Comment on: "Mini-magnetospheric plasma propulsion: tapping the energy of the solar wind for spacecraft propulsion" by R. Winglee et al.

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1 Introduction

In a recent paper [Winglee et al., 2000] (henceforth W2000) propose an interesting new type of magnetic propulsion concept for accelerating spacecraft to high velocities by using only little fuel and energy. It has been known for a longer time that by setting up a strong artificial magnetic field around a spacecraft, the solar wind dynamic pressure excerts a force on the spacecraft, thus producing thrust; however, the magnetic field has to be quite strong in order to produce significant levels of thrust [Zubrin, 1993]. To remedy this, W2000 suggest that if one fills the artificial mini-magnetosphere with plasma, it will increase in size and thus form a larger obstacle for the solar wind flow, which supposedly increases the force that the solar wind excerts to the system. They also present innovative magnetohydrodynamic (MHD) simulation results that quantify their idea. This is an interesting and timely idea. However, the idea seems to be based on an incorrect way of calculating the force on the spacecraft. The only force that can actually push the spacecraft is the Lorentz force due to asymmetric magnetic field at the spacecraft. Estimation of the Lorentz force gives numbers which are up to ten orders of magnitude smaller than what are obtained in W2000. This casts doubt on the relevance of the results as a viable spacecraft engine.

2 Plasma-free mini-magnetosphere

By the plasma-free case we mean a spacecraft with artificial strong quasi-dipolar magnetic field [Zubrin, 1993]. In such a setting, the solar wind excerts a force on the magnetopause, which is the same force as that acting on the spacecraft because there is nothing else in the system that could take away additional momentum. The solar wind force $F_{\rm SW}$ is equal to the solar wind dynamic pressure $P_{\rm dyn}$ times the cross-sectional area of the magnetosphere, $F_{\rm SW} = R_{\rm MP}^2 P_{\rm dyn}$. Here $R_{\rm MP}$ is the subsolar distance of the magnetopause which is obtained from the MHD force balance condition $P_{\rm dyn} = B_{\rm MP}^2/\mu_0$ and $B_{\rm MP} = \mu_0 M/R_{\rm MP}^3$ where M is the dipole moment,

$$R_{\rm MP} = \left[\frac{\mu_0 M^2}{P_{\rm dyn}}\right]^{1/6}. \qquad (1)$$

For simplicity, we drop all numerical factors of order unity and use the equality sign to significy proportionality.

Alternatively, one may calculate the force as the Lorentz force acting on an idealized dipole representing the spacecraft magnet. It is easy to show that the Lorentz force acting on a z-directed dipole is $F = M dB_z/dx$ where B_z is the z

component of the magnetic field produced by all other current systems in the artificial magnetosphere except the dipole itself, and x is sunward. The most important contributor to B_z is the dayside magnetopause current sheet, which produces a magnetic field $B_{\rm MP} = \sqrt{\mu P_{\rm dyn}}$. The derivative of this field at the spacecraft is estimated as $B_{\rm MP}/R_{\rm MP}$, because $R_{\rm MP}$ is the scale size of the current system. Thus we obtain

$$F = M\frac{B_{\rm MP}}{R_{\rm MP}} = \mu_0^{1/3}(MP_{\rm dyn})^{2/3}. \tag{2}$$

On the other hand the force excerted by the solar wind on the magnetosphere is

$$F_{\rm SW} = R_{\rm MP}^2 P_{\rm dyn} = \mu_0^{1/3}(MP_{\rm dyn})^{2/3}, \tag{3}$$

i.e. $F = F_{\rm SW}$ as expected: the momentum transferred from the solar wind (i.e. the force acting on the magnetopause) is the same as the Lorentz force acting on the spacecraft. This must be so because there is nothing else in the system that could carry away momentum.

3 Plasma-filled mini-magnetosphere

If the magnetosphere is filled with plasma, the mini-magnetosphere effectively turns into a "mini-heliosphere", where the magnetic field decays slower than $1/r^3$ as was pointed out by W2000. Consequently, the magnetosphere will become larger ($R_{\rm MP}$ increases) and so the force $F_{\rm SW}$ excerted by the solar wind to the magnetopause also increases. W2000 assumed that the force acting on the spacecraft is essentially same as $F_{\rm SW}$. While this was true in the plasma-free case as we pointed out above, this is unfortunately no longer necessarily so in the plasma-filled case, since the escaping plasma may transfer away part of the momentum. However, also in the presence of plasma it is still so that the only force that can accelerate the spacecraft is the Lorentz force that a dipole residing in an inhomogeneous magnetic field feels. Using the same argumentation as in the plasma-free case, the Lorentz force F is again given by

$$F = M\sqrt{\mu_0 P_{\rm dyn}}/R_{\rm MP} \tag{4}$$

where $R_{\rm MP}$ is the size of the magnetosphere. Thus the force acting on the spacecraft will rather diminish if the magnetosphere is made larger, which is contrary to what W2000 asserts. The reason is that the magnetospheric current systems (especially the dayside magnetopause) will be farther away from the spacecraft and thus the spatial gradient of its magnetic field at the spacecraft location is smaller (the magnitude of the magnetopause magnetic field $B_{\rm MP}$ depends only on the solar wind dynamic pressure $P_{\rm dyn}$ and thus is the same in the plasma-free and plasma-filled cases).

One might argue that in the plasma-filled case the magnetopause current system is perhaps not the main contributor to gradients of B at the spacecraft. While this may be partly true, it is difficult to see how the net force could change qualitatively. In order to produce a strong force, significant currents should reside very close to the spacecraft in a non-symmetrical fashion (for example, a ring current type symmetric current would produce zero net force). Since the only source of day/night asymmetry is the solar wind, estimating dB_z/dx as $B_{\rm MP}/R_{\rm MP}$ is a plausible order of magnitude estimation also in the plasma-filled case.

4 Numerical estimates

In the example given in W2000, the magnetic field of 0.015 T at 10 cm distance from the nominal center is used, which corresponds to $M = 150$ A m^2 dipole moment.

In the plasma-free case, using Eq. (2) we find that such a dipole would generate a thrust of $\sim 10^{-6}$ N, and the size of the magnetosphere would be $R_{\rm MP} \sim 10$ m. In the plasma-filled case, if we assume that the system size will increase to $R = 20$ km as in W2000, the force excerted by the solar wind on the magnetopause is $F_{\rm SW} = \pi R^2 P_{\rm dyn} = 2.5$ N (we use $P_{\rm dyn} = 2$ nPa as in W2000), which is in agreement with the estimate of W2000 which is 2.5-5 N. Using Eq. (4), however, we find that the Lorentz force acting on the spacecraft is only $\sim 4 \times 10^{-10}$ N, i.e. ten orders of magnitude smaller than $F_{\rm SW}$.

To see in another way where the problem lies, consider that according to W2000 the mass flux of the plasma emitted from the spacecraft is 7.5×10^{-6} kg s^{-1}. The escaping plasma is finally accelerated to the solar wind velocity 450 km/s. This requires a force which is given by $7.5 \times 10^{-6} \times 4.5 \times 10^5$ kg m s^{-2} = 3.4 N, which is of the same order as $F_{\rm SW}$ which was 2.5-5 N according to W2000. Thus the force taken by the escaping plasma is comparable in magnitude to the force tapped from the solar wind but of opposite sign, so that the remainder force that can be used to accelerate the spacecraft is smaller (and indeed very small, as we showed above).

It would be possible and straightforward for the authors of W2000 to compute the force acting on the spacecraft by evaluating their simulated dB_z/dx at the spacecraft location, e.g. by first computing the current density by numerical differentiation in their MHD simulation box and then using Biot-Savart numerical integration in a stencil of points around the origin to be able to evaluate dB_z/dx there. Thus far they have only computed the force $F_{\rm SW}$ acting on the magnetopause (by several different methods [Winglee, 2002, private communication]). It would be interesting to see what the calculation of $F = M dB_z/dx$ would give if applied to the MHD simulation results of W2000.

5 Conclusion

In our view, W2000 contains a very interesting idea and has already proved its value because it has revived interest in alternative propulsion techniques and has also lead to suggestions of using an artificial magnetosphere for plasma research purposes. However, it seems to us that the authors of W2000 base their estimates on solar wind thrust on the force which is not excerted on the spacecraft but acts on the magnetopause: the majority of the transferred momentum goes into accelerating the escaping plasma and only a negligible part goes into accelerating the spacecraft, since we showed that the Lorentz force acting on the spacecraft is actually smaller if the magnetosphere is made larger by inserting plasma. In other words, one cannot increase the thrust by introducing plasma in an artificial magnetosphere. While our argument here is based on rough estimation only, the difference between the force as estimated by us and that arrived at by W2000 is easily about ten orders of magnitude. Such a large gap cannot be attributed to the numerical factors that we for simplicity dropped from our analysis.

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References

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