

Electrodynamics, spring 2008

Harjoitus 4 (14.2., 15.2.; Friday group in English)

1. There are two point charges q_1 and q_2 at the distance d from each other. The energy density of the total field is proportional to $E(\mathbf{r})^2 = E_1(\mathbf{r})^2 + E_2(\mathbf{r})^2 + 2\mathbf{E}_1(\mathbf{r}) \cdot \mathbf{E}_2(\mathbf{r})$. The contribution of E_1^2 and E_2^2 diverges when integrated over the whole space (this problem can be handled in QED). Show that the interaction term $2\mathbf{E}_1 \cdot \mathbf{E}_2$ provides the desired result. Tip: $\mathbf{E} = -\nabla\varphi$ and suitable integration formulas.
2. Determine the pressure on a uniformly charged spherical shell. Tip: the pressure on a surface element da is equal to the normal component of the force on the element divided by its area.
3. A lightning stroke hits the ground whose conductivity is $10^{-3} \Omega^{-1}m^{-1}$. Assuming that the current is uniformly distributed in the half-space, determine the voltage between two points at a distance of 1 m from each other, when the distance from the stroke is a) 100 m b) 1 km. Assume a direct current whose amplitude is 10 kA (a typical value in Finland, although even more than 100 kA is possible).
4. The inner and outer radii of a hollow spherical conductor are R_1 and R_2 , respectively. Determine the resistance between the inner and outer surfaces, when the conductivity is σ .
5. Show by a direct calculation that the magnetic flux density expressed by Biot and Savart law (Eq. 5.30 in lectures) is divergence-free.
6. Extra problem (one point): The wife of a "lighthouse" asked from Kirsti (the omniscient wizard of Helsingin Sanomat, the largest Finnish newspaper) the following question: "Does a 40 W glow lamp consume more energy than a 20 W lamp? If I changed the four 40 W lamps in my chandelier to smaller ones would I notice that in my electricity bill? What if I just removed one lamp and used three of them? My husband claims that the consumption would be equal for both 4 and 3 lamps, because in the case of 3 lamps the rest would have to do the work of the missing lamp too. So the 3 lamps would burn out faster." Advise the wife and enlighten her husband.

Return the answers until Tuesday 12.2. 12 o'clock.