Electrodynamics, spring 2003

Exercise 3 (Thu 13.2., Fri 14.2.)

1. Assume that function φ is harmonic in a volume containing a sphere B of radius R centred at $\mathbf{r}$. Show that
$$\varphi(\mathbf{r}) = \frac{1}{4\pi R^2} \int_{\partial B} \varphi(\mathbf{r}')dS'$$
where ∂B is the surface of the sphere. Green III may be useful:
$$\begin{aligned} \varphi(\mathbf{r}) &= -\frac{1}{4\pi} \int_V dV' \frac{\nabla^2 \varphi(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \\ &\quad + \frac{1}{4\pi} \oint_S dS' \left[\frac{1}{|\mathbf{r}-\mathbf{r}'|} \frac{\partial \varphi(\mathbf{r}')}{\partial n'} - \frac{\partial}{\partial n'} \left(\frac{1}{|\mathbf{r}-\mathbf{r}'|} \right) \varphi(\mathbf{r}') \right] \end{aligned}$$
where $\partial/\partial n' = \mathbf{n}' \cdot \nabla'$.
2. Two parallel conducting planes are located at $z = 0$ ja $z = d$. Between them there is a uniform insulating body whose cross-section in the xy-plane is arbitary. The height of the body is d and its permittivity is ϵ. The planes are at potentials V_1 and V_2. Calculate the electric field and the electric displacement between the planes. Note: the shape of the cross-section is not essential.
3. The inner radius of a coaxial cable is a. It is surrounded by insulating material (relative permittivity ϵ_r) to radius b. Next there is an air gap to the outer conductor at radius c. The voltage between the inner and outer conductors is V. What is the largest electric field in this system?
4. An insulating sphere of radius R has a permanent constant polarisation $\mathbf{P}_0$, and there are no other sources of the electric field. Calculate $\mathbf{E}$ and $\mathbf{D}$ inside and outside of the sphere (the material outside is air). The most straighforward method is evidently a direct integration of the polarisation charge density, with the help of spherical harmonics. Another possibility is to note that potential satisfies Laplace equation, which is then to be solved.
5. In a regular cubic lattice there are molecules with identical dipole moments. Show that at each lattice point the electric field due to other dipoles vanishes. Assume a general dipole moment vector with all three components.

Return answers until Tuesday 11.2. at 12 o'clock.

Exercises on Thursday mornings since 6.2. will start at 8.30.