Kallio 20100530

Motion of a charged particle in an electromagnetic field

One important component in a hybrid code is a particle propagator, or a particle mover, which propagates the velocity and the position of a changed particle which feels the Lorenz force. Boris-Buneman algorithm is a commonly used algorithm to propagate ions in a numerical simulations. Study the motion of a proton in the solar wind by using (1) Boris-Buneman algorithm and compare the result to the exact (2) analytical solution.

You can assume the following:

- Magnetic field, $\mathbf{B}$, is a constant: $\mathbf{B} = (B_x, B_y, B_z) = (0, 0, 1)$ nT
- Electric field, $\mathbf{E}$, is a constant: $\mathbf{E} = -\mathbf{U}_{sw} \times \mathbf{B}$ where $\mathbf{U}_{sw}$ is the constant solar wind velocity of $(U_x, 0, 0) = (-400, 0, 0)$ km/s.
- Initially, at t=0, the ion is at the point $\mathbf{r}_o = (x_o, y_o, z_o) = (10000, 0, 0)$ km
- Use $\mathbf{v}_o = (v_{xo}, 0, 0)$ to start the propagation
- ion motion is non-relativistic

1. ----------- Task: Numeric Lorenz force integrator: Boris-Buneman -----------

1.1. Implement the Boris-Buneman algorithm to integrate ion path numerically. You can find the algorithm, for example, from the attached file (see p.1). Discussion about the properties of the algorism and its basic properties can be found, for example, at Hockney and Eastwood: Computer Simulation Using Particles (see pages 2-5 in the attached file).

1.2. Write a program which gives these three different type of plots:

1) Ion trajectory, $\mathbf{r} = \mathbf{r}(t)$, in the 3D space
2) Ion velocity, $\mathbf{v} = \mathbf{v}(t)$, in the 3D velocity space
3) Time-velocity plot: (t,v_x), (t,v_y), (t,v_z), $(t,|v|)$,

Derive these plots by using different time steps, dt, at least these time steps:

dt = 0.1 s, 0.5 s, and 1 s.

Use different initial starting velocities, such as:

$v_{xo} = 0, U_x/2, 2{*}U_x, 10{*}U_x$

Use, if possible, Matlab or Octave, but if that is not possible you can use other language.

1.3. Extra exercise: How the ion moves if you use dt = 5 s, 21 s, 100 s or even larger dt?

2. ----------- Task: Analytical (exact) solution -----------

2.1. Implement an algorithm which gives an analytical solution for the ion velocity, $\mathbf{v}=\mathbf{v}(t)$ and ion trajectory $\mathbf{r}=\mathbf{r}(t)$ for a given constant $\mathbf{E}$ and $\mathbf{B}$ field (A hint: You may want to consider to solve first $\mathbf{v}$ and $\mathbf{r}$ in the coordinate frame which moves along $\mathbf{U}_{sw}$. In that frame of reference $\mathbf{E} = 0$ and, therefore, the speed of ion is a constant.)

2.2. Compare the exact solutions with the numerical solutions which you derived by using Boris-Buneman algorithm in Task 1.2.

2.3. Extra exercise: Study the role of non-zero B_x. How the speed of the ion and its path in the 3D velocity space looks like if you assume that, say, $\mathbf{B} = (1,0,1)$ nT?