Electrodynamics, spring 2003

Exercise 12 (Fri 2.5.)

1. Starting from the field tensor $(F^{\alpha\beta})$ expressed in terms of the electric and magnetic fields, show that the homogenous Maxwell equations can be written as

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0$$

2. Define a 4-component fully antisymmetric permutationsymbol ϵ so that $\epsilon^{0123} = 1$ and $\epsilon^{\alpha\beta\gamma\mu}$ is antisymmetric with respect to a change of any pair of indices. Using this, define the components of the dual field tensor $G^{\alpha\beta}$ by

$$G^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\mu\nu}F_{\mu\nu}$$

a) Calculate these components and compare them to the components of $F^{\alpha\beta}$.
b) Show that the homogenous Maxwell equations are

$$\partial_\beta G^{\alpha\beta} = 0$$

3. Show that the quantities $\mathbf{E}\cdot\mathbf{B}$ and $\mathbf{E}^2 - c^2\mathbf{B}^2$ are Lorentz-invariant.
4. In an infinitely long straight thin wire the linear charge density is constant (λ). An observer moves with a constant velocity v with respect to the wire and parallel to it at the distance d. Calculate the field seen by the moving observer.
5. The generalised Maxwell stress tensor is

$$T^{\nu\mu} = \frac{1}{\mu_0}(F_\alpha{}^\nu F^{\alpha\mu} - \frac{1}{4}g^{\nu\mu}F_{\alpha\beta}F^{\alpha\beta}) = T^{\mu\nu}$$

a) Show that

$$\partial_\nu T^{\nu\mu} = \frac{1}{\mu_0}(\partial_\nu F_\alpha{}^\nu)F^{\alpha\mu} = f^\mu$$

where f is the density of the four-force.
b) Show that $T^{0i} = -S^i/c$, where $\mathbf{S}$ is Poynting's vector.

Return answers until Tuesday 29.4. at 14 o'clock.

Note. Due to Vappu (1.5.), the only exercise group is on Friday!