Electrodynamics, spring 2008
Exercise 8 (3.4., 4.4.; Friday group in English)

1. A stationary current I flows along a straight long wire with a circular cross-section (radius a). Show that the energy flux into the wire equals to Ohmic losses, when the conventional Poynting vector $\mathbf{E} \times \mathbf{H}$ is applied.

2. Show that $\mathbf{E} \times \mathbf{H} + \nabla \times (\varphi \mathbf{H})$ is also an acceptable Poynting vector in problem 1, where φ is the electrostatic potential. Consider again the energy flux.

3. There is an infinitely long wire along the z-axis with a uniform line charge density $-\lambda$. It is surrounded by an insulating cylinder (radius a), whose moment of inertia per unit length is j. The surface charge density of the cylinder is $\lambda/(2\pi a)$. The cylinder is initially at rest in an external uniform magnetic field $\mathbf{B}_0 \mathbf{e}_z$. At $t = 0$ the external field starts to vanish slowly, and finally it is zero. What is then the angular velocity of the cylinder? (Neglect radiation losses and friction.) Note that the final total magnetic field is non-zero, although there is no more any external field.

4. Show by a careful direct differentiation that

$$\varphi(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\mathbf{r}', t - |\mathbf{r} - \mathbf{r}'|/c)}{|\mathbf{r} - \mathbf{r}'|} \, dV'$$

   fulfills the wave equation of the scalar potential.

5. a) Show that a possible gauge condition of the vector potential is $\mathbf{r} \cdot \mathbf{A} = 0$.
   b) Express $\mathbf{A}$ by using the magnetic flux density $\mathbf{B}$ in this gauge. Tip: consider $\mathbf{r} \times \mathbf{B}$.

Return the answers until Tuesday 1.4. 12 o'clock.