Mariehamn Summer School Compressible MHD Project Task

The HLL (Harten-Lax-van Leer) Riemann solver is one of the simplest general-purpose Riemann solvers. The HLL Riemann solver is very robust (which in case of MHD means that it does not produce negative pressure or density), is fast to compute and its implementation does not require any other knowledge except the ability to compute the flux function (i.e., there is no need for an eigenvalue solution as in e.g. the Roe solver).

The idea of the HLL solver is to compute rigorously the spatial average of the exact Riemann problem solution inside the Riemann fan, i.e. the region bounded by the fastest left and right-moving waves emanating from the initial condition discontinuity. Perhaps surprisingly, the spatial Riemann-fan average of the exact solution can be computed analytically as soon as the flux function is known. On physical grounds, any decent hyperbolic system has the property that a weighted average of physical states is a physical state. Thus the HLL intermediate state is physical i.e. it is guaranteed to have positive pressure and density. In case of MHD, this holds true for any Riemann problem whose left and right states obey the basic requirement that there is no jump in the interface-normal magnetic field component (because the existence of such a jump would violete Maxwell's third equation div(B)=0).

The HLL flux is

fHLL = (bR*fL - bL*fR + bL*bR*(uR-uL))/(bR-bL)

where bL and bR are the maximum left and right-moving wave speeds (typically bL<0 and bR>0), respectively, and fL and fR are the flux function values at the left and right input states. Estimates for bL and bR must be obtained separately. If the real maximum and minimum wave speeds are outside the range [bL,bR], the positivity of the pressure is no longer guaranteed by the HLL solver.

Task: Program the HLL solver for 1-D MHD equations, using first-order time integration and a uniform spatial grid. Develop reasonable estimates for the needed bL and bR. Remember to consider also special cases where both bL and bR
may have the same sign. Test your solver by the Sod shock tube problem and other similar problems where a single discontinuity is present in the grid in the initial state. Despite its large number of desirable robustness etc. properties, the HLL solver is not typically used in real-world simulations. Why? Contemplate the benefits and drawbacks of the HLL solver and try to invent good applications for it.

**Before the school**: Find out what is an HLL Riemann solver and what it is used for.