Elektrodynamics, spring 2003

Exercise 1 (Thu 30.1., Fri 31.1.)

1. Prove the following useful vector identities:
$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$
$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = (\nabla \times \mathbf{A}) \cdot \mathbf{B} - (\nabla \times \mathbf{B}) \cdot \mathbf{A}$$
$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$
Tip: It is worth learning to use the permutation symbol (Levi-Civita symbol). For example, $(\mathbf{A} \times \mathbf{B})_i = \epsilon_{ijk} A_j B_k$, where summation goes through indices appearing twice. An especially useful result is $\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$. Note also that using the short notation, $\mathbf{A} \cdot \mathbf{B} = A_i B_i$.

2. Let
$$\mathbf{A}(\mathbf{r}) = \int_{R^3} \frac{\mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \, dV'$$
On which physically reasonable mathematical conditions it holds that
$$\nabla \cdot \mathbf{A}(\mathbf{r}) = \int_{R^3} \frac{\nabla' \cdot \mathbf{J}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \, dV'$$

3. About half of the mass of a human body consists of protons. Let us remove one percent of electrons of a human being. Set two such persons of 70 kg at a distance of one meter from each other.
a) How large is the repulsion between them?
b) Make the result understandable in some illustrative way.

4. A sphere of radius R is uniformly charged so that the total charge is Q. Inside this sphere there is a hollow space, which is also a sphere of radius a and whose centre is at distance d from the centre of the larger sphere ($R \geq d + a$). Calculate the electric field everywhere.

5. A hollow conducting sphere of radius one meter is uniformly charged until the electric field at the surface is 100 kV/m. Voltage between the surface and the centre is measured by a voltmeter whose sensitivity is 1 μV. The result is zero volts. What limits are obtained for the parameter λ ($|\lambda| << 1$) if Coulomb's law were as $r^{-2-\lambda}$, and the superposition principle is assumed to be valid?

Return answers until Tuesday 28.1. at 14 o'clock (electrodynamics box on the 2nd floor). Exercise times: Thu 8-10 (E206), Thu 14-16 (E206), Fri 10-12 (D114). The teaching language is Finnish, but you may present your contribution in English.