1. An infinitely long time-harmonic \((e^{-i\omega t})\) sheet current flows to the \(y\) direction on the \(xy\)-plane. The current density transverse to the flow direction is \(f(x)\). The whole space has constant electromagnetic material parameters.

a) Starting from the wave equation of the vector potential, show that the fields are

\[
E_y = \frac{\omega \mu_0 I(\omega)}{4} \int_{-\infty}^{\infty} dx' f(x') H_0^{(2)}(-k\sqrt{(x' - x)^2 + z^2})
\]

\[
B_x = \frac{i \mu_0 k I(\omega)}{4} \int_{-\infty}^{\infty} dx' f(x') \frac{zH_1^{(2)}(-k\sqrt{(x' - x)^2 + z^2})}{\sqrt{(x' - x)^2 + z^2}}
\]

\[
B_z = \frac{i \mu_0 k I(\omega)}{4} \int_{-\infty}^{\infty} dx' f(x') \frac{(x' - x)H_1^{(2)}(-k\sqrt{(x' - x)^2 + z^2})}{\sqrt{(x' - x)^2 + z^2}}
\]

b) Show by a direct calculation that the Maxwell equations are fulfilled.

c) Show that at the static limit the expressions of the magnetic field reduce to the Biot and Savart law.

2. A time-harmonic line current flows above a uniform earth at the height \(h\). Assuming low frequencies, use the complex image method to study the behaviour of the surface fields and the ratio \(E_{\text{hor}}/B_{\text{hor}}\) at large distances from the current.
3. A line current $I$ flows above a layered earth at height $z = -h$. The earth’s surface is the $xy$-plane and $z$ is positive downwards.

a) Using the integrals 5.38-5.40 and omitting the displacement current, show that the surface fields can be written as

$$B_x(x, \omega) = \frac{\mu_0 I}{2\pi} \int_0^\infty \left( e^{-bh} + Re^{-bh} \right) \cos bx \, db$$

$$B_z(x, \omega) = -\frac{\mu_0 I}{2\pi} \int_0^\infty \left( e^{-bh} - Re^{-bh} \right) \sin bx \, db$$

$$E_y(x, \omega) = \frac{i\omega \mu_0 I}{2\pi} \int_0^\infty \left( 1 - R \right) \frac{e^{-bh}}{b} \cos bx \, db$$

where the reflection coefficient is

$$R = \frac{i\omega \mu_0 / Z + b}{i\omega \mu_0 / Z - b}$$

b) Show that the complex image depth is

$$p = -\frac{Z}{i\omega \mu_0}$$

assuming that $|pb|^3 \ll 1$. Tip: apply the binomial expansion to

$$R = 1 - \frac{2pb}{1 + pb}$$

and compare to the power series of $e^{-2pb}$. 