Electrodynamics, spring 2008
Exercise 7 (27.3., 28.3.; Friday group in English)

1. Let's practice vector calculus. Derive Green's formulas I-III starting from the divergence theorem (page 34 in the lecture notes).

2. The geomagnetic dipole field at the Earth's surface at the (magnetic) equator is 30 μT.
   a) Calculate the total energy of the field outside of the Earth.
   b) Illustrate the numeric value by some understandable way.

3. A current distribution $\mathbf{J}_0(\mathbf{r})$ creates a magnetic field $\mathbf{B}_0$ in an otherwise empty space (permeability μ_0). A magnetising body (permeability μ) is set under the influence of this field. Assume that $\mathbf{J}_0(\mathbf{r})$ remains unchanged. Show that the change of the magnetic energy is $\int \frac{1}{2}\mathbf{M} \cdot \mathbf{B}_0$, where $\mathbf{M}$ is the magnetisation of the body and the integration volume contains only the body. Tip: electrostatic analogy.

4. There are two identical lamps (resistance R) connected in parallel. In series with another lamp, there is a coil whose inductance is L and whose resistance is negligible.
   a) A DC voltage V is switched on at $t = 0$. Calculate the currents through the lamps as a function of time.
   b) After a long time, the voltage is switched off. Calculate the currents after the disconnection. Show that the magnetic energy is dissipated as Ohmic losses.

5. Two parallel circular plates form a capacitor. It is filled by a material whose permittivity is ϵ and conductivity σ. The initial charges at the plates are $\pm Q$. Determine the charge of the capacitor as a function of time. What is the time constant of discharging for quartz ($\epsilon = 4.3\epsilon_0, \sigma = 10^{-13}\ \Omega^{-1}m^{-1}$)? Calculate the magnetic field inside the capacitor. Show that the energy dissipated in the Joule heating equals the original electrostatic energy.

Return the answers until Tuesday 25.3. 12 o'clock.
Due to the Easter time, there are no lectures on Thursday 20.3. and Monday 24.3.
The study group is held on Monday 17.3.