Electrodynamics, spring 2008
Exercise 9 (10.4., 11.4.; Friday group in English)

1. a) Show that the plane wave $\mathbf{E} = E_0 \mathbf{e}_x \sin(kz - \omega t)$ satisfies the wave equation in vacuum.
   b) Find a potential representation for the wave in the Lorenz gauge.

2. The half-space $z > 0$ is the air and $z < 0$ is the earth, whose permeability is μ_0 and whose conductivity σ is constant. Assume that at the earth's surface the field is harmonic in time: $\mathbf{B}(z = 0, t) = B_0 e^{-i\omega t}\, \mathbf{e}_x$ (B_0 constant, $\omega > 0$, $\omega\epsilon/\sigma \ll 1$). Calculate the magnetic and electric fields in the earth.

3. The electric field $\mathbf{E}(\mathbf{r}, t)$ and the electric displacement can be given as Fourier integrals:
$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega\ \mathbf{E}(\mathbf{r}, \omega) e^{-i\omega t}$$
   and $\mathbf{D}(\mathbf{r}, t)$ in the same manner.
   a) If the permittivity only depends on frequency then $\mathbf{D}(\mathbf{r}, \omega) = \epsilon(\omega)\mathbf{E}(\mathbf{r}, \omega)$. What is then the relationship between $\mathbf{D}(\mathbf{r}, t)$ and $\mathbf{E}(\mathbf{r}, t)$?
   b) In a simple version of the model by Drude and Lorentz
$$\epsilon(\omega) = \epsilon_0 (1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - i\omega\gamma})$$
   where $\gamma > 0$ and ω_0 and ω_p are constants. Prove that the relationship derived in a) is causal, i.e. that $\mathbf{D}$ at time t only depends on previous (and simultaneous) values of $\mathbf{E}$. You may need calculus of residues to solve this problem.

4. The maximum power of the Earth's auroral kilometric radiation (AKR) is about 1 GW. The power of the infrared radiation emitted by the Earth is about 10^8 GW. The wavelengths of these radiations are about 3 km and 10 μm, respectively.
   a) Calculate the number of photons per unit time for these radiation types.
   b) Assume that an exoplanet at a distance of 100 light years emits AKR as described above. How many photons would a detector of an area 100 m^2 observe in one second close to the Earth?
   c) AKR radiated by an exoplanet may be screened due to radiation by its mother star. However, the planetary AKR can then be observed using interferometry. One way to perform this is to place two detectors in an orbit circulating our Sun. Determine the required distance of these instruments if the aim is to observe exoplanet AKR. Assume that the exoplanet orbit has a radius of 1 AU. (Credit: Pekka Janhunen, FMI.)

5. Baywatch man Kari Kukko observes a swimmer to get a dangerous cramp. Their distances from the straight shoreline are 10 m and 30 m, respectively. Their distance along the shoreline is 50 m. The baywatcher runs at speed 10 m/s and swims at speed 2 m/s.
   a) What is the optimal point for Kari Kukko to jump into the water?
   b) What has this problem to do with electrodynamics?

Return the answers until Tuesday 8.4. 12 o'clock.