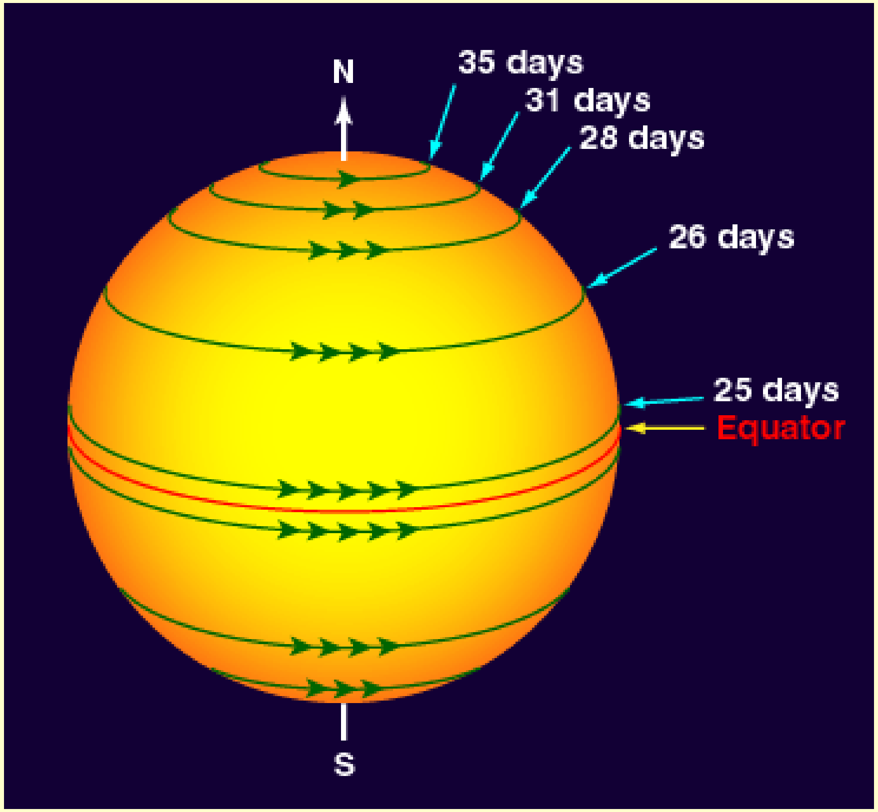
Group A doing field-work



- Time point series with weights and errors in timing.
- Who killed disnosaurs, if not lensed light?

Group B





- Flip-flop or differential rotation?
- Can we find out from photometry?

$$
\begin{aligned}
S^2 &= \frac{1}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (y_i - y_j)^2 = \\
&= \frac{1}{N-1} \sum_{i=1}^{N} (y_i - \mu)^2.
\end{aligned}
$$

Nonparametric hunt

for changing frequencies.

Can we beat

the Doppler effect?

Group C severely punished!

Why?

Solution is simple, just use Period*n instead of t_{max}, where n is required number of coherent cycles

Smoother periodograms

$$D^2(\nu) = \frac{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} G^*(t_i, t_j, \nu)\left[f(t_i) - f(t_j)\right]^2}{2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} G^*(t_i, t_j, \nu)}$$

$$G^*(t_i, t_j, \nu) = G(t_i, t_j, \nu) W(t_i, t_j)$$

$$W(t_i, t_j) = \begin{cases} 1, & |t_i - t_j| \leq t_{\max} \\ 0, & \text{otherwise}, \end{cases}$$

2007

See you soon!