Electrodynamics, spring 2008
Exercise 1 (Thu 24.1., Fri 25.1.; Friday group in English)

1. Prove the following useful vector identities:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = (\nabla \times \mathbf{A}) \cdot \mathbf{B} - (\nabla \times \mathbf{B}) \cdot \mathbf{A}$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

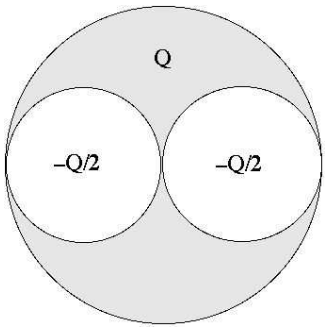
   Tip: It is worth learning to use the permutation symbol (Levi-Civita symbol). For example, $(\mathbf{A} \times \mathbf{B})_i = \epsilon_{ijk} A_j B_k$, where summation goes through indices appearing twice. An especially useful result is $\epsilon_{ijk}\epsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$. Note also that using the short notation, $\mathbf{A} \cdot \mathbf{B} = A_i B_i$.

2. Two spherical water droplets collide. What is the electric field and potential at the surface of the resulting droplet? Before the collision, the radii were R_1 ja R_2 and charges Q_1 ja Q_2, respectively.

3. A point charge q is put into a cavity inside a neutral ungrounded conductor.
   a) Determine qualitatively the electric field in the cavity, in the conductor and outside of the conductor. What are the total charges at the surface of the cavity and at the outer surface of the conductor?
   b) How does the situation change if the conductor is connected to the ground?

4. Charge Q is uniformly distributed at the surface of a sphere (radius R). Outside of the sphere, there is such a charge density $\rho = \rho(r)$ that the amplitude of the electric field is constant. Determine $\rho(r)$.

5. Two uniformly charged spheres (charge $-Q/2$, radius $R/2$) are inside a larger sphere (radius R) as shown in the figure. The rest of the larger sphere is also uniformly charged (charge Q). Determine the leading behaviour of the electric field far away from this charge distribution.



Return the answers until Tuesday 22.1. 12 o'clock.