

Electrodynamics, spring 2003

Exercise 7 (Thu 13.3., Fri 14.3.)

1. The geomagnetic dipole field at the earth's surface at the (magnetic) equator is $30 \mu\text{T}$.
 - a) Calculate the total energy of the field outside of the Earth.
 - b) Illustrate the result by comparing it to some everyday example.
2. A square-shaped circuit (side a) moves with a constant velocity $\mathbf{v}$ into the x -direction. In the region $x > 0$ there is a uniform magnetic field $B_0 \mathbf{e}_z$ transverse to the plane of the circuit, and in the region $x < 0$ there is no field. Two sides of the circuit are parallel to the x -axis (and two others parallel to the y -axis).
 - a) Calculate the induced current in the circuit as a function of time if the circuit enters the field at $t = 0$. The resistance of the circuit is R and inductance L .
 - b) The circuit is completely inside the field at $t = a/v$. Show that the stored magnetic energy will be dissipated as ohmic losses at $t > a/v$.
3. Show that the product of the self-inductances L_1 and L_2 of two circuits is always at least as large as the square of their mutual inductance M : $L_1 L_2 \geq M^2$. Tip: prove first that the energy of the system is always positive.
4. Part of a thin paramagnetic slab is inside a uniform magnetic field $\mathbf{B}$ and another part is outside. The area of the cross-section of the slab is A and susceptibility is χ_m . Show that the magnetic force on the slab is

$$F = \frac{B_0^2 \chi_m A}{2\mu_0(1 + \chi_m)}$$

Calculate the magnitude of the force for titanium ($\chi_m = 1.8 \cdot 10^{-4}$), if $A = 1 \text{ cm}^2$ and $B = 0.25 \text{ T}$.

5. A current distribution $\mathbf{j}_0(\mathbf{r})$ creates a magnetic field $\mathbf{B}_0$ in an otherwise empty space. Then a body with permeability μ is set into the space. Assuming that $\mathbf{j}_0(\mathbf{r})$ is not affected, show that the change in the magnetic energy is $\int \frac{1}{2} \mathbf{M} \cdot \mathbf{B}_0$, where $\mathbf{M}$ is the magnetisation of the body, and the integration region contains only the body. Tip: an analogous electrostatic situation.

Return answers until Tuesday 11.3. at 12 o'clock.

The first exam (välikoe) is on Friday, March 21, at 9.00-13.00 (in D101).

Material: sections 1-8 of lecture notes and exercises 1-7.

Lecture on Thursday 13.3. is exceptionally in E207. During the exam week, there are no exercises, but lectures are at their normal times. On the lecture of Thursday 20.3., we will go through what we have learned in sections 1-8 of the lectures.

Misprint catalogue (in Finnish) is available in the library copy of lectures (also: <http://www.geo.fmi.fi/~viljanen/ed2003/>).