- LSQ calculation of power spectra
- Noise in the Power Spectrum
- The Spectral window function
- Multi-site campaigns and observations from space
- Merging data from different telescopes
- Statistical weights

# Calculating the Power Spectrum

- The Power Spectrum is calculated as

$$P(\nu) = A_i^2 = \alpha(\nu)^2 + \beta(\nu)^2$$

$$\alpha(\nu) = \frac{s \cdot cc - c \cdot sc}{ss \cdot cc - sc^2}$$

$$\beta(\nu) = \frac{c \cdot ss - s \cdot sc}{ss \cdot cc - sc^2}$$

$$s = \sum_{j=1}^{N} x_j \sin(\nu_i t_j)$$

$$c = \sum_{j=1}^{N} x_j \cos(\nu_i t_j)$$

$$ss = \sum_{j=1}^{N} \sin^2(\nu_i t_j)$$

$$cc = \sum_{j=1}^{N} \cos^2(\nu_i t_j)$$

$$sc = \sum_{j=1}^{N} \sin(\nu_i t_j) \cdot \cos(\nu_i t_j)$$

# Calculating the Power Spectrum

- The *weighted* Power Spectrum is calculated as

$$P(\nu) = A_i^2 = \alpha(\nu)^2 + \beta(\nu)^2$$

$$\alpha(\nu) = \frac{s \cdot cc - c \cdot sc}{ss \cdot cc - sc^2}$$

$$\beta(\nu) = \frac{c \cdot ss - s \cdot sc}{ss \cdot cc - sc^2}$$

$$s = \sum_{j=1}^{N} w_j \cdot x_j \sin(\nu_i t_j)$$

$$c = \sum_{j=1}^{N} w_j \cdot x_j \cos(\nu_i t_j)$$

$$ss = \sum_{j=1}^{N} w_j \cdot \sin^2(\nu_i t_j)$$

$$cc = \sum_{j=1}^{N} w_j \cdot \cos^2(\nu_i t_j)$$

$$sc = \sum_{j=1}^{N} w_j \cdot \sin(\nu_i t_j) \cdot \cos(\nu_i t_j)$$

# Calculating the Power Spectrum

- Then the Power Spectrum is calculated as

$$P(\nu) = A_i^2 = \alpha(\nu)^2 + \beta(\nu)^2$$

$$\alpha(\nu) = \frac{s \cdot cc - c \cdot sc}{ss \cdot cc - sc^2}$$

$$\beta(\nu) = \frac{c \cdot ss - s \cdot sc}{ss \cdot cc - sc^2}$$

$$(x_1, t_1, \sigma_1), \ldots, (x_N, t_N, \sigma_N)$$

$$s = \sum_{j=1}^{N} w_j \cdot x_j \sin(\nu_i t_j)$$

$$c = \sum_{j=1}^{N} w_j \cdot x_j \cos(\nu_i t_j)$$

$$ss = \sum_{j=1}^{N} w_j \cdot \sin^2(\nu_i t_j)$$

$$cc = \sum_{j=1}^{N} w_j \cdot \cos^2(\nu_i t_j)$$

$$sc = \sum_{j=1}^{N} w_j \cdot \sin(\nu_i t_j) \cdot \cos(\nu_i t_j)$$

# The Statisical Weights

- Should reflect the actual noise properties of the data
- Must be completely independent of the signal
- Can be obtained in various ways
- No two data sets are identical – choose the best method for your data

→ Try out different methods and choose the one that one that optimizes the science return

# The Statisical Weights

- The weight of a data point should reflect its quality;

$$w_i = \frac{1}{\sigma_i^x}$$

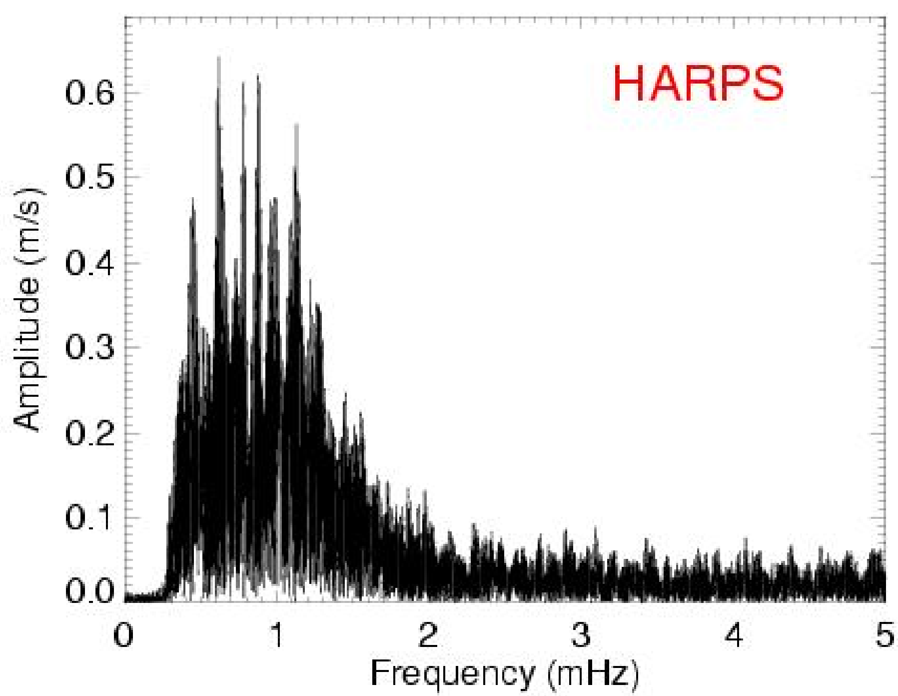
- The inverse variance would be expected statistically;

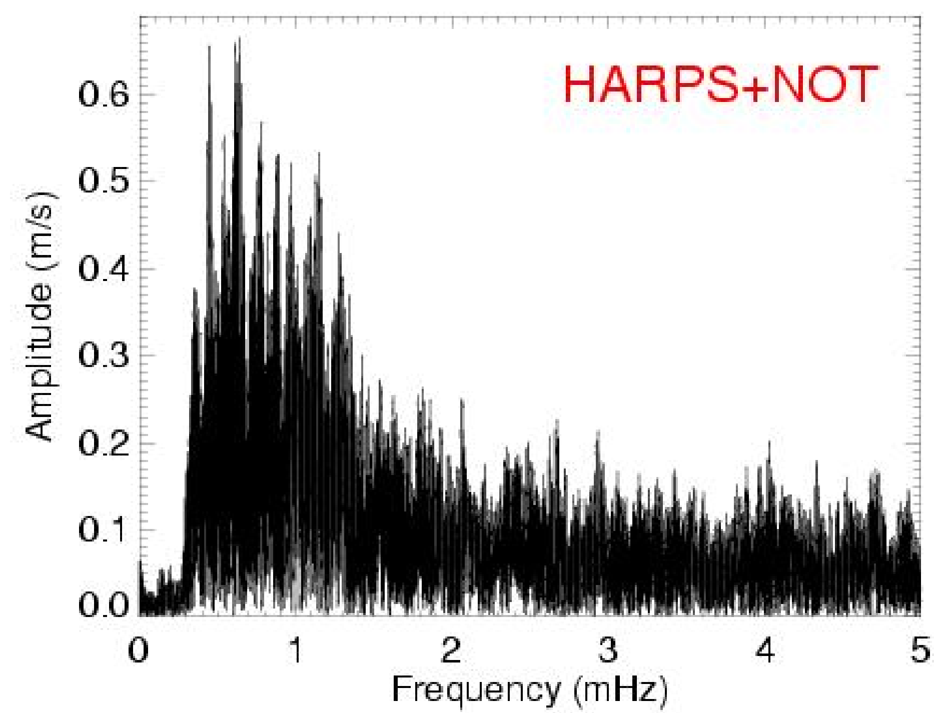
$$w_i = \frac{1}{\sigma_i^2}$$

- This is, however, not always the case – test it!

# Why do we need Statisical Weights

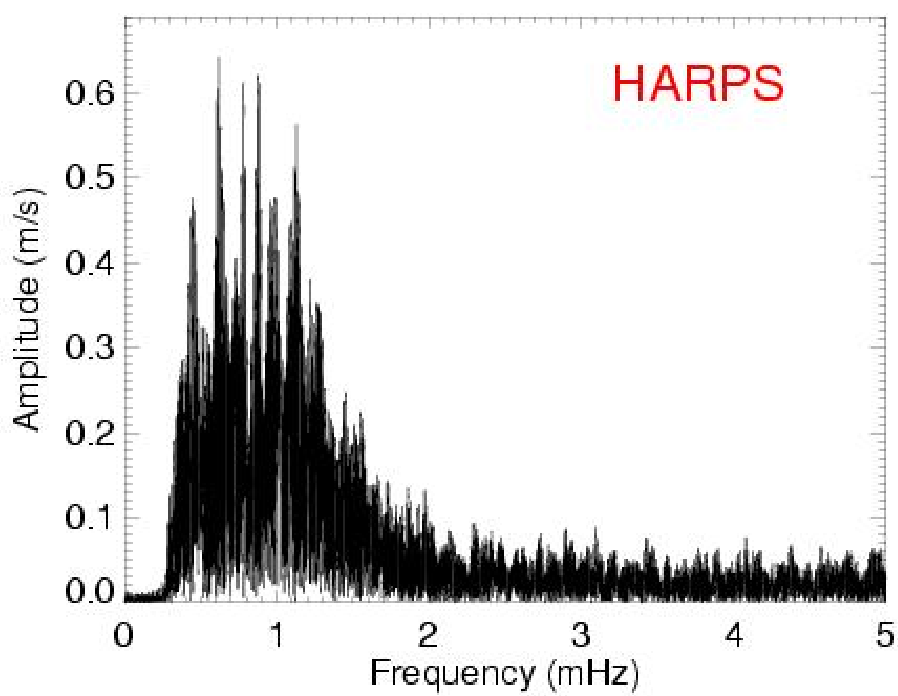
- Good data may be degraded when combined with bad data!

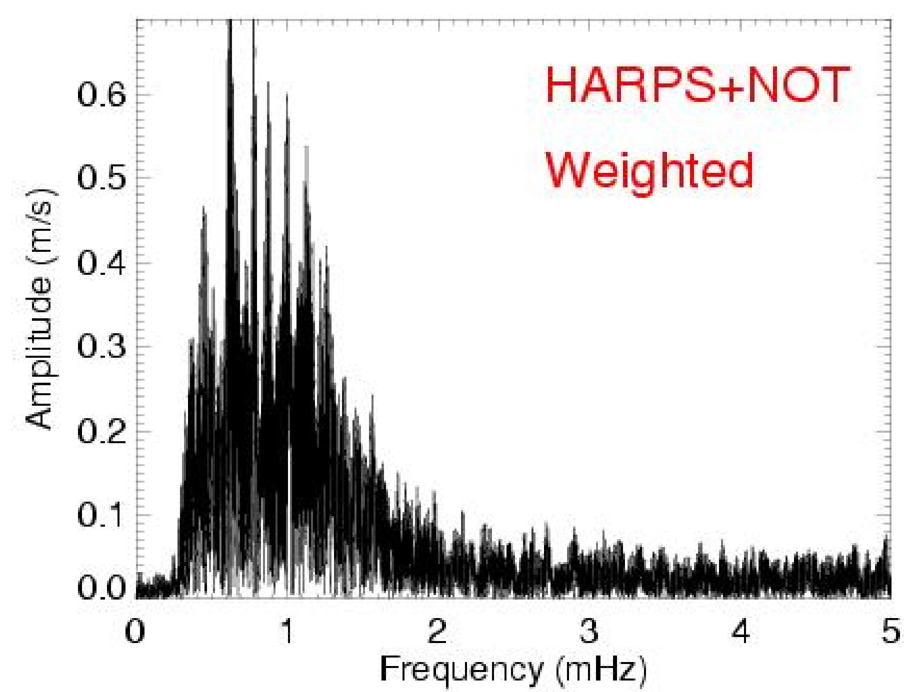


# Why do we need Statisical Weights

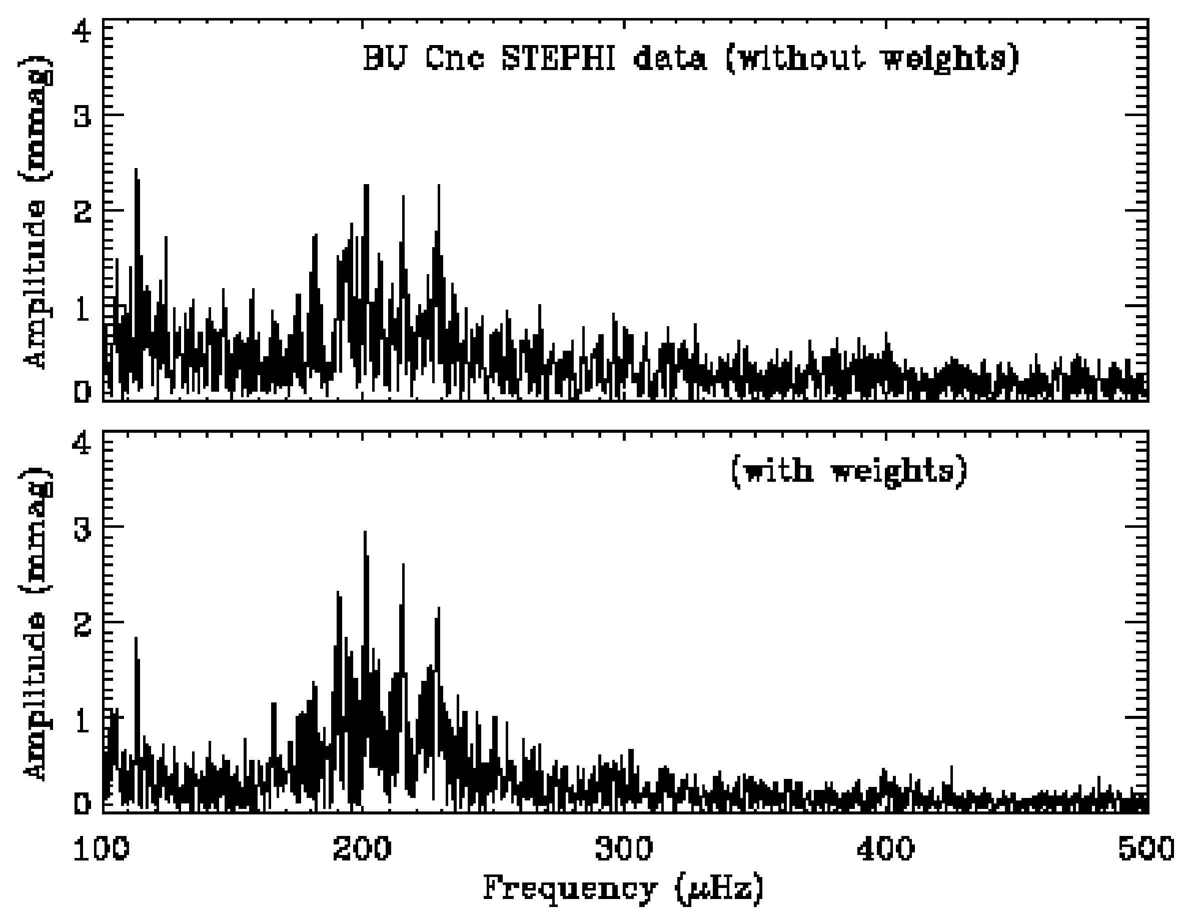
- Good data may be degraded when combined with bad data!



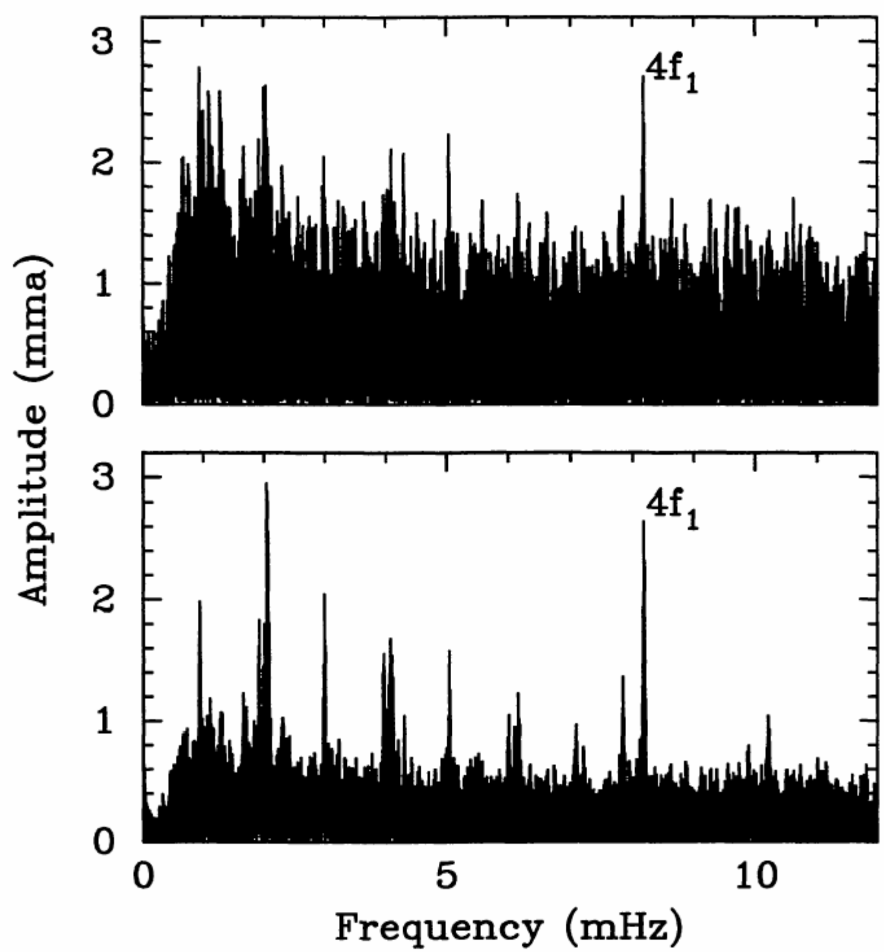
# Why do we need Statisical Weights

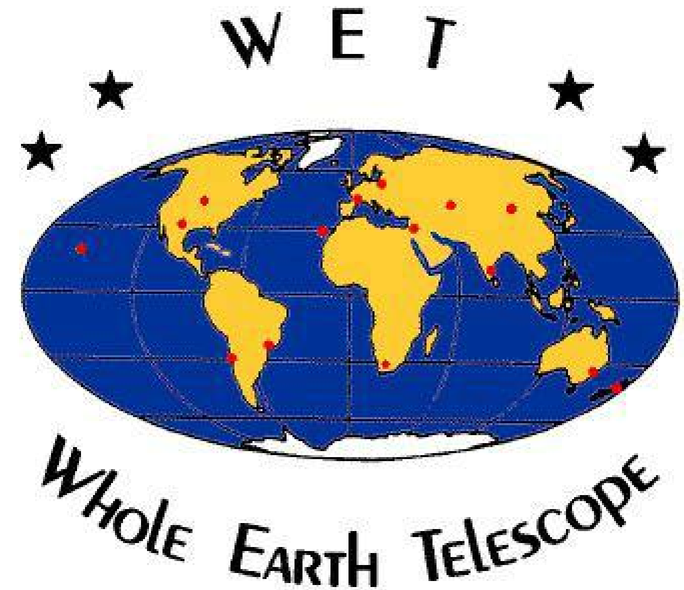
- Weights can have a large effect on detection sensitivity



# Why do we need Statisical Weights

- Weights can have a large effect on detection sensitivity

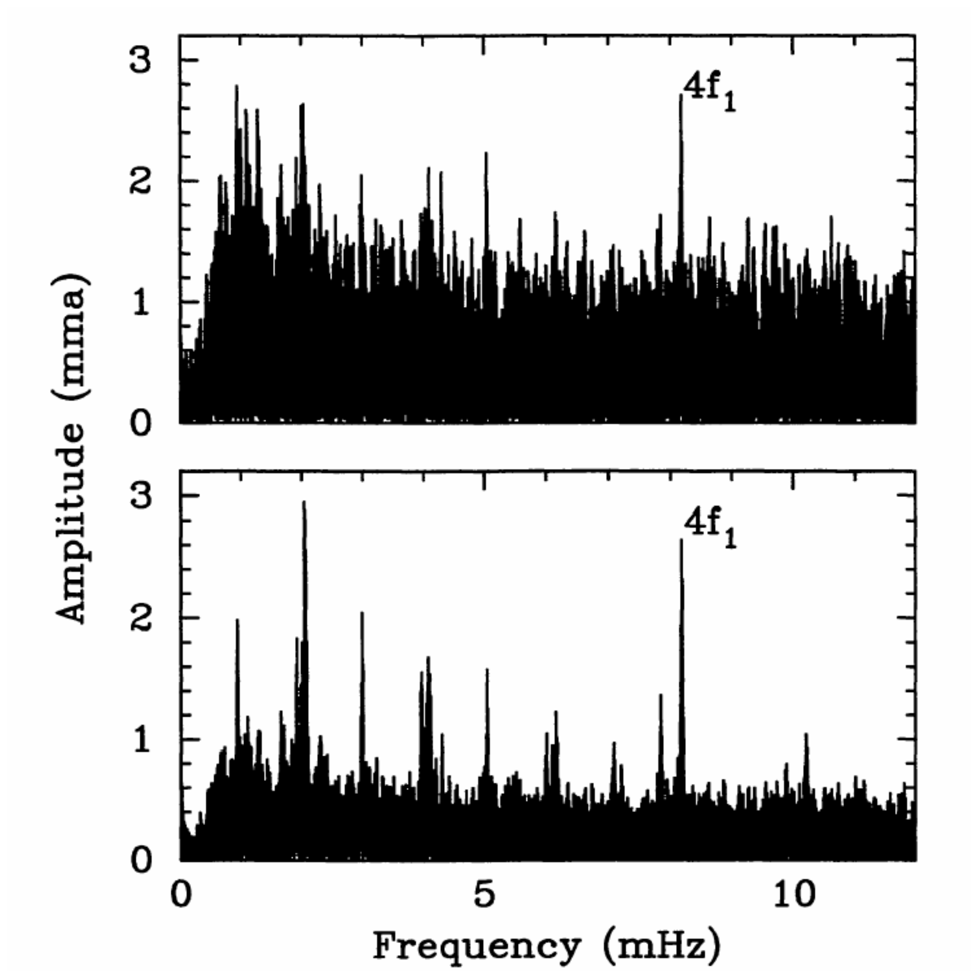


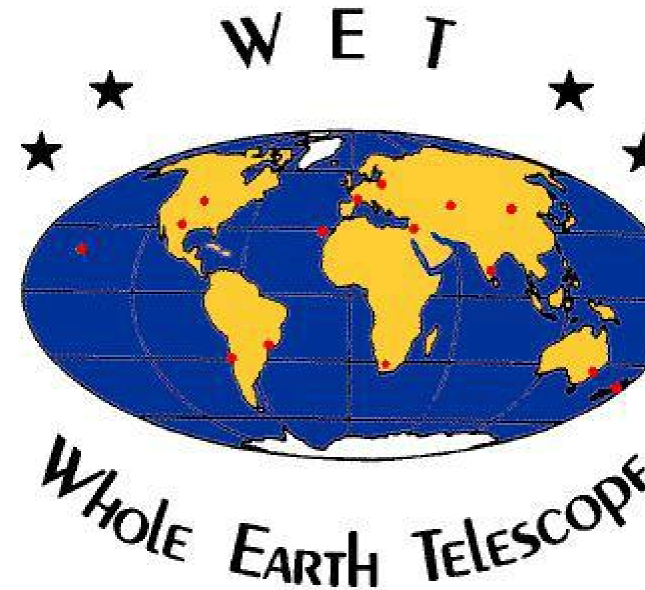




(Handler 2003, Baltic Astronomy v. 12, 253)

# Why do we need Statisical Weights

- Weights can have a large effect on detection sensitivity




$$S / N \geq 4$$

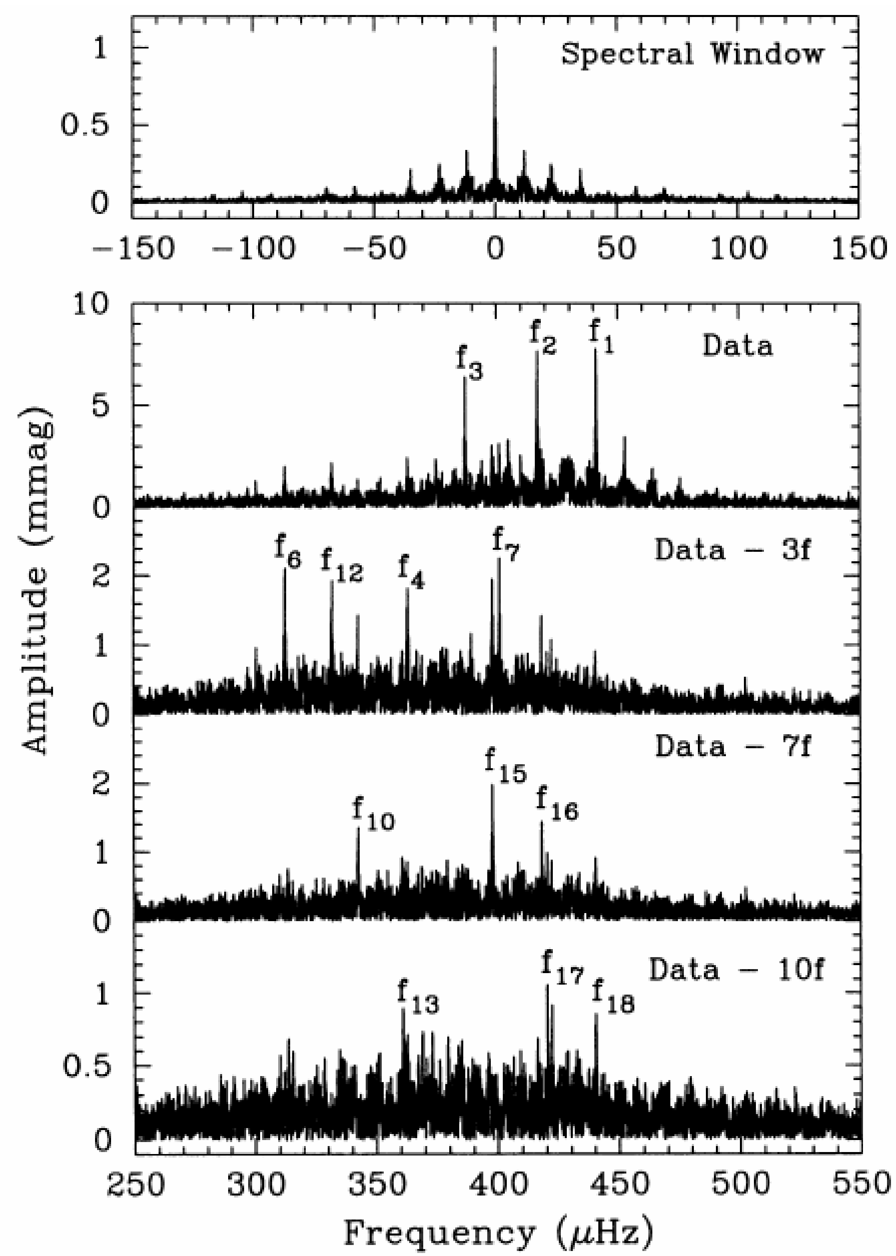
# How do we get the Statisical Weights?

- Could get them from the data reduction software
  - must be checked
- Could get them from photon statistics
- Could get them via the power spectrum
- Could get them directly from the time series
  - but watch out for the oscillation signal

# From Data Reduction Software

- Good if estimated uncertainties are trustworthy
- Check anyway that they actually reflects the noise of the data
- This can be done from a time series cleaned for signal so it only contains noise

# Removing the oscillation signal



# Removing the oscillation signal

- A set of N observations, $(x_1, t_1), \ldots, (x_N, t_N)$

- Set up a model of the observations at each frequency

$$x_j = \alpha_i \cdot \cos(2\pi \cdot f_i \cdot t_j) + \beta_i \cdot \sin(2\pi \cdot f_i \cdot t_j)$$

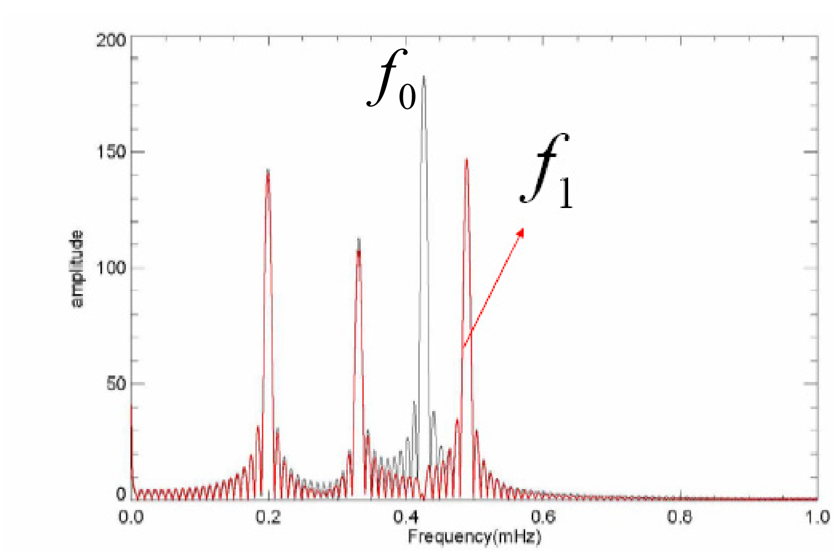
$$P(f_i) = \alpha_i^2 + \beta_i^2$$

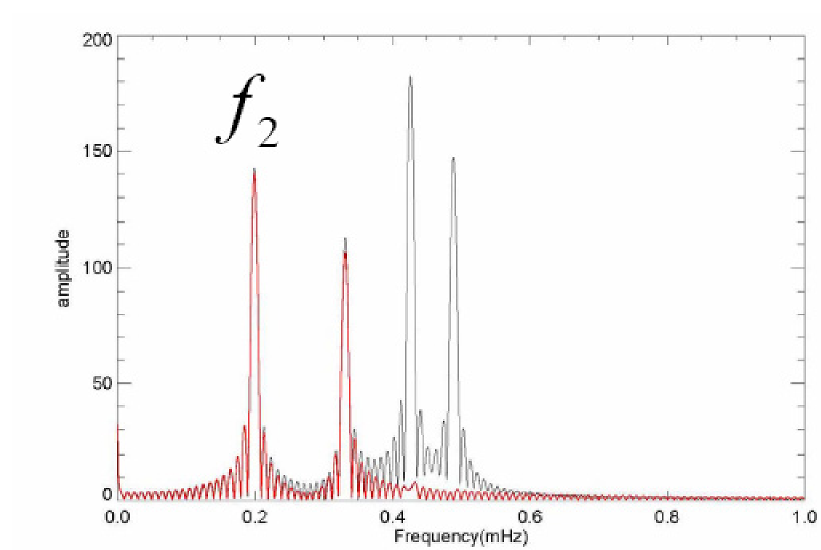
- The Time Series is described by $\alpha$ and $\beta$!

# Removing the oscillation signal

- The data can now be CLEANed for individual frequencies

$$data(t) - \alpha(f_0)\cdot\sin(2\pi\cdot f_0\cdot t) - \beta(f_0)\cdot\cos(2\pi\cdot f_0\cdot t)$$
$$- \alpha(f_1)\cdot\sin(2\pi\cdot f_1\cdot t) - \beta(f_1)\cdot\cos(2\pi\cdot f_1\cdot t)$$

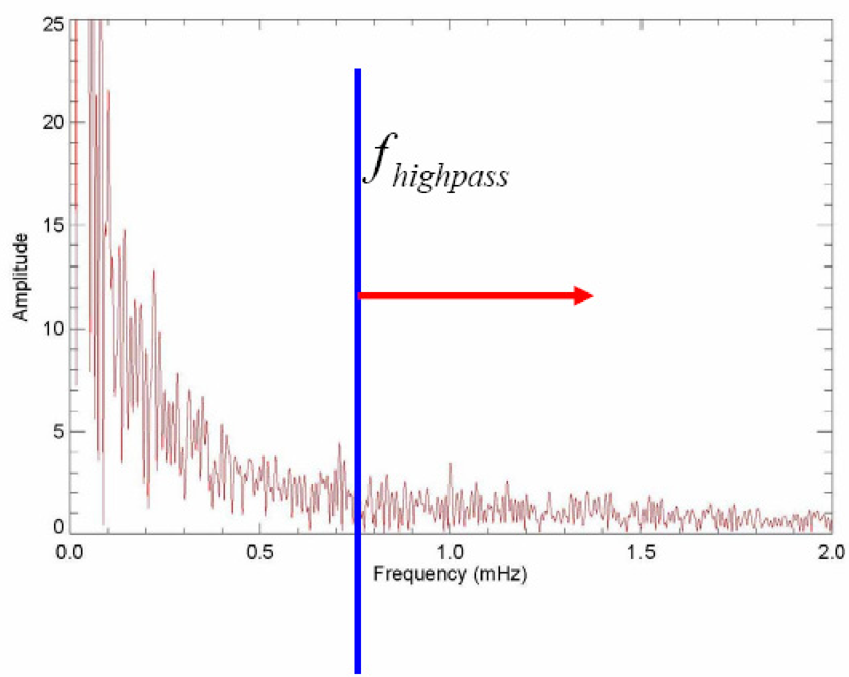


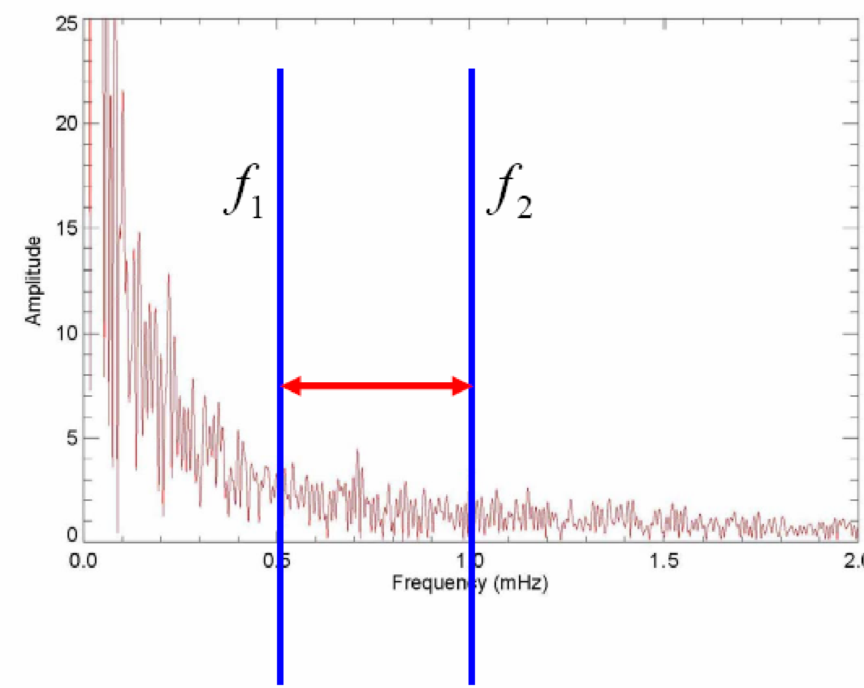
# Removing the oscillation signal

- … or we can just remove everything using a bandpass-filter

High-pass filter



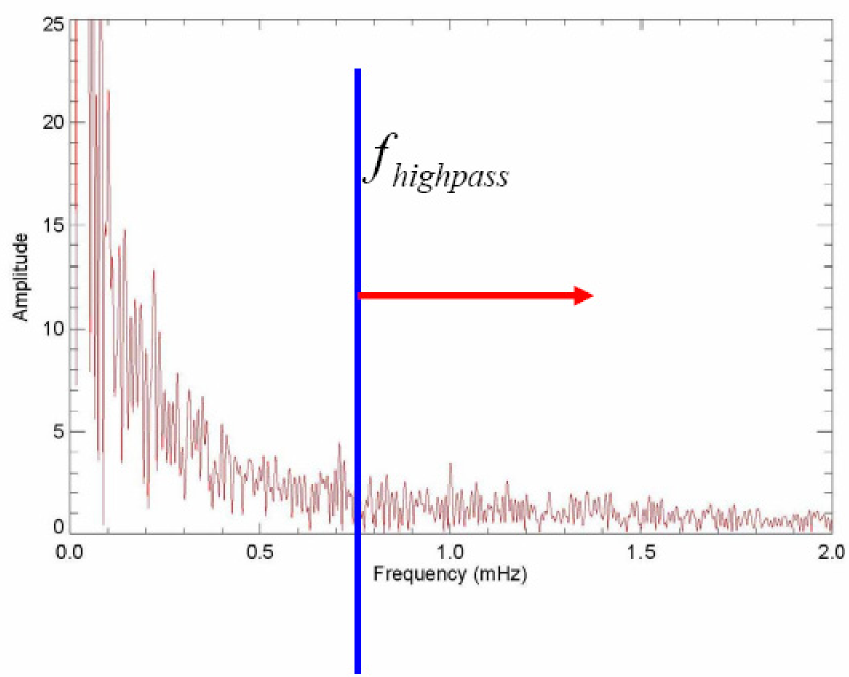

Band-pass filter



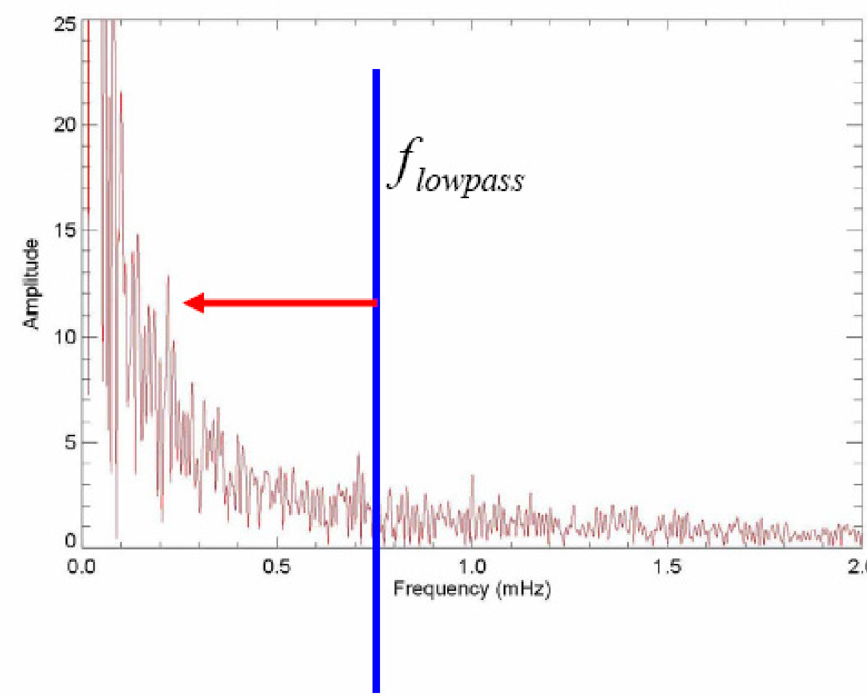
# Removing the oscillation signal

- … or we can just remove everything using a bandpass-filter

High-pass filter




Low-pass filter



# Removing the oscillation signal

- The High-pass filter:

$$highpass(j) = data(j) - \frac{\sum_{i=1}^{n(f_{highpass})} \alpha(i) \cdot \sin(2\pi \cdot f_i \cdot t_j) + \beta(i) \cdot \cos(2\pi \cdot f_i \cdot t_j)}{\sum_{i=1}^{N} P_{window}(i)}$$

# Removing the oscillation signal

- The band-pass filter:

$$data2(j) = data(j) - \frac{\sum_{i=n(f_1)}^{n(f_2)} \alpha(i) \cdot \sin(2\pi \cdot f_i \cdot t_j) + \beta(i) \cdot \cos(2\pi \cdot f_i \cdot t_j)}{\sum_{i=1}^{N} P_{window}(i)}$$

$$bandpass(j) = data(j) - data2(j)$$

# Removing the oscillation signal

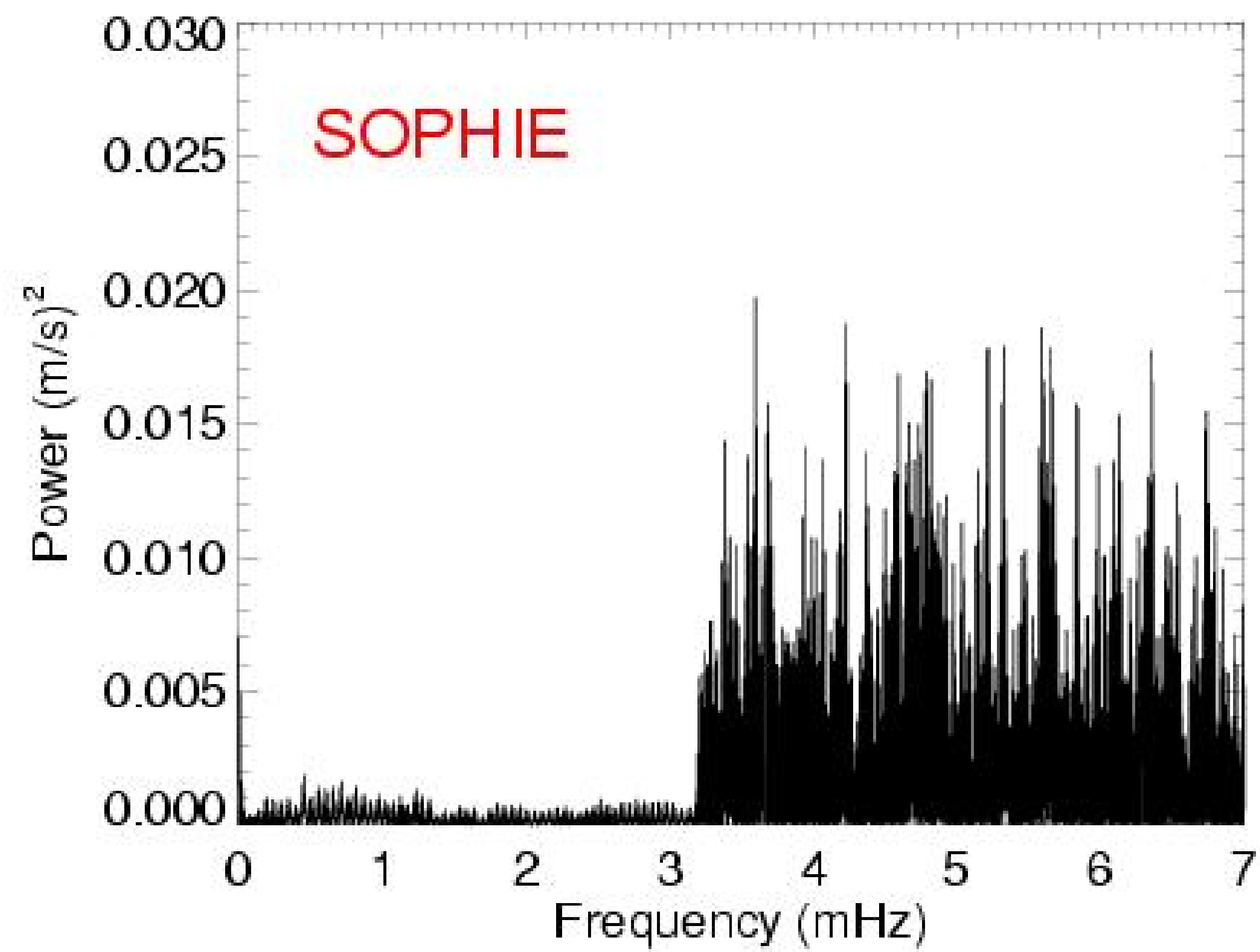
- To remove everything in a frequency band:

$$data2(j) = data(j) - \frac{\sum_{i=n(f_1)}^{n(f_2)} \alpha(i) \cdot \sin(2\pi \cdot f_i \cdot t_j) + \beta(i) \cdot \cos(2\pi \cdot f_i \cdot t_j)}{\sum_{i=1}^{N} P_{window}(i)}$$

$$bandpass(j) = data(j) - data2(j)$$

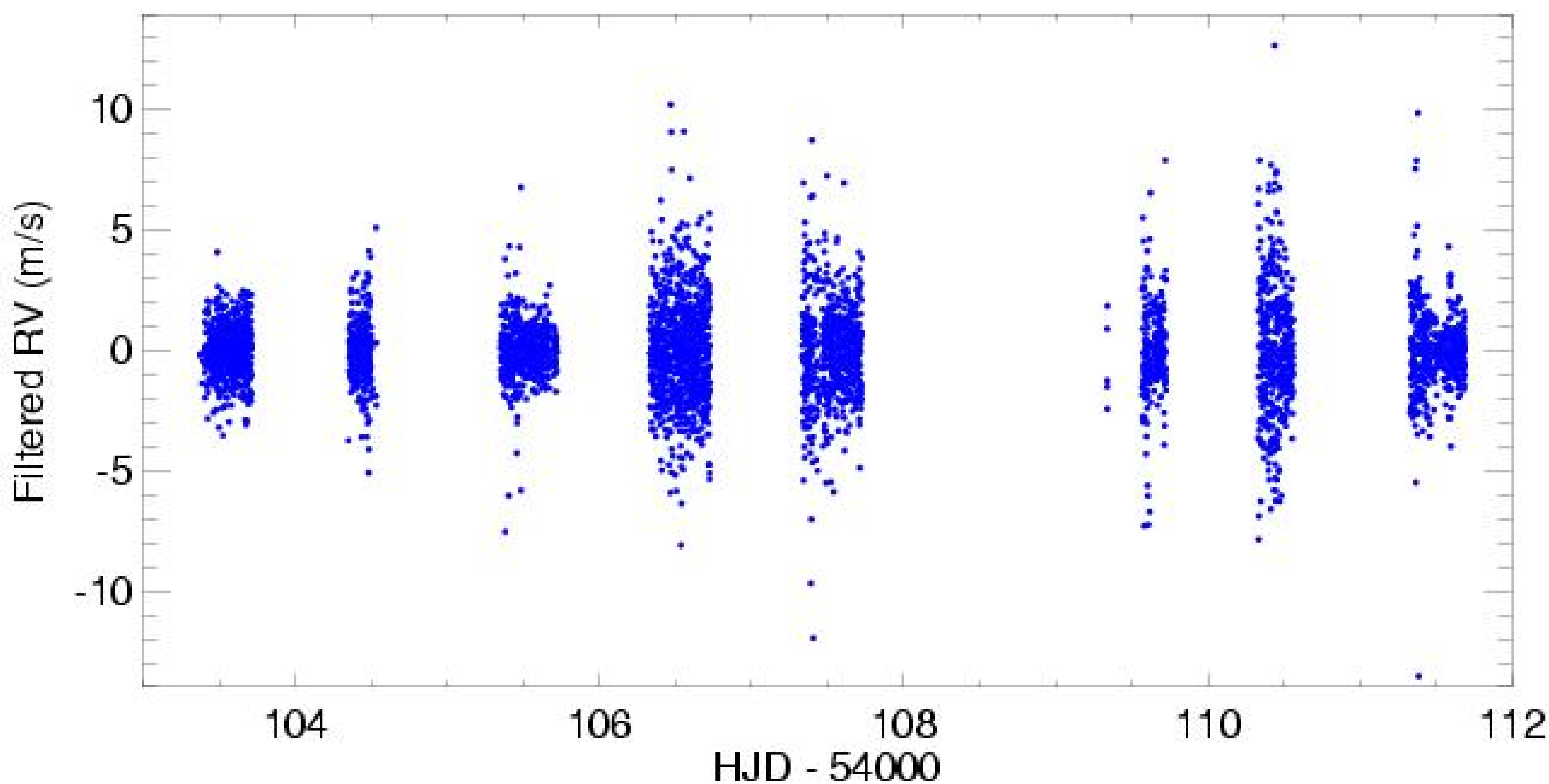
# Removing the oscillation signal

- The High-pass filter:



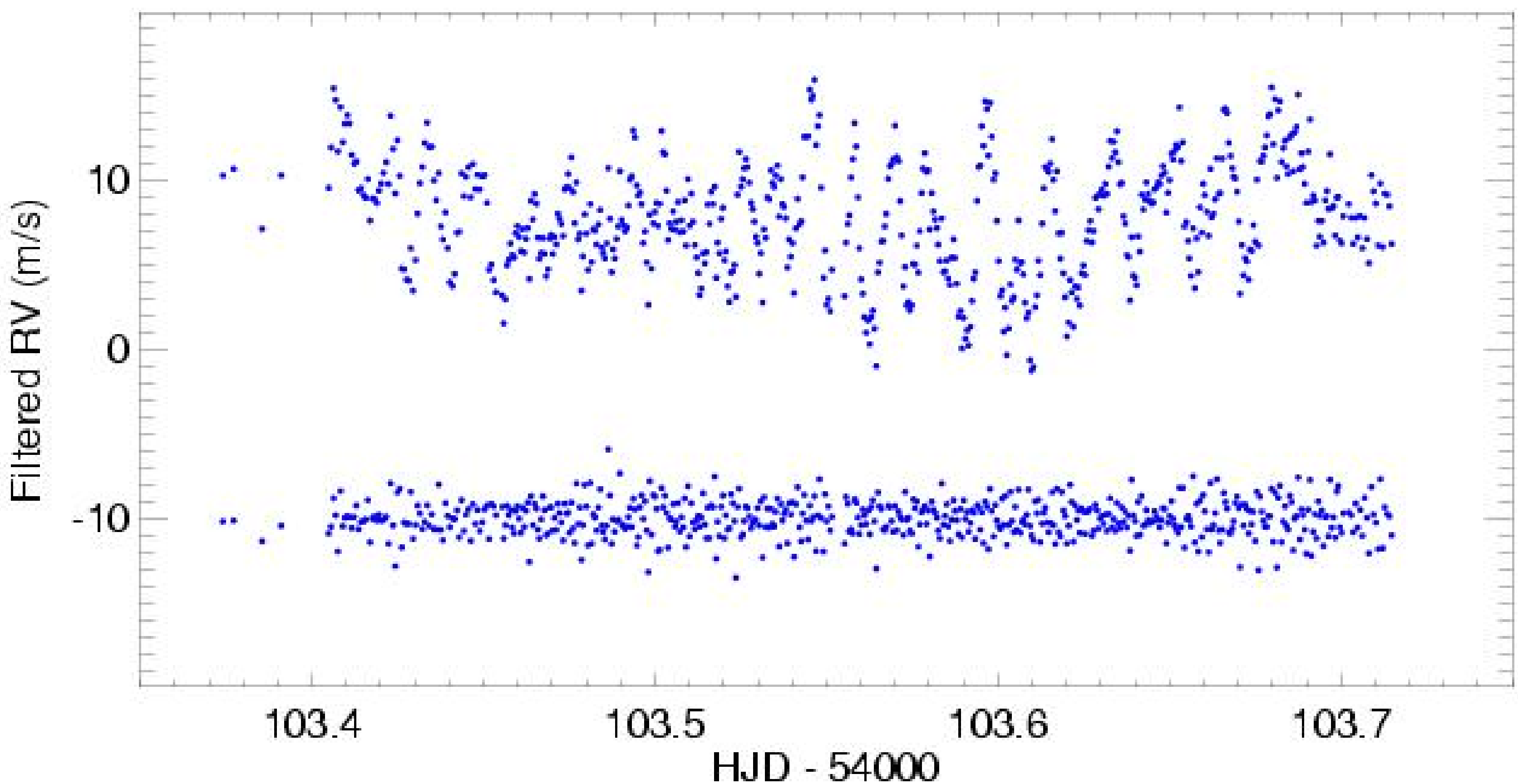
# Removing the oscillation signal

- The High-pass filter:



# Removing the oscillation signal

- The High-pass filter:

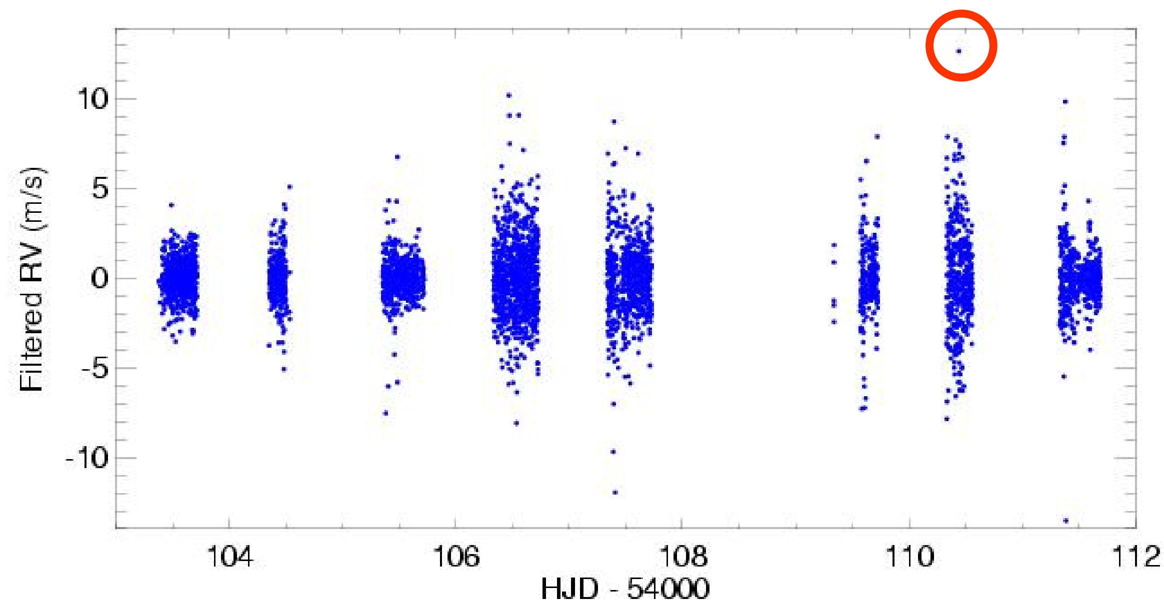


# From Data Reduction Software

- Check for each data-point that

$$| r_i / \sigma_i | \leq\sim 2.5$$

(from experience)




- Adjust the σ of poor data-points – it really helps!

(See Butler et al. 2004, ApJ 600, L75-L78)

# From Data Reduction Software

- Finally, calibrate the uncertainties to match reality

The mean variance of the time-series: $\sum_{i=1}^{N} \sigma_i^2 w_i / \sum_{i=1}^{N} w_i$

The variance from the amplitude spectrum: $\sigma_{amp}^2 N / \pi$

… should be equal $\left(\sigma_{amp} = \sqrt{\frac{\pi}{N}} \cdot \sigma_{TimeSeries}\right)$

# From Data Reduction Software

- Finally, calibrate the uncertainties to match reality

$$\sigma_{amp}^2 \sum_{i=1}^{N} \sigma_i^{-2} = \pi$$

- Adjust software-$\sigma$ on a night-by-night basis

# Fourier Weights

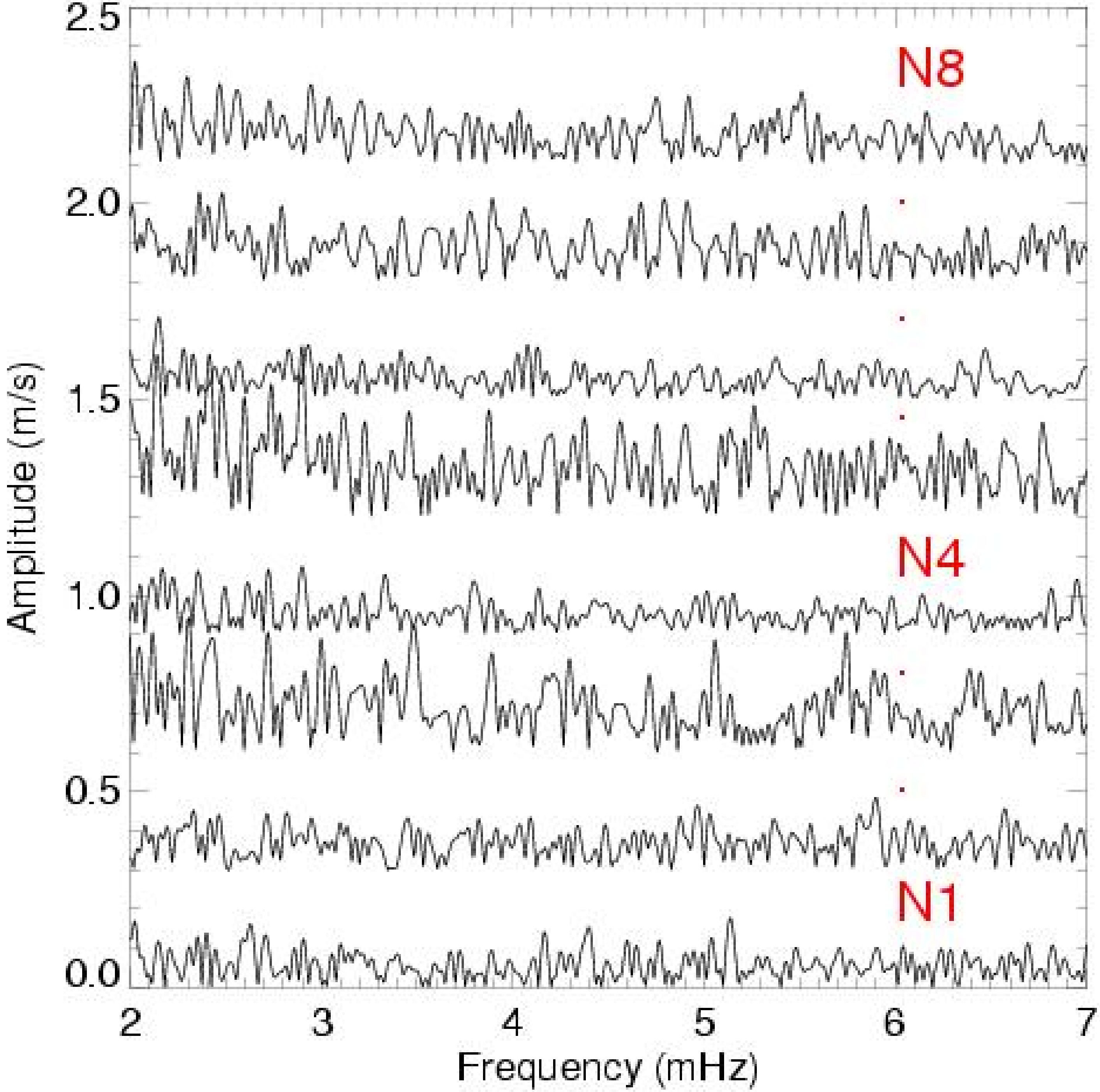
- Obtain noise levels from Fourier Spectrum – typically as the noise level at high frequencies

- Limited time-resoluton – but independent of signal

- A single weight per telescope or per telescope per night

- The weights are used in the time-domain: take into account the number of data-points used for calculating the spectrum

$$\sigma_{amp} = \sqrt{\frac{\pi}{N}} \cdot \sigma_{TimeSeries}$$



N8
N4
N1
Amplitude (m/s)
Frequency (mHz)
2.5
2.0
1.5
1.0
0.5
0.0
2
3
4
5
6
7

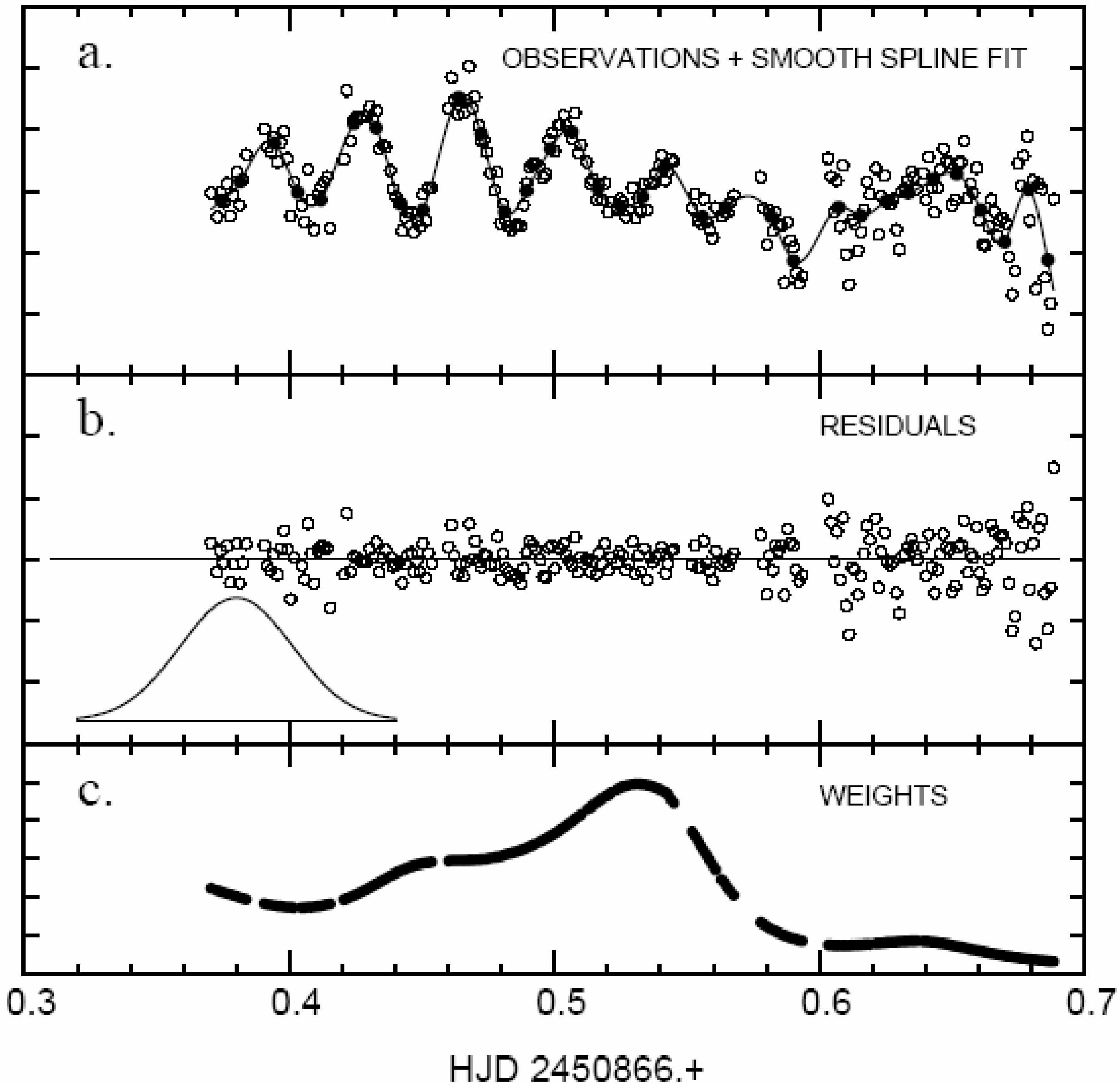
# Directly from the Time Series

- Remove all signal from the time series and hereby obtain a time series that contains only noise

- Calculate (a robust) local variance using a box-car smoothing of the residuals and use for weights

- Advantages:
  - uses the observed light curves directly
  - high time resolution

- Disadvantages: Must be sure to remove all signal

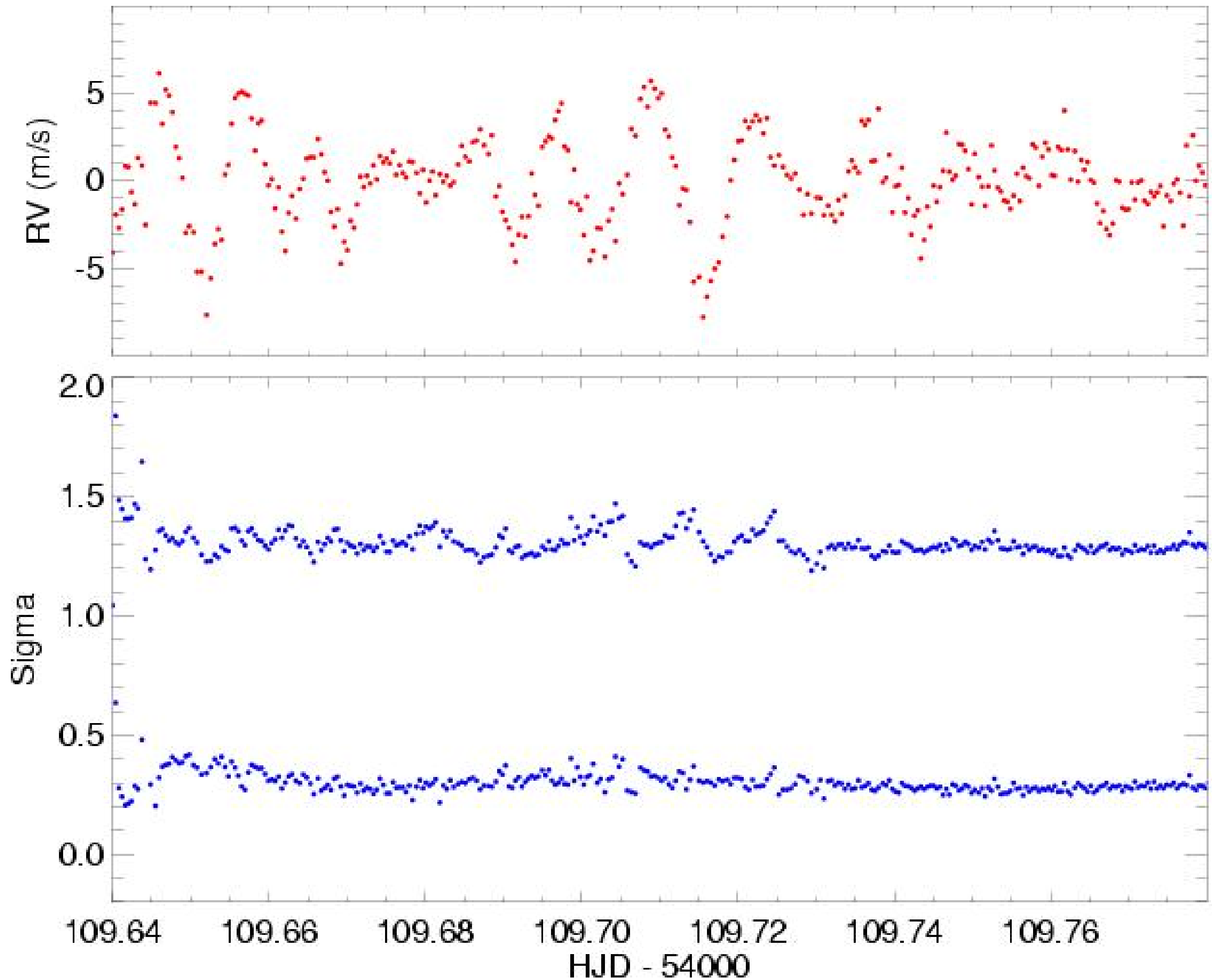

a.
OBSERVATIONS + SMOOTH SPLINE FIT
b.
RESIDUALS
c.
WEIGHTS
0.3
0.4
0.5
0.6
0.7
HJD 2450866.+

# Directly from the Time Series – II

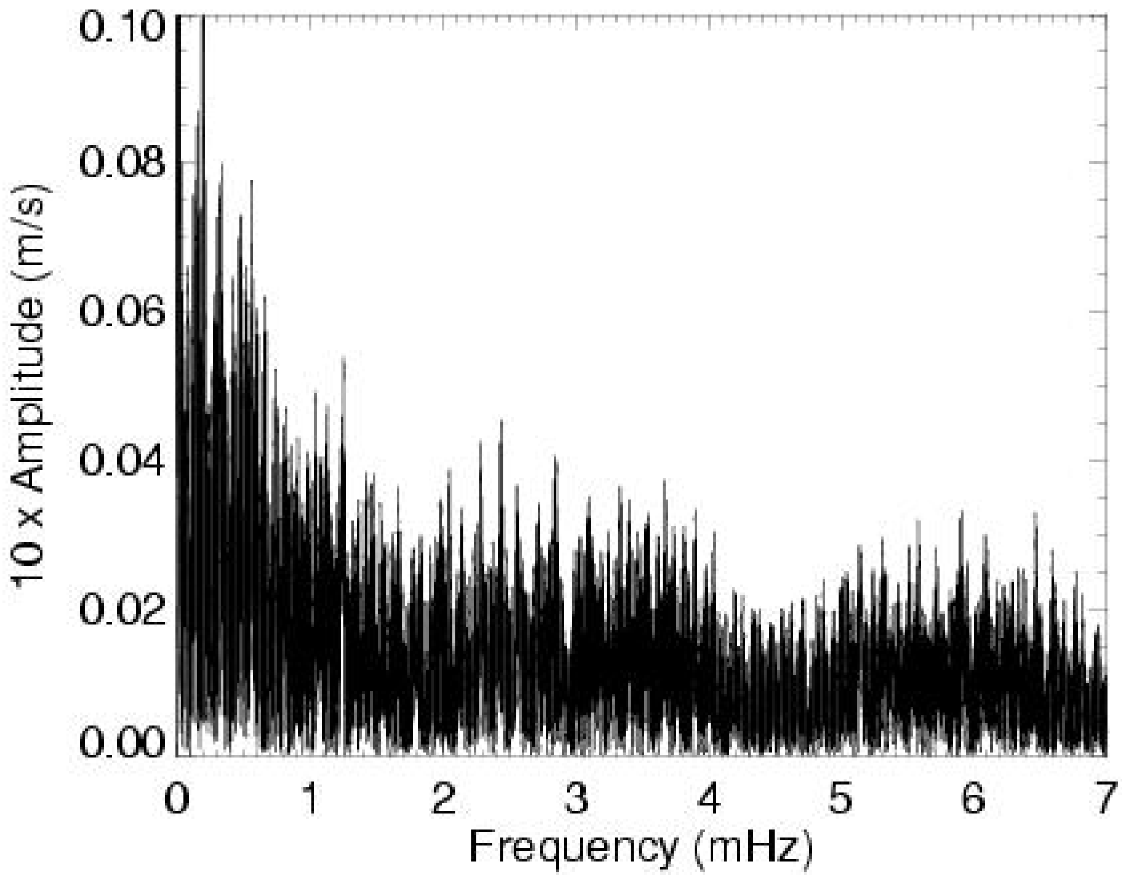
- Remove all signal from the time series and hereby obtain a time series that contains only noise
- Calculate mean standard deviation $\sigma_{avg}$
- Use weight-scheme:

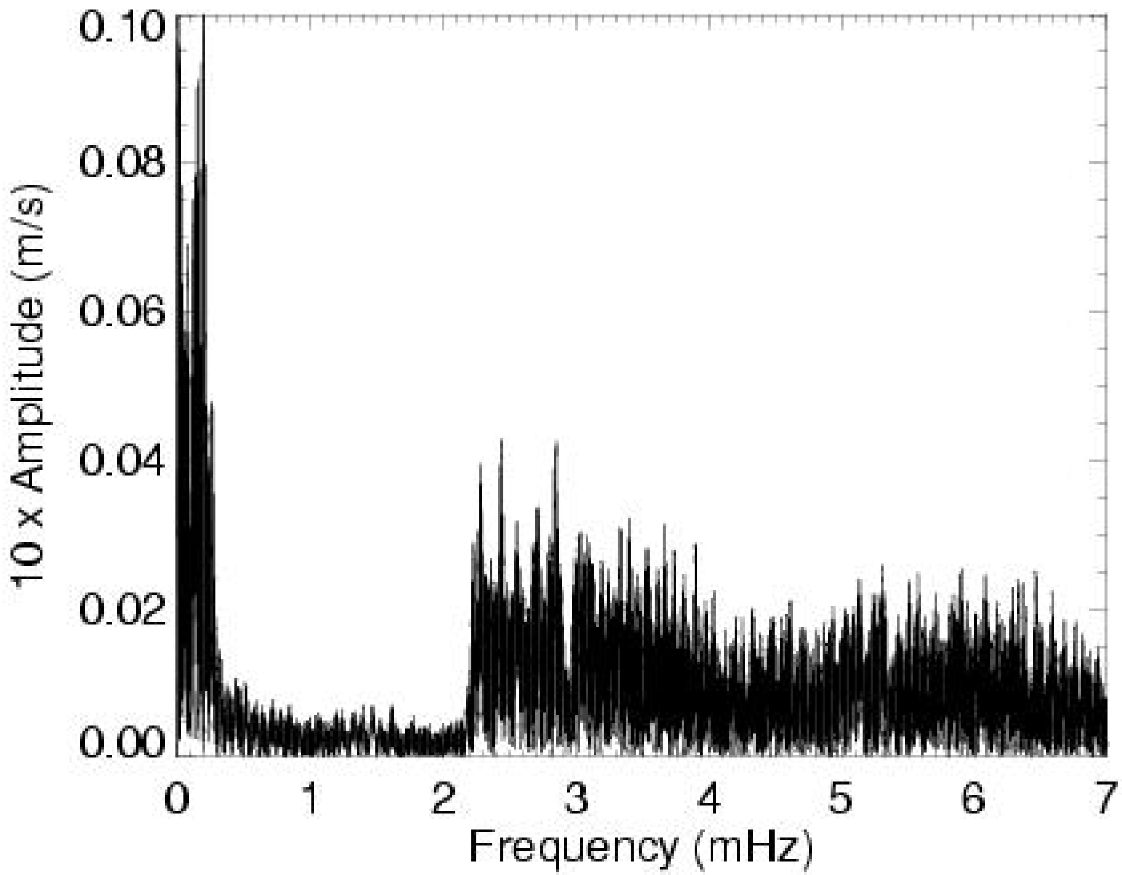
$$\sigma_i \le 2\sigma_{avg} : \quad w_i = 1$$

$$\sigma_i > 2\sigma_{avg} : \quad w_i = \left(2\sigma_{avg} / \sigma_i\right)^2$$

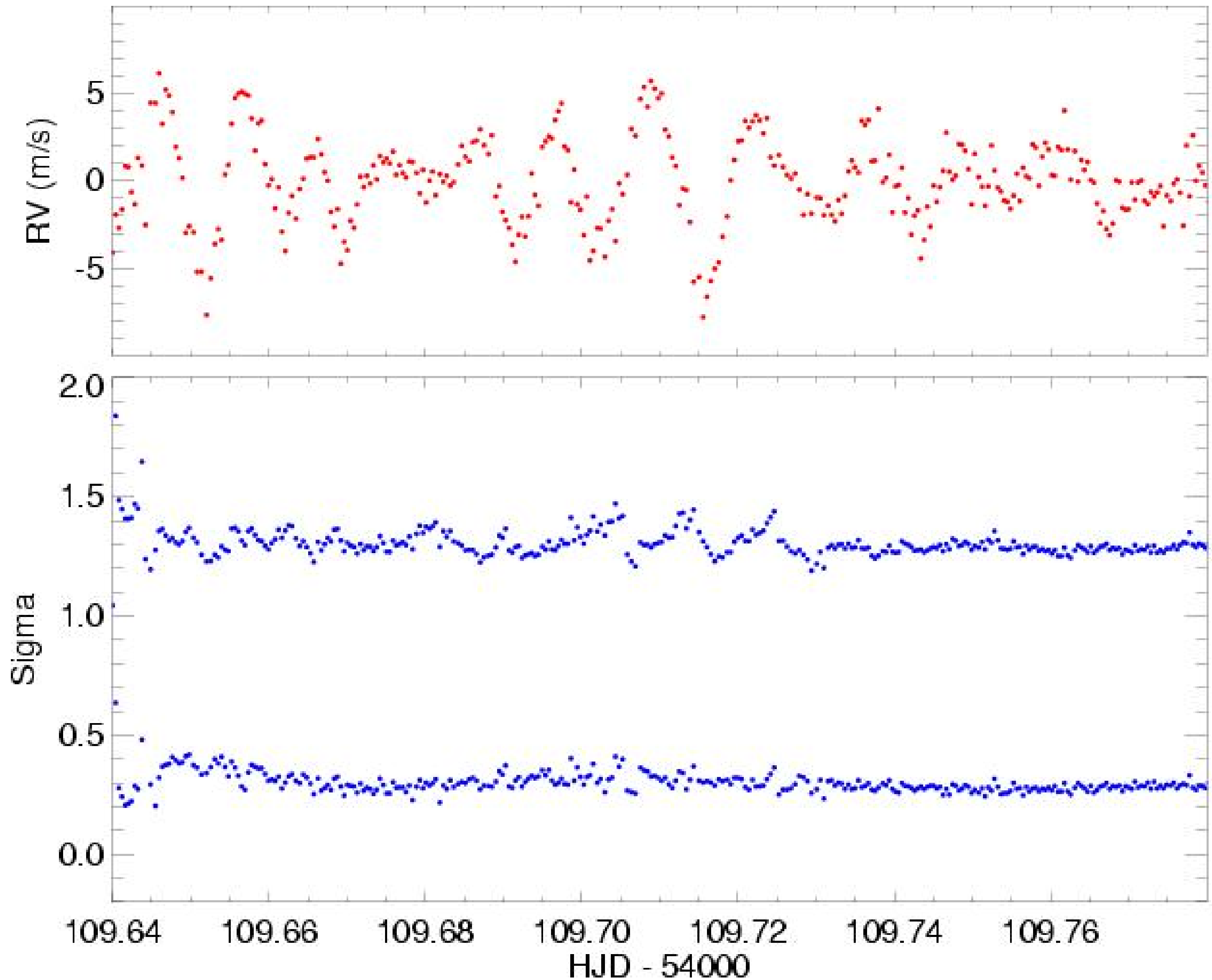



Check weights visually!


0.10
0.08
0.06
0.04
0.02
0.00
10 x Amplitude (m/s)
0
1
2
3
4
5
6
7
Frequency (mHz)

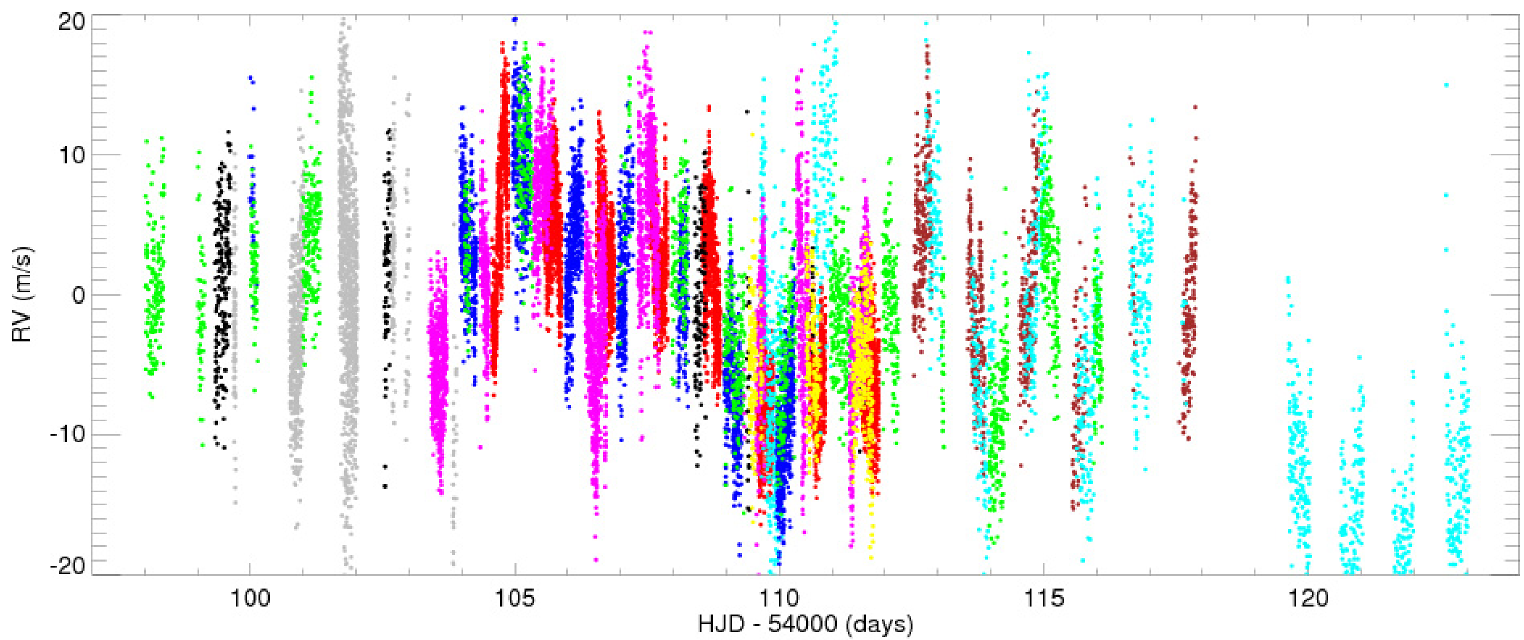


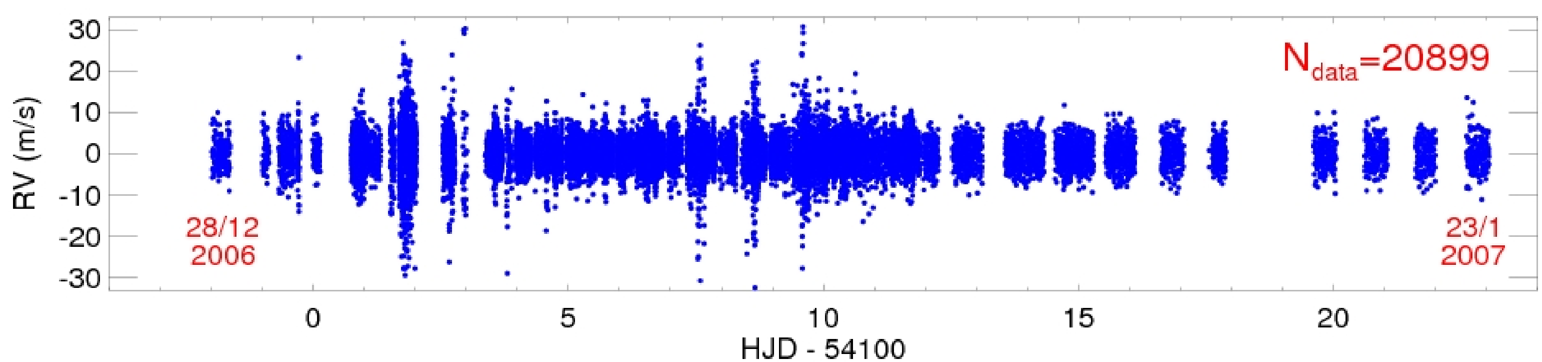
10 x Amplitude (m/s)
0.10
0.08
0.06
0.04
0.02
0.00
0
1
2
3
4
5
6
7
Frequency (mHz)

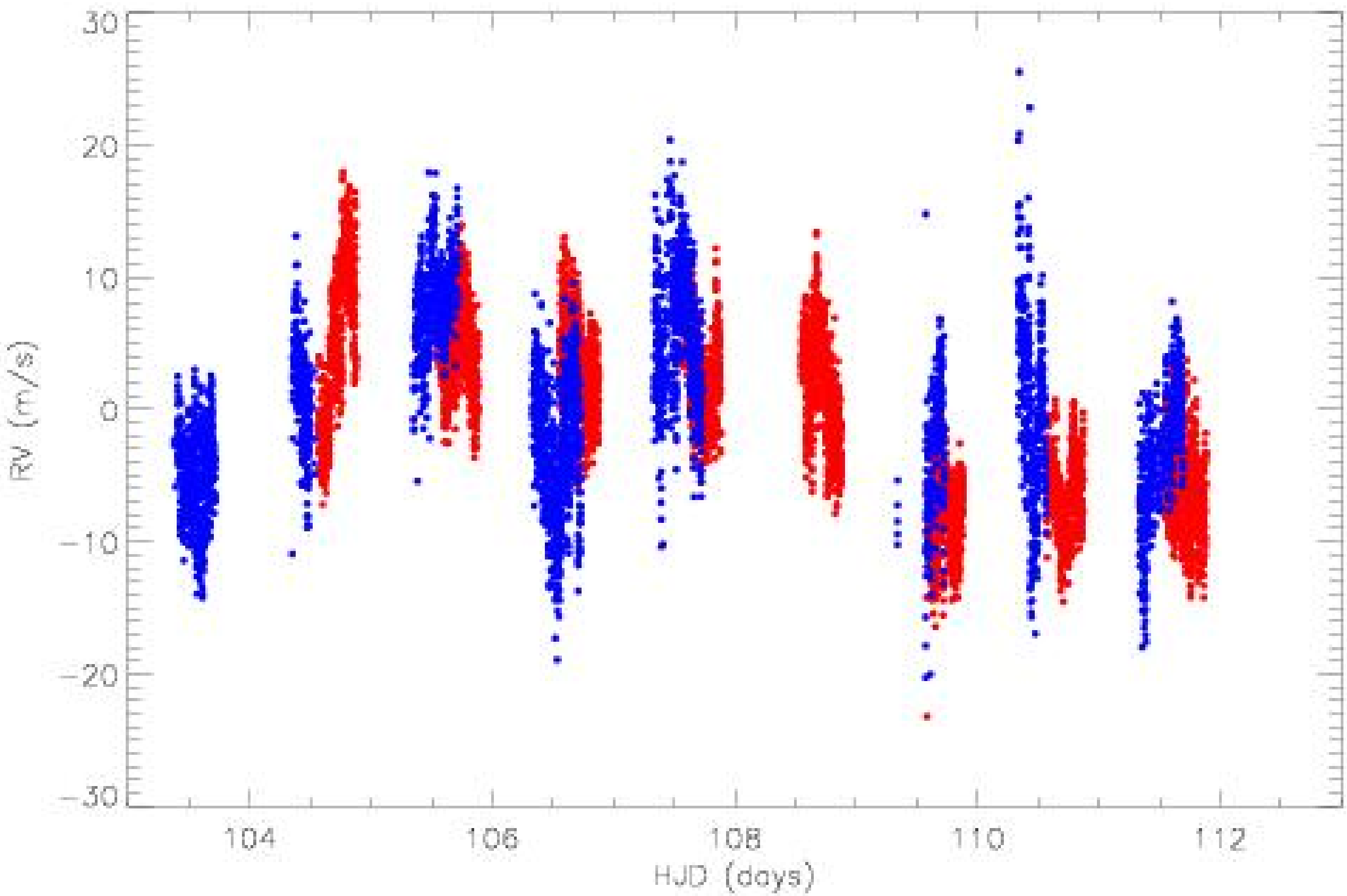


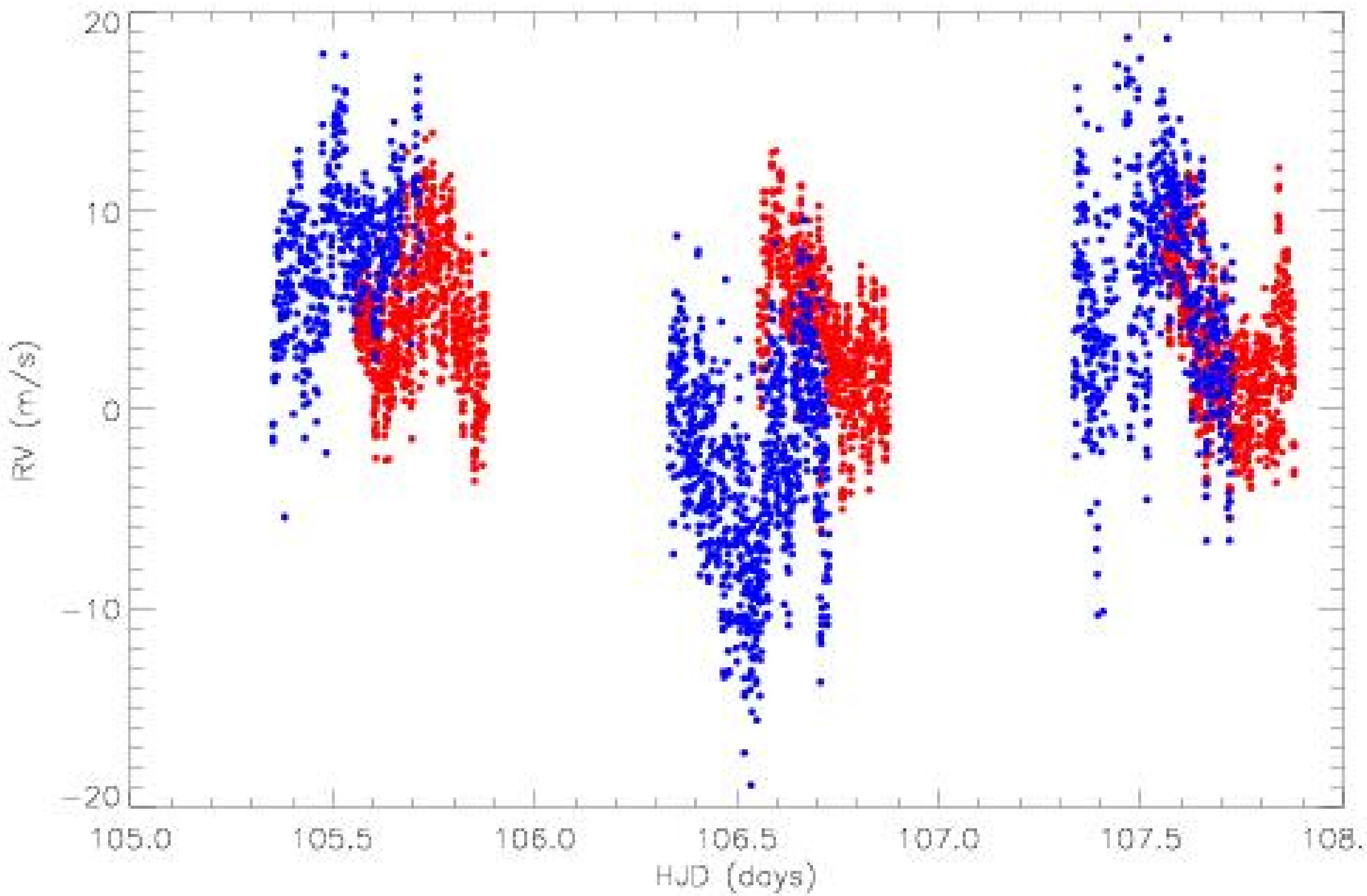

The weights contain noise!

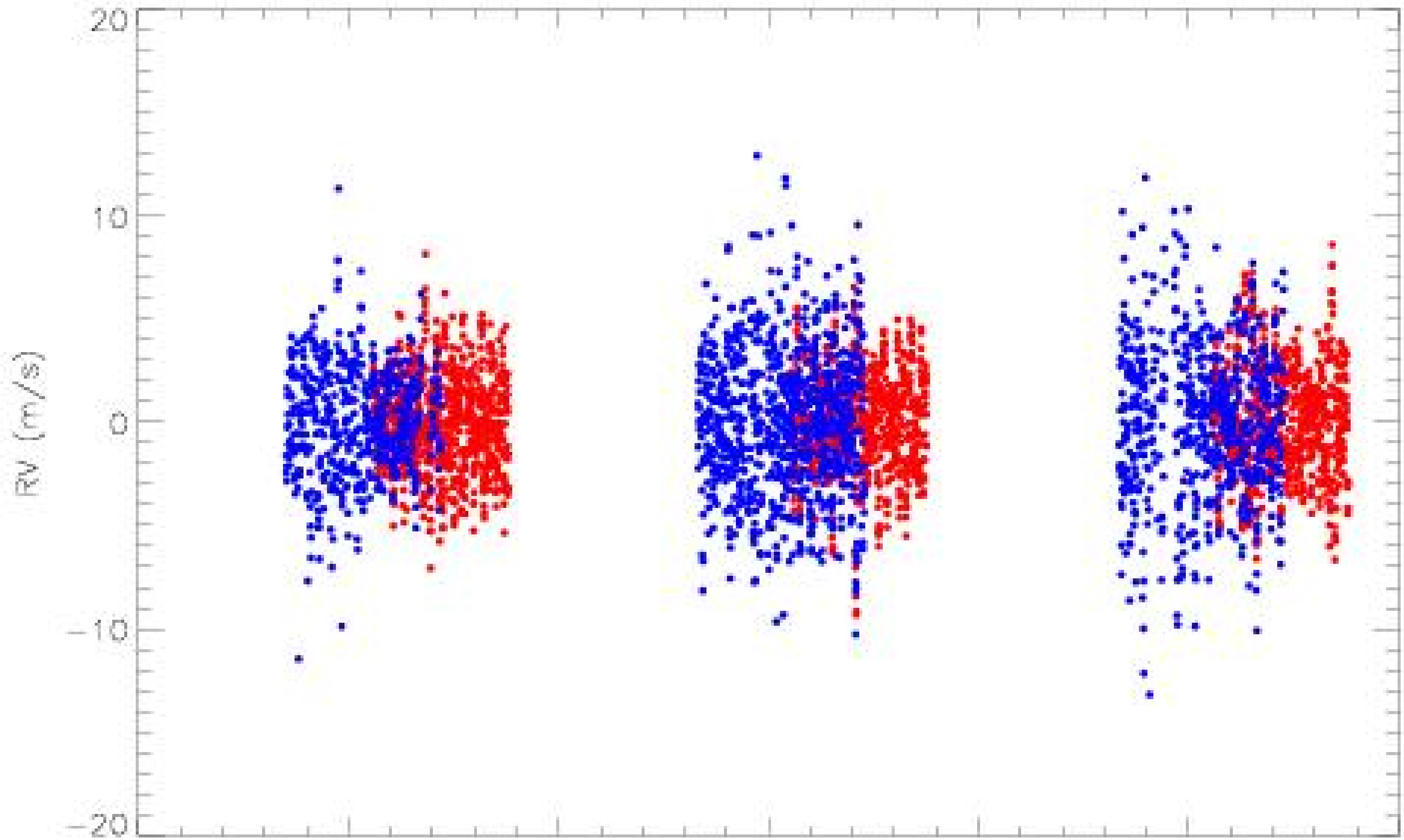
# High-pass filter to merge data:



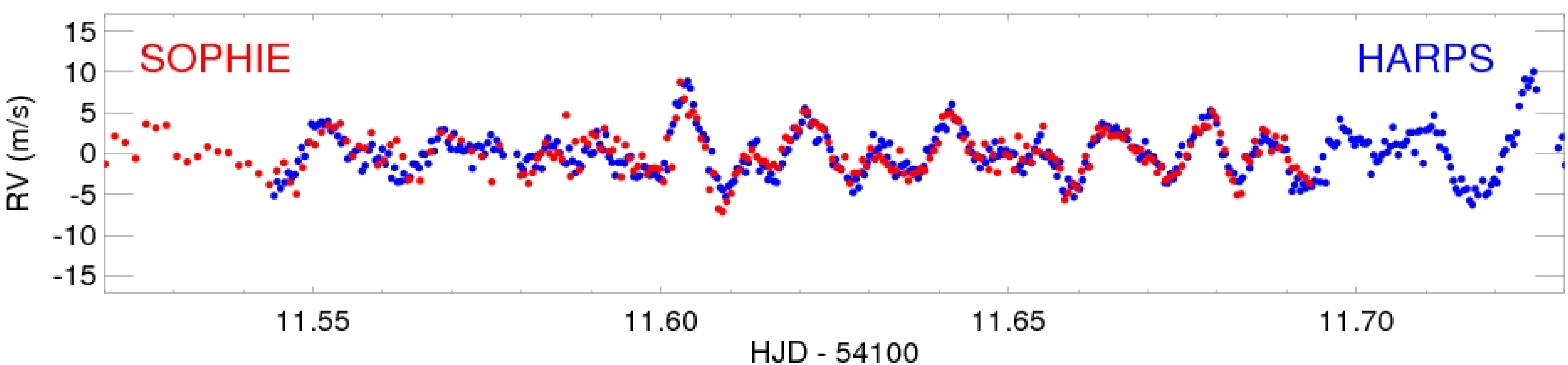




RV (m/s)
HJD (days)
30
20
10
0
−10
−20
−30
104
106
108
110
112

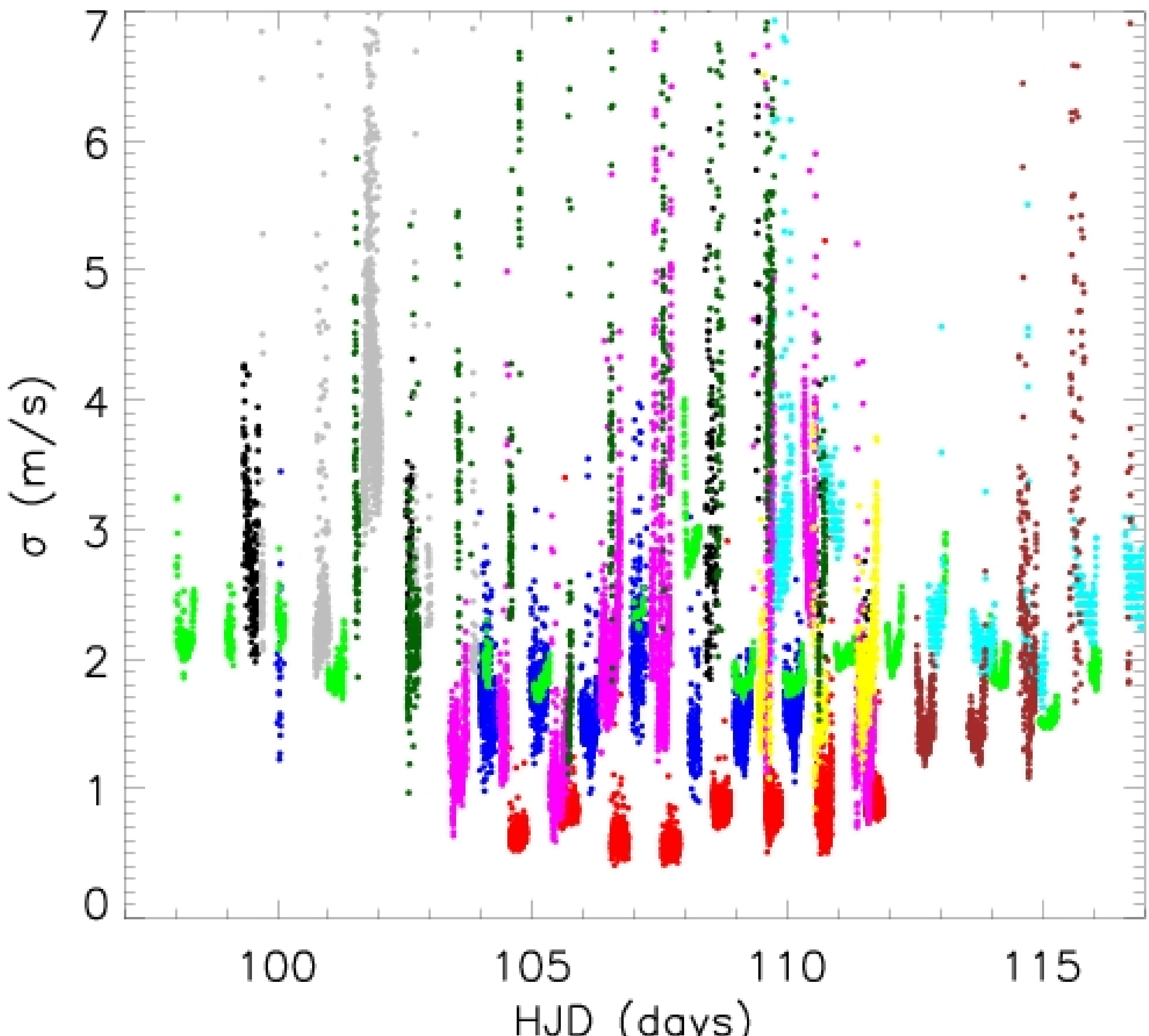

RV (m/s)
HJD (days)
20
10
0
−10
−20
105.0
105.5
106.0
106.5
107.0
107.5
108.0



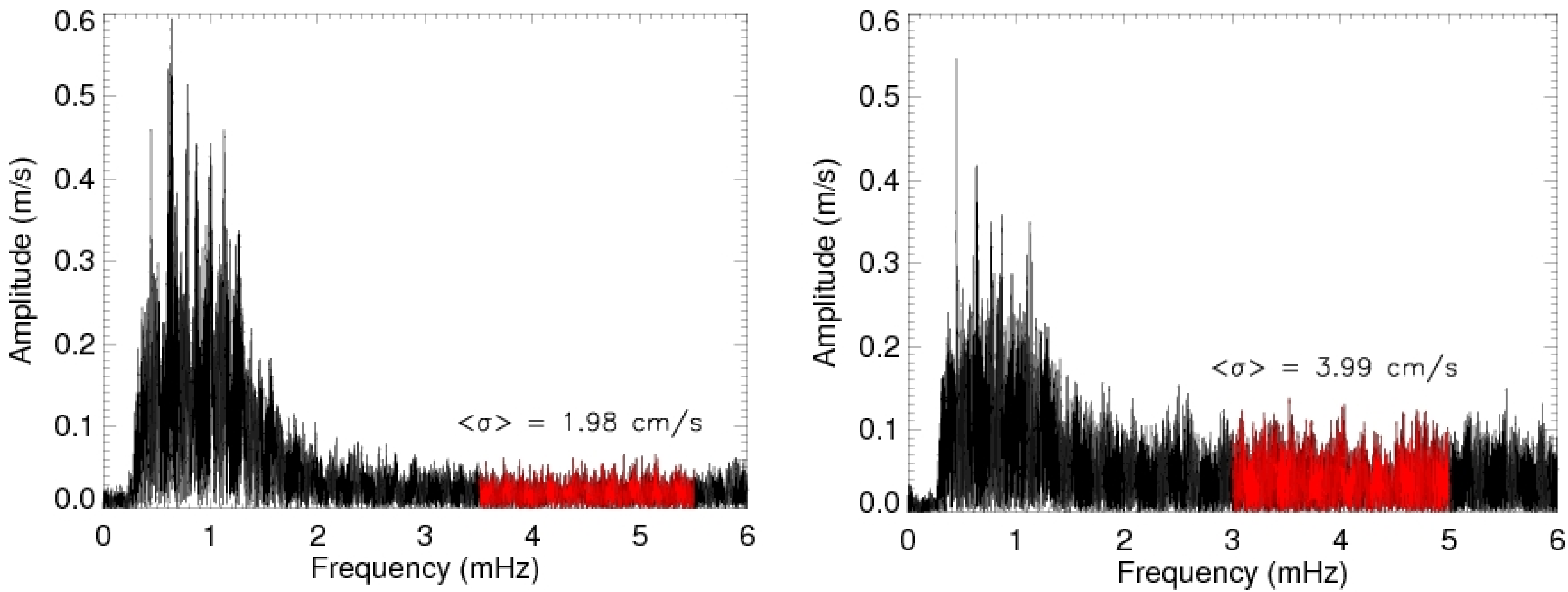
20
10
0
−10
−20
RV (m/s)

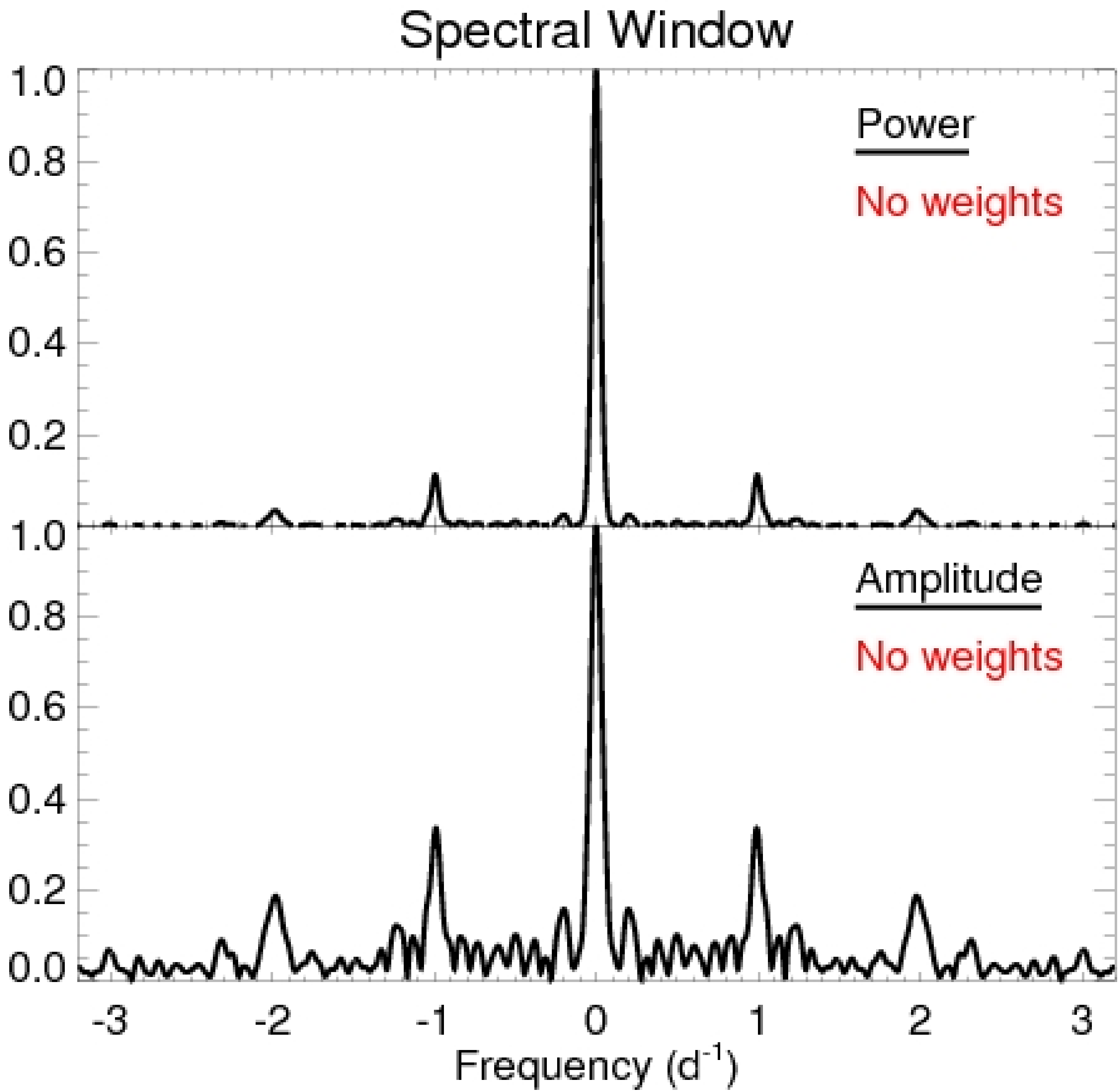



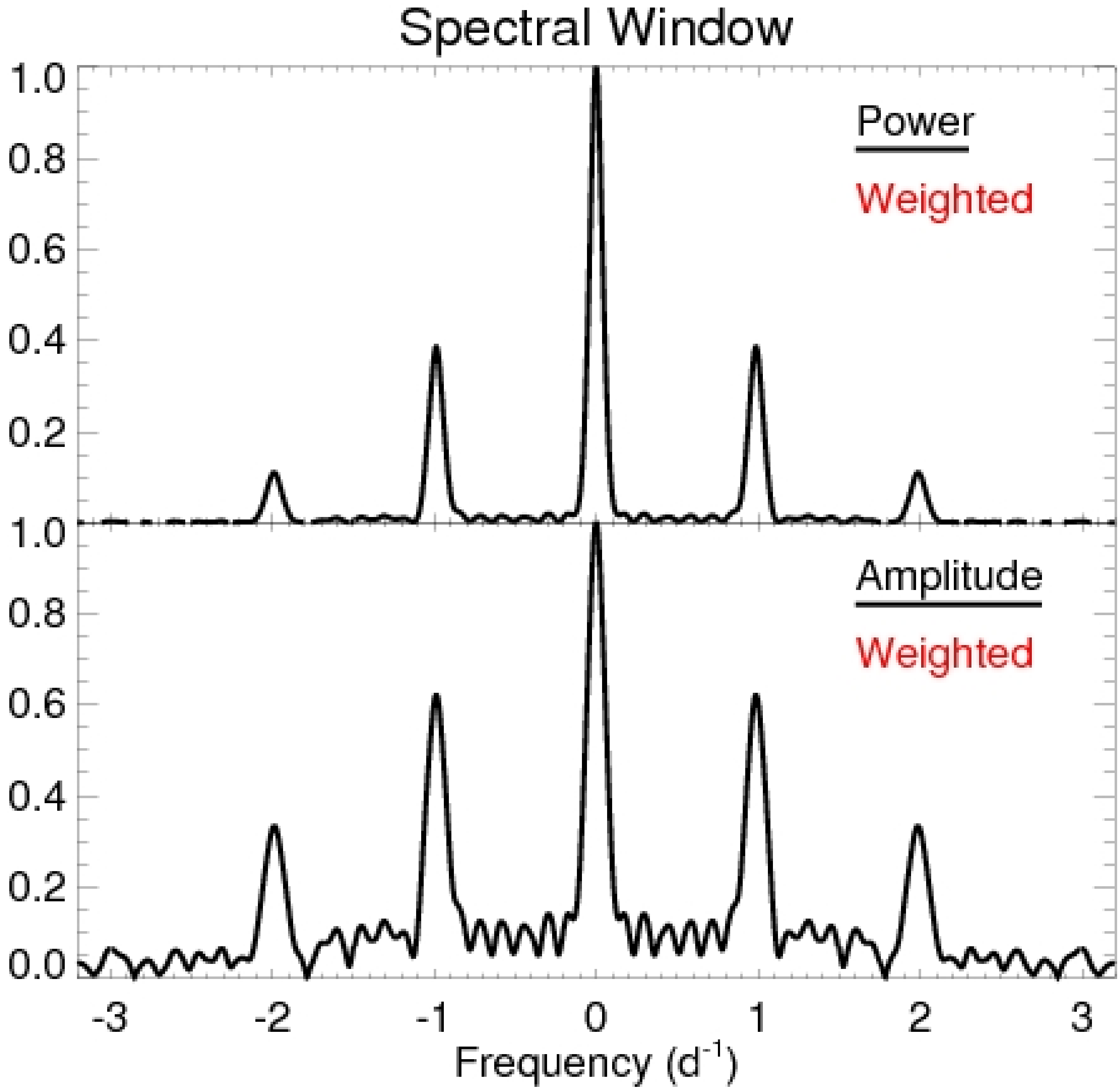
SOPHIE
HARPS
15
10
5
0
-5
-10
-15
RV (m/s)
11.55
11.60
11.65
11.70
HJD - 54100



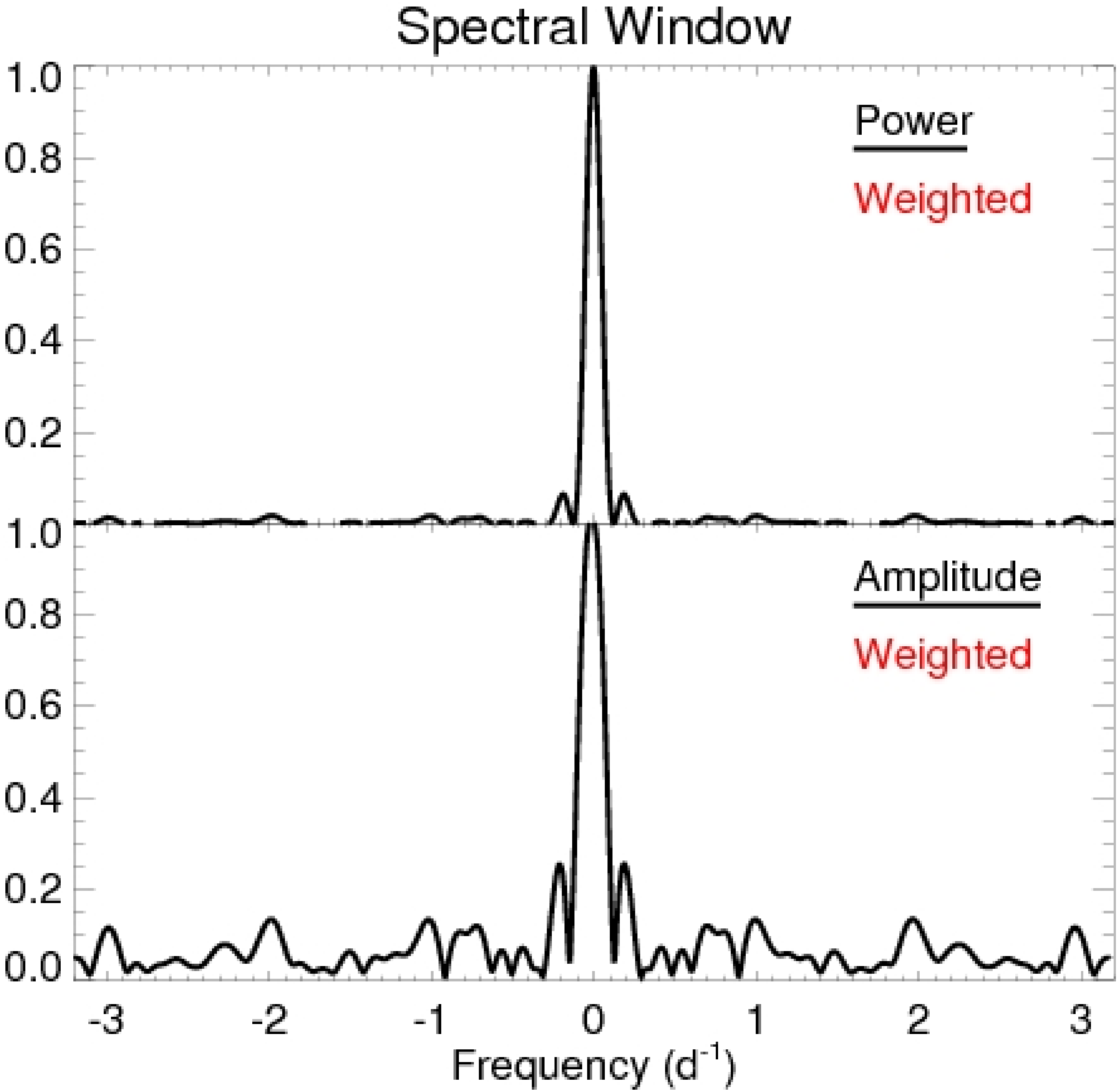
σ (m/s)
HJD (days)
0
1
2
3
4
5
6
7
100
105
110
115


Amplitude (m/s)
Frequency (mHz)
<σ> = 1.98 cm/s
Amplitude (m/s)
Frequency (mHz)
<σ> = 3.99 cm/s


Spectral Window
Power
No weights
Amplitude
No weights
1.0
0.8
0.6
0.4
0.2
0.0
-3
-2
-1
0
1
2
3
Frequency (d$^{-1}$)



Spectral Window
1.0
0.8
0.6
0.4
0.2
Power
Weighted
1.0
0.8
0.6
0.4
0.2
0.0
Amplitude
Weighted
-3
-2
-1
0
1
2
3
Frequency ($d^{-1}$)




’Window Optimized’

(Kjeldsen et al. 2005)

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

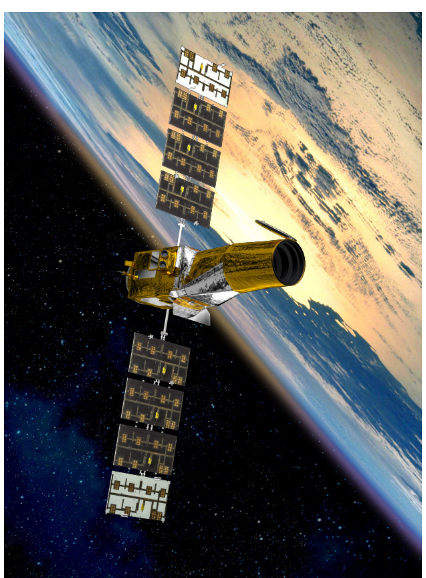
CENTRE NATIONAL D'ÉTUDES SPATIALES

Paris, 3rd of May, 2007

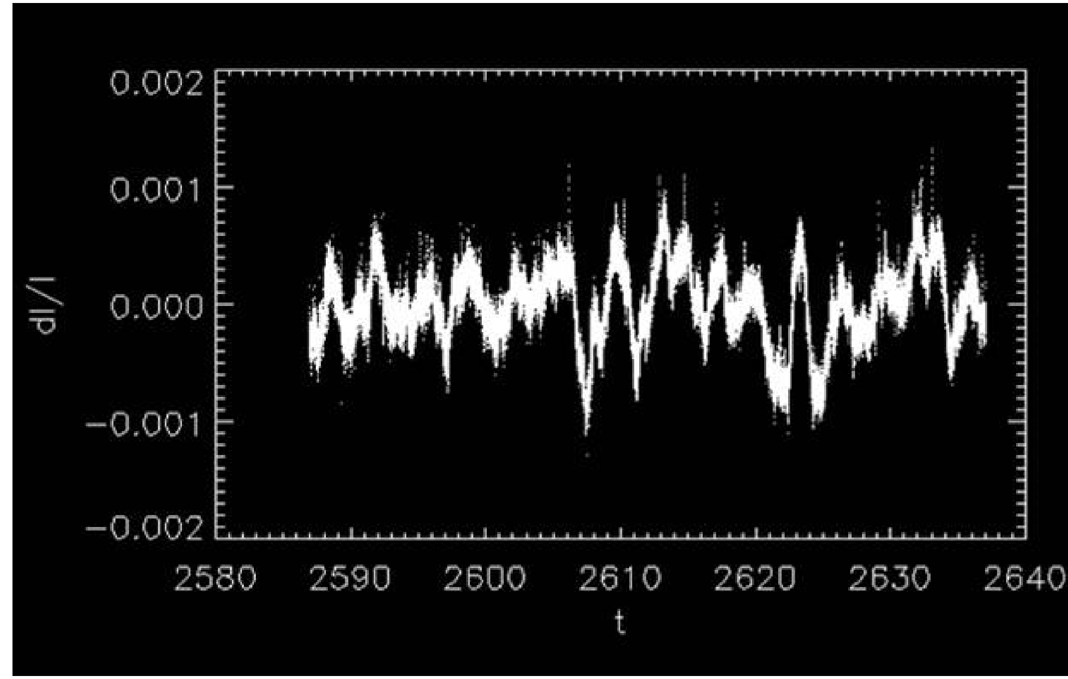
# Press release

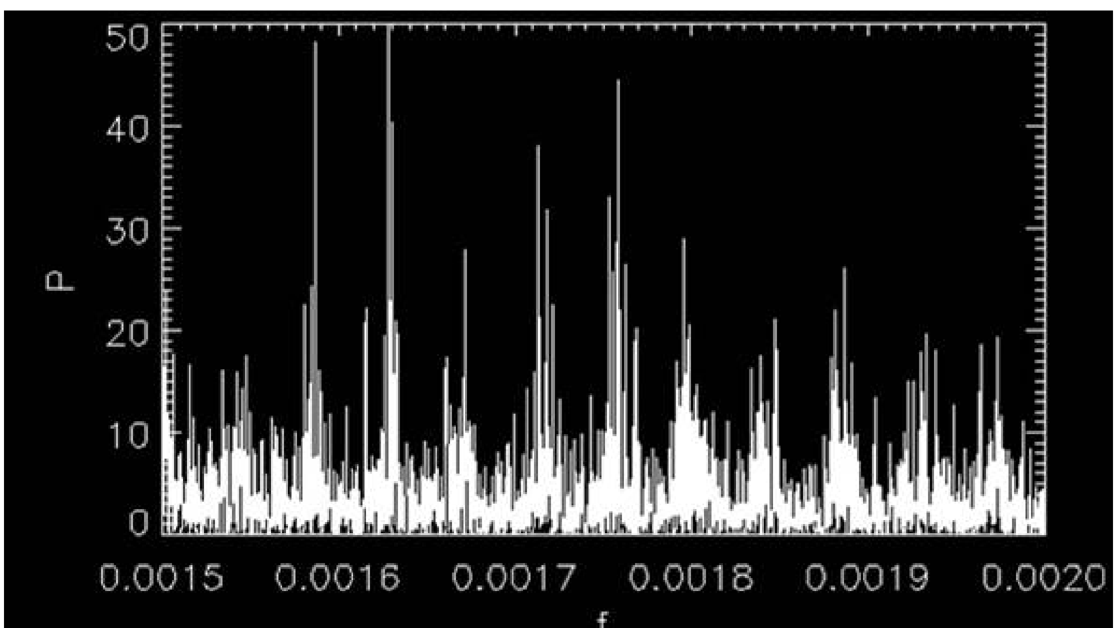
**ASTRONOMY**

## SUCCESS OF THE FIRST COROT SATELLITE OBSERVATION:
## FIRST EXOPLANET AND FIRST STELLAR OSCILLATIONS



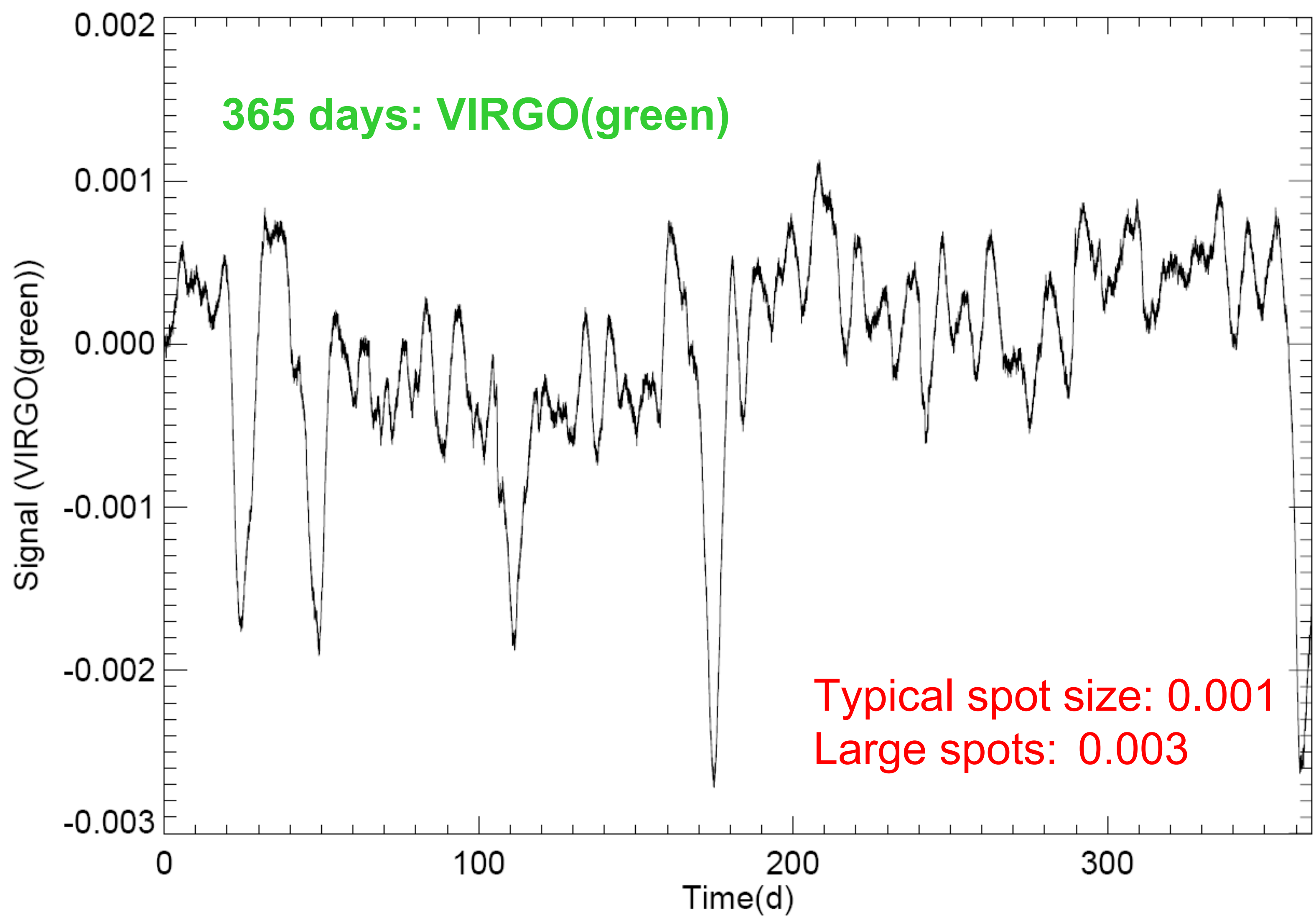
# Activity and Oscillations in a Solar-like star



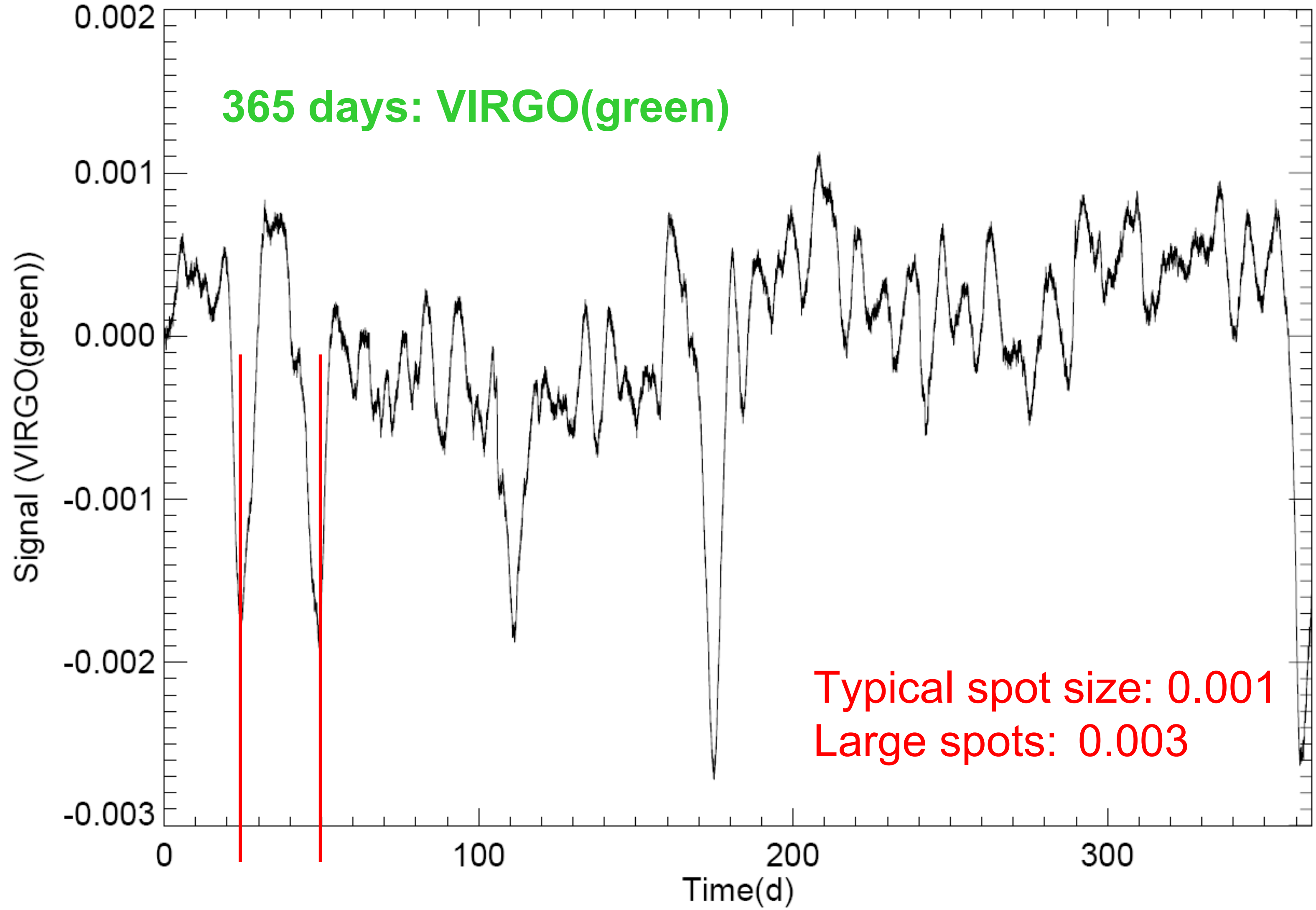
# Activity in the Sun:

dI/I



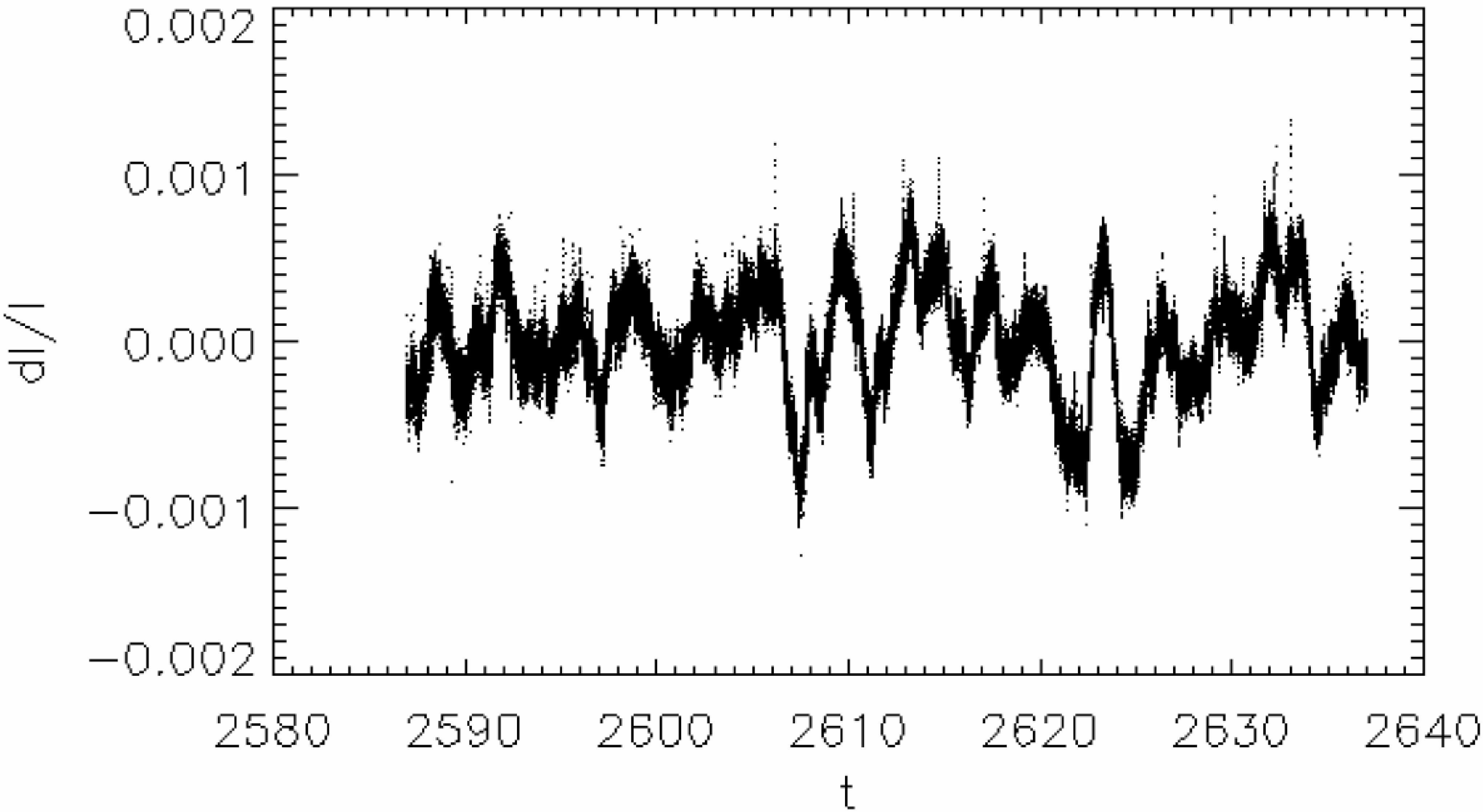
# Activity in the Sun:

dI/I



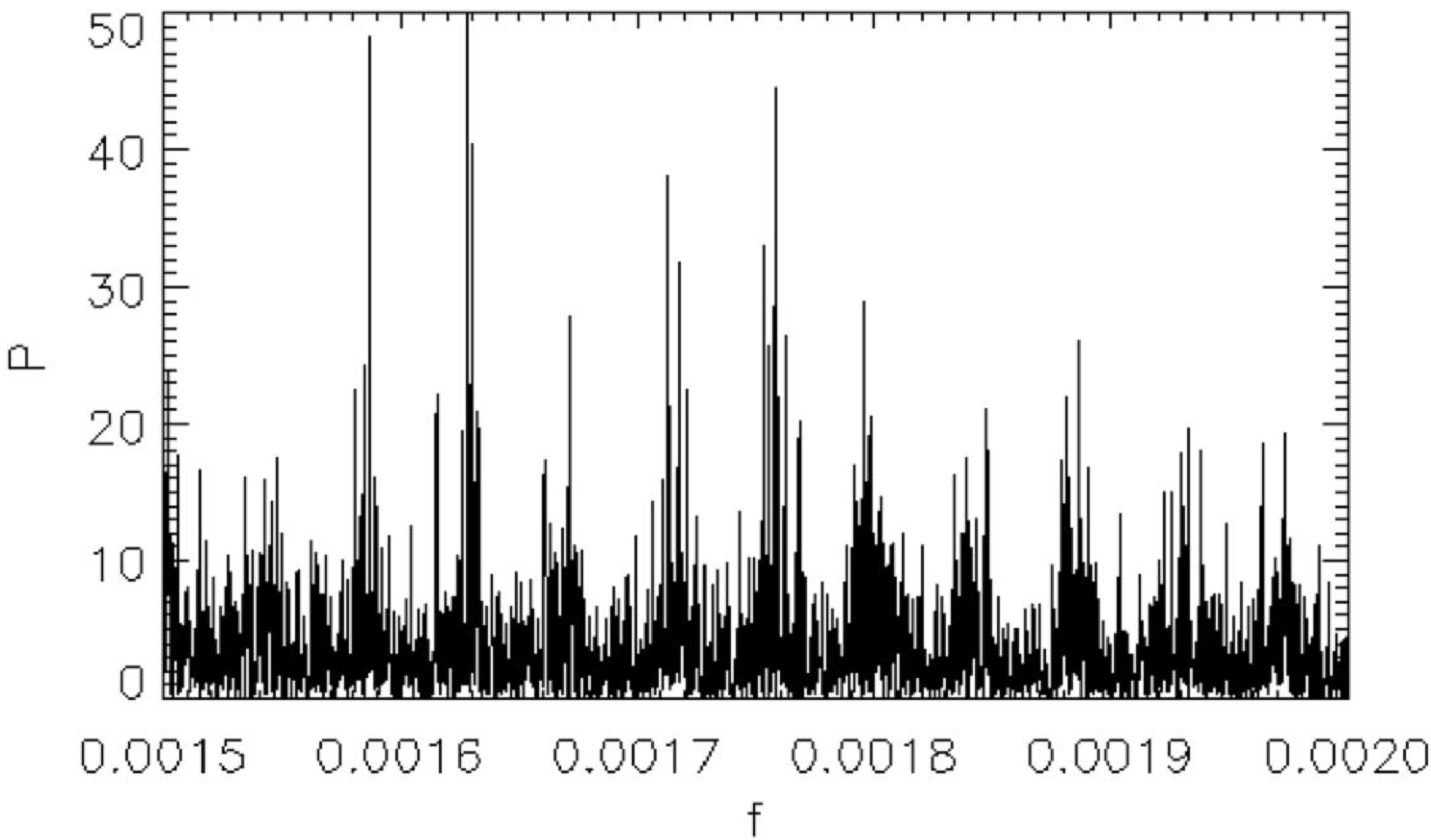

$P_{rot} \sim 24$ days

HD49933:

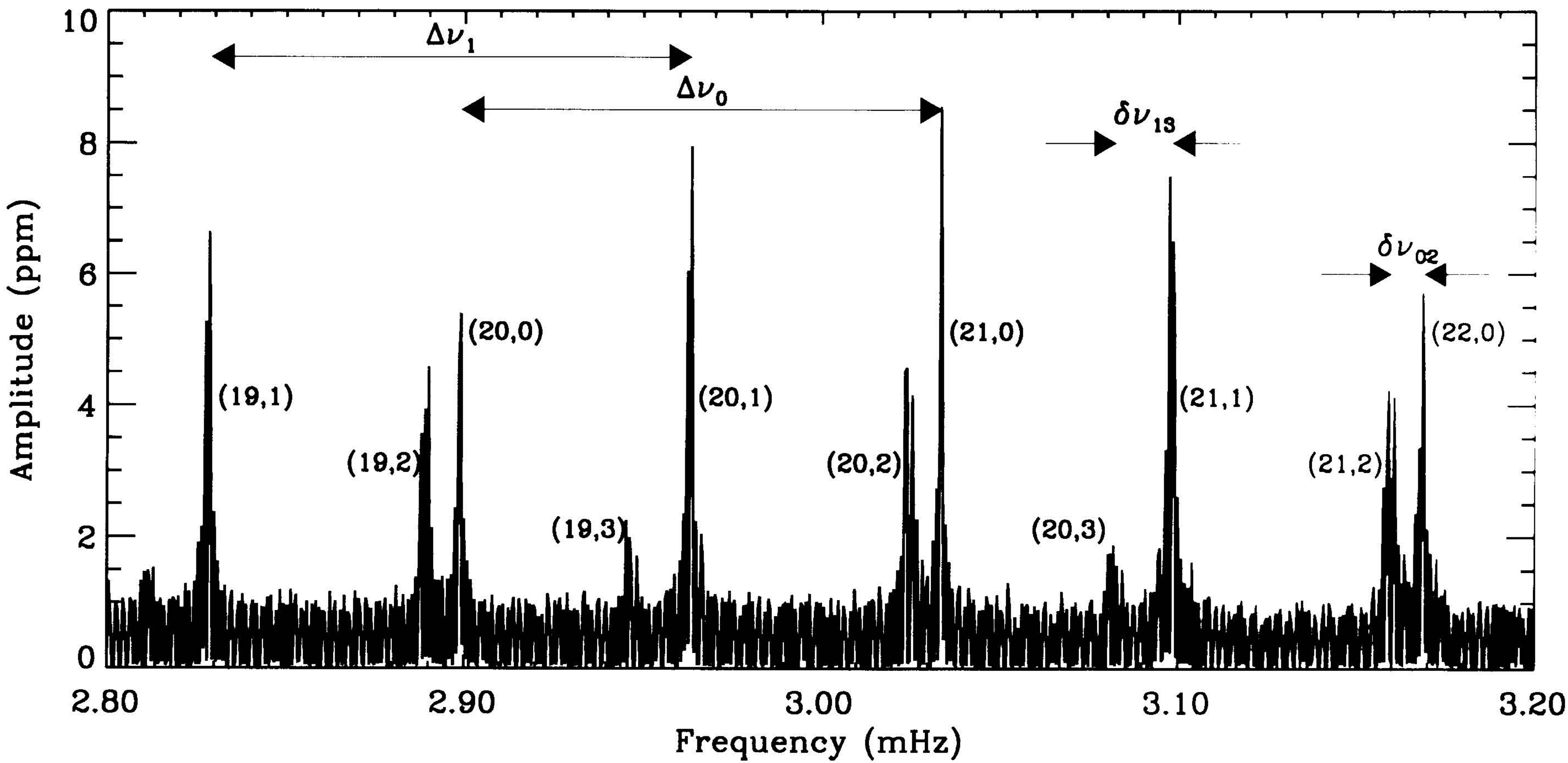


0.002
0.001
0.000
−0.001
−0.002
dI/I
2580
2590
2600
2610
2620
2630
2640
t

# HD49933: Dominant modes are l=0 and l=1



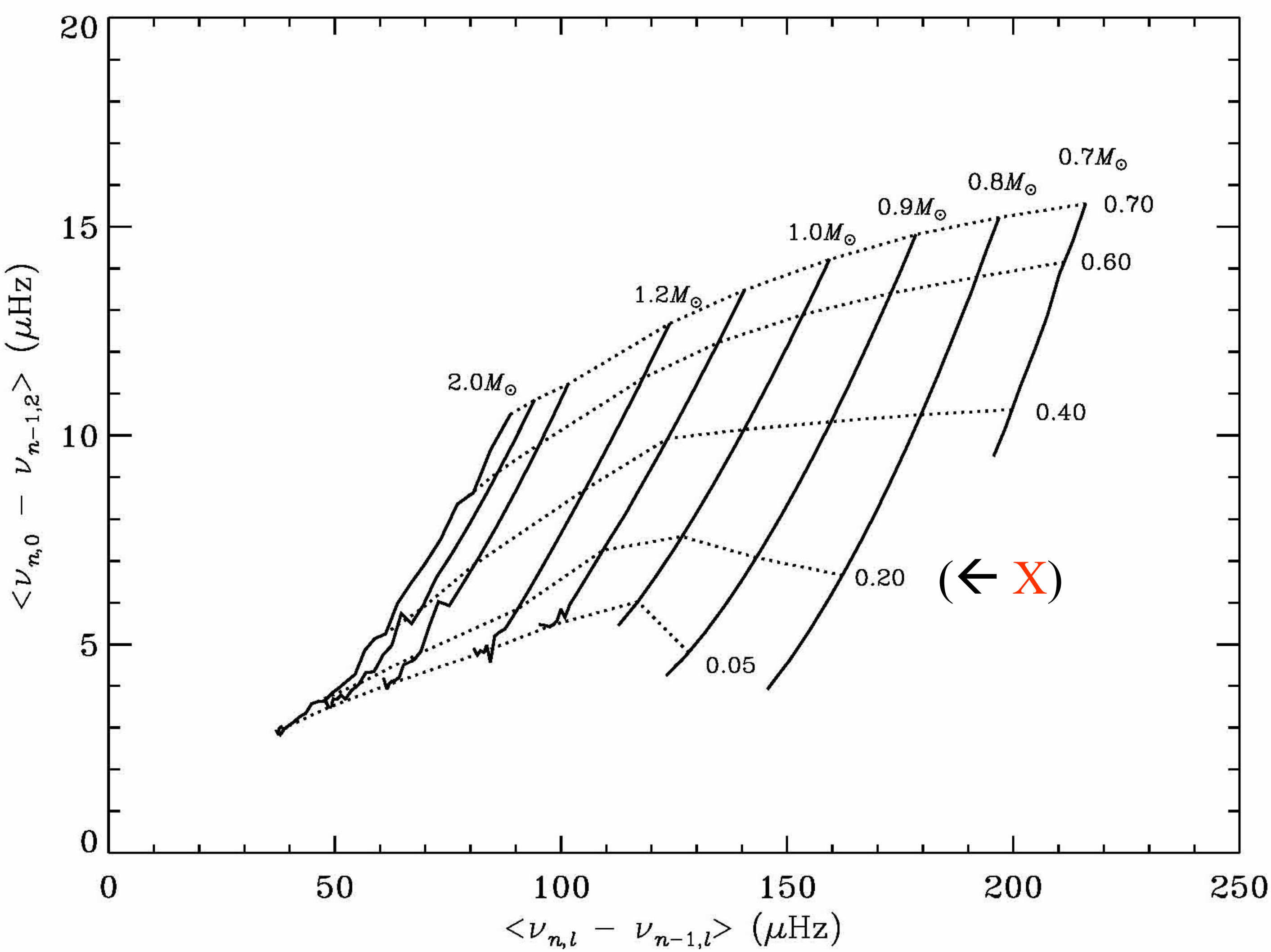
$$\nu_{n,l} = \Delta\nu\left(n + \frac{1}{2}l + \varepsilon\right) - l(l+1)D_0$$




$$D_0 = \frac{1}{6}\delta\nu_{02} = \frac{1}{2}\delta\nu_{01} = \frac{1}{10}\delta\nu_{13}$$

# Theoretical Separations from stellar models:

Small Separation ($\delta_{02}$)




Large Frequency Separation

# Theoretical Separations from stellar models:

$$T = \frac{10 - 14.3X}{M^3} \cdot 10^9\, yr$$

$$\Delta \nu_0 \approx 135 \mu Hz \sqrt{M / R^3}$$ (in Solar units)

$$\frac{\Delta \nu_0}{\Delta \nu_{Sun}} = \sqrt{\frac{\bar{\rho}}{\rho_{Sun}}}$$

# HD49933:

$$T = \frac{10 - 14.3X}{M^3} \cdot 10^9\, yr$$

$$V \sin(i) = 10.9\,\mathrm{km/s}$$

$$R \approx 1.5 R_{Sun} \rightarrow P_{rot} \approx 7.0\,\mathrm{d}$$

$$\Delta\nu_0 \approx 88.7\,\mu\mathrm{Hz}$$

$$\varepsilon \approx 1$$

$$\Delta\nu_0 \approx 135\,\mu Hz \sqrt{M / R^3}$$

$$\nu_{n,l} = \Delta\nu(n + \frac{1}{2} l + \varepsilon) - l(l+1) D_0$$

Identify the dominant modes in the spectrum

What is its Mass, age and Radius?
What is its rotation period and inclination?

dI/I ~ 0.001– 0.003 for Sunspots
Are the spots on HD49933 of similar size?