Electrodynamics, spring 2008
Exercise 10 (17.4., 18.4.; Friday group in English)

1. Consider a magnetic storm in a region whose area is $L_x L_y$. Assume the earth to be a uniform half-space ($\omega\epsilon/\sigma \ll 1$). Assume that the electromagnetic field in the earth ($z < 0$) is a downwards propagating time-harmonic plane wave.
   a) Show that the time-averaged power production per unit volume in the earth due to induced currents is
   $$\langle\frac{dw}{dt}\rangle = \frac{\omega|B_0|^2}{2\mu_0} \cdot e^{2z/\delta}$$
   where δ is the skin depth and B_0 is the amplitude of the magnetic field at the earth's surface.
   b) Calculate the total power in the earth using the following values which are possible in the auroral region: L_x = 500 km, L_y = 1000 km, $\sigma = 10^{-2}\ \Omega^{-1}m^{-1}$, T = 60 s, B_0 = 100 nT.
   c) Would it be a good business idea to sell such "ground heat"?

2. A linearly polarised electromagnetic wave propagates in vacuum into the z-direction and arrives perpendicularly at a perfectly conducting wall (plane $z = 0$).
   a) Determine the average energy flux in vacuum. The amplitude of the electric field of the incident wave is assumed to be known.
   b) Calculate the charge and current densities at the surface of the conductor.

3. A waveguide has a rectangular cross-section, with side lengths a and b into the x- and y-directions ($a > b$), and whose walls are perfectly conducting. Calculate the cut-off frequency of the lowest TM mode. Start from the Maxwell equations.

4. A microwave oven is basically a box ($0 \le x \le a, 0 \le y \le b, 0 \le z \le c$), whose walls are perfect conductors. Determine the eigenfrequencies of the system using the wave equation of the electric field.

5. Show that the electric field of a moving charge given in lectures is the same as
   $$\mathbf{E}(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0}\,(\frac{\mathbf{R}}{R^3} + \frac{R}{c}\frac{d}{dt}(\frac{\mathbf{R}}{R^3}) + \frac{1}{c^2}\frac{d^2}{dt^2}(\frac{\mathbf{R}}{R}))$$
   where $\mathbf{R} = \mathbf{R}(t')$ is the radius vector from the location of the particle at $t' = t - R(t')/c$ to the observation point $\mathbf{r}$.

6. Three extra points are available for solving the following problems:
   a) Why a wet sand is darker than a dry sand?
   b) Why Sun and Moon seem to flatten close to the horizon?
   c) Why does the blue sky get lighter close to the horizon?

Return the answers until Tuesday 15.4. 12 o'clock.
Note: the morning exercise group (8-10) on Thursday 17.4. is cancelled.