Electrodynamics, spring 2003

Exercise 4 (Thu 20.2., Fri 21.2.)

1. Thunderstorms maintain a negative charge at the earth's surface so that the vertical electric field at the surface is about 100 V/m.
   a) How long could the energy stored at the surface produce a power of 14 GW, which is the highest value ever measured in the Finnish power grid (January 2003)?
   b) Why do we not get electric shocks, although the field is quite large?
2. Two point charges q_1 and q_2 are separated by a distance d. The energy of the total field is proportional to the quantity $E(\mathbf{r})^2 = E_1(\mathbf{r})^2 + E_2(\mathbf{r})^2 + 2\mathbf{E}_1(\mathbf{r}) \cdot \mathbf{E}_2(\mathbf{r})$. E_1^2 and E_2^2 integrated over the whole space clearly diverge (the infinite self-energy can be properly handled in quantum electrodynamics). Show that the interaction term $2\mathbf{E}_1 \cdot \mathbf{E}_2$ yields the desired result. Tip: $\mathbf{E} = -\nabla\varphi$ and suitable integration theorems.
3. Complete the equivalence of the electrostatic volume force density and the surface force density concerning torque. In other words, show that

   $$\int_V \mathbf{r} \times \mathbf{f} \; dV = \int_S \mathbf{r} \times \mathbf{f}^S \; dS$$

   where S is the boundary of V and

   $$f_i = \sum_{j=1}^{3} \partial_j T_{ij}, \; f_i^S = \sum_{j=1}^{3} T_{ij} n_j, \; T_{ij} = \epsilon_0 (E_i E_j - \frac{1}{2}\delta_{ij}\mathbf{E}^2)$$

4. Calculate the pressure on a uniformly charged spherical shell. The pressure on a surface element da is the normal component of the force on the element divided by its area. The easiest solution is based on the use of Maxwell's stress tensor.
5. Consider a pair of metal plates (distance d) between which the voltage is V. Assume that an unlimited number of electrons can leave the cathode (zero potential) and go to the anode. Finally, a steady state is reached when the charge density is independent of time. Solve the electric field and the charge density between the plates in the steady state using the continuity equation and Poisson's equation. Show also that Ohm's law is not valid.

Return answers until Tuesday 18.2. at 12 o'clock.

Exercises on Thursday mornings start at 8.30.