

Electrodynamics, spring 2008

Exercise 12 (2.5., only one group)

1. Define a 4-component fully antisymmetric permutation symbol ϵ so that $\epsilon^{0123} = 1$ and $\epsilon^{\alpha\beta\gamma\mu}$ is antisymmetric with respect to a change of any pair of indices. Using this, define the components of the dual field tensor $G^{\alpha\beta}$ by

$$G^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\mu\nu} F_{\mu\nu}$$

- a) Calculate these components and compare them to the components of $F^{\alpha\beta}$.
- b) Show that the homogenous Maxwell equations are

$$\partial_\beta G^{\alpha\beta} = 0$$

- c) Show that $\mathbf{E} \cdot \mathbf{B} = -\frac{c}{4} F_{\alpha\beta} G^{\alpha\beta}$.
2. Show that the quantities $\mathbf{E} \cdot \mathbf{B}$ and $\mathbf{E}^2 - c^2 \mathbf{B}^2$ are Lorentz-invariant.
 3. Observer K thinks that in an infinitely long straight wire there is a line charge density λ and a constant current I . Observer K' moves with respect to K at a constant velocity v parallel to the wire at a distance d from it. Determine the charge and current densities and the electromagnetic field as observed by K' .
 4. Solve the relativistic equation of motion of a charged particle (mass m , charge q) initially at rest in a uniform electric field \mathbf{E} (ignore radiation losses).
 5. A charged particle enters the half-space $x > 0$ at $t = 0$ with a velocity $v_0 \mathbf{e}_x$. There is uniform magnetic field $B_0 \mathbf{e}_z$ (transverse to the initial velocity vector) in the region $x > 0$. Solve the relativistic equation of motion (ignore radiation losses).
 6. Go through all exercises of the course and select the three most educational and/or meaningful problems as well as the two really unnecessary and/or unpleasant problems. Justify your choice. With a good luck, some of these problems will be in the second exam. Remember also to answer to the course survey:
<http://www.physics.helsinki.fi/suomi/opetus/kurssipalaute/>
(The survey for this course seems to be available only in Finnish. A fellow student may help you in translation.)

Return the answers until Tuesday 29.4. 12 o'clock.

You may return the answer to problem 6 until 2.5. (during the exercise group or directly to Elina).

Due to the First of May, the only exercise group is on Friday 2.5.

The last lecture is on Monday 5.5. at 10-12 (we will revise the topics of the second exam (in Finnish)).

The second exam is on Monday 12.5. at 9-13 in D101. Content: chapters 8-15 and exercises 7-12.