Electrodynamics, spring 2003

Exercise 5 (Thu 27.2., Fri 28.2.)

1. Current I flows in a wire along the z axis to the positive direction from $-\infty$ to 0. At the origin, the current diverges radially and uniformly on the xy plane. Calculate the magnetic field ($\mathbf{B}$) everywhere. Tip: Ampere's law in integral form.
2. Calculate the vector potential and the magnetic field due to two infinitely long parallel wires carrying opposite currents $\pm I$. The distance between the wires is a. Consider also the limiting case with $a \to 0, I \to \infty$ so that aI = constant. Tip: compare to a uniformly charged wire. (It is also possible to solve this problem by brutal calculations.)
3. Consider the magnetic dipole field in spherical coordinates. The dipole axis is the z axis, so the field does not depend on the azimuth angle ϕ.
   a) The field lines are defined by a differential equation $\mathbf{dl} \times \mathbf{B} = 0$, where $\mathbf{dl}$ is a small displacement along the field line. Derive the expression of the field line in the form $r = f(\theta)$.
   b) The geomagnetic field can be resonably modelled by setting a dipole at the Earth's centre. How large is the field in the (magnetic) north pole when it is known to be 30000 nT at the surface at the geomagnetic equator? How about Finland (magnetic latitudes 57-67)?
4. Assume that a stationary current density $\mathbf{J}(\mathbf{r})$ is non-zero only inside a sphere of radius R. Show that
   $$\int_{r \leq R} \mathbf{B}(\mathbf{r}) dV = \frac{2}{3} \mu_0 \mathbf{m}$$
   where $\mathbf{m}$ is the magnetic dipole moment of the current system. You may need the formula
   $$\int_{4\pi} \frac{d\Omega'}{|\mathbf{r} - \mathbf{r}'|} = \frac{4\pi}{max(r, r')}$$
5. Derive in detail the expression that gives the magnetic field due to a magnetised material using the scalar potential ψ and magnetisation $\mathbf{M}$ ($\mathbf{B}(\mathbf{r}) = -\mu_0 \nabla \psi(\mathbf{r}) + \mu_0 \mathbf{M}(\mathbf{r})$). Start from the vector potential
   $$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{M}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \, dV'$$
   Some guidelines are given in lecture notes.

Return answers until Tuesday 25.2. at 12 o'clock.

Possibly useful observations about exercises (in Finnish):
http://www.geo.fmi.fi/~viljanen/ed2003/harjoituksista2003.html